Control of Crossed Genes Ratio for Directed Mating in Evolutionary Constrained Multi-Objective Optimization

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ABSTRACT

As an evolutionary approach to solve constrained multi-objective optimization problems (CMOPs), an algorithm using the two-stage non-dominated sorting and the directed mating (TNSDM) has been proposed. To generate offspring, the directed mating utilizes useful infeasible solutions having better objective values than feasible solutions in the population. The directed mating achieves higher search performance than the conventional mating which avoids using infeasible solutions in several CMOPs. However, since the directed mating uses infeasible solutions, generated offspring tend to be infeasible compared with the conventional mating. To further improve the effectiveness of the directed mating by improving the feasibility of generated offspring, in this work we propose a method to control the crossed genes ratio in the directed mating. In this method, we control the amount of genes copied from infeasible parents to offspring in the directed mating. Experimental results using m-objective k-knapsack problem with 2-4 objectives show the contribution of the directed mating for the search performance is further improved by controlling crossed genes ratio.

Categories and Subject Descriptors

I.2.8 [**Artificia Intelligence**]: Problem Solving, Control Methods, and Search—*Heuristic methods*; G.1.6 [**Numerical Analysis**]: Optimization

Keywords

multi-objective optimization, constraint-handling, directed mating

1. INTRODUCTION

Evolutionary algorithms are suited to solve multi-objective optimization problems (MOPs) since a set of solutions to approximate Pareto front can be simultaneously obtained from the population in a single run [1]. To solve constrained MOPs (CMOPs) involving constraints, several constraint-handlings including the death penalty approach, repair approaches of infeasible solutions, and approaches to evolve infeasible solutions into feasible ones have been proposed [2]. We have focused on the last approach to evolve infeasible solutions into feasible ones and proposed an algorithm based on the two-stage non-dominated sorting and the directed mating (TNSDM) [3].

TNSDM classifie the entire population into several fronts by the two-stage non-dominated sorting based on constraint violation values and objective values, and the parent population is selected from upper fronts. In the reproduction, TNSDM uses the directed mating utilizing useful infeasible solutions having better objective function values. To generate one offspring, a primary parent is selected from the parent population, and solutions dominating the primary parent are picked as candidate solutions for its secondary parent from the entire population including infeasible solutions. Then, a secondary parent is selected from the picked candidates by using a binary tournament selection, and genetic operators are applied to the primary and secondary parents. In this way, the directed mating utilizes valuable genetic information of infeasible solutions to enhance the convergence of each primary parent toward Pareto front.

Our previous study [3] showed that TNSDM using the directed mating achieved higher search performance than CNSGA-II [4] using the conventional mating which avoids using infeasible parents. So far, we have focused on ways to select parents on the concept of the directed mating and proposed some variants of the directed mating [5, 6]. However, there is a problem in the crossover operator combined with the directed mating, and this work focuses on it. Since the directed mating utilizes infeasible solutions as parents, generated offspring tend to become infeasible solutions compared with the conventional mating which avoids selecting infeasible solutions as parents. If the feasibility of offspring generated by the directed mating can be improved, we can expect to further improve the contribution of the directed mating for the search performance in evolutionary constrained multi-objective optimization.

To further improve the effectiveness of the directed mating by improving the feasibility of generated offspring, in this work we propose a method to control the crossed ratio of genes in the directed mating. In our previous studies [3, 5, 6], we have used the conventional uniform crossover [7] with the directed mating. In this crossover, almost half genes of an offspring are copied from an infeasible secondary parent, and other genes are copied from a feasible primary parent. If many genes of the infeasible secondary parent are copied to the offspring, it would be infeasible. Therefore, to increase the feasibility of offspring generated by the directed mating, in this work we introduce a concept of the control of the number of crossed genes (CCG) [8] and control the amount of genes copied from infeasible secondary parents to offspring in the directed mating. We use m-objective k-knapsack problems with 2-4 objectives and verify effects of the controlling crossed ratio of genes in the directed mating.

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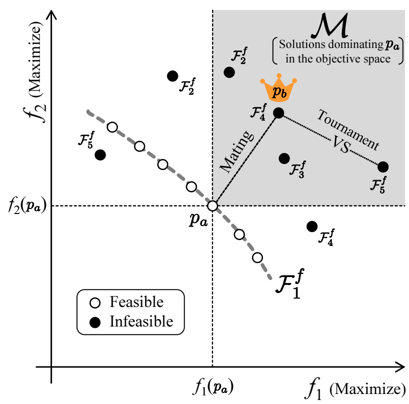


Figure 1: Directed Mating [3]

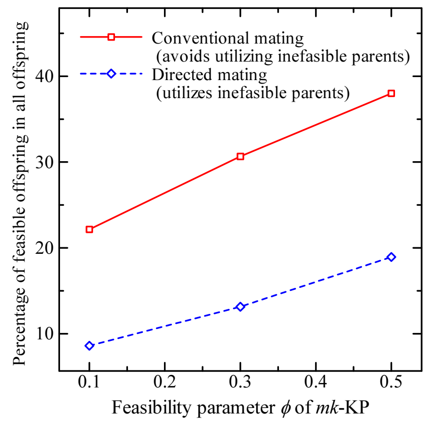


Figure 2: Percentage of feasible offspring in all generated offspring (mk-KPs with $m = 2$ objectives and $k = 6$ knapsacks)

2. EVOLUTIONARY CONSTRAINED MULTI-OBJECTIVE OPTIMIZATION

Constrained MOPs (CMOPs) are concerned with findin solutions $\boldsymbol{x}$ maximizing m kinds of objective functions f_i $(i = 1, 2, \ldots, m)$ subject to satisfy k kinds of constraints g_j $(j = 1, 2, \ldots, k)$. CMOP is define as

$$\left\{ \begin{array}{lll} \text{Maximize} & f_i(\boldsymbol{x}) & (i = 1, 2, \ldots, m) \\ \text{Subject to} & g_j(\boldsymbol{x}) \geq 0 & (j = 1, 2, \ldots, k). \end{array} \right. \quad (1)$$

Solutions satisfying all k constraints are said to be *feasible*, and solutions not satisfying all k constraints are said to be *infeasible*. The constraint violation vector $\boldsymbol{v}(\boldsymbol{x})$ is define as

$$v_j(\boldsymbol{x}) = \left\{ \begin{array}{ll} |g_j(\boldsymbol{x})|, & \text{if } g_j(\boldsymbol{x}) < 0 \\ 0, & \text{otherwise} \end{array} \right. (j = 1, 2, \ldots, k). \quad (2)$$

Next, *Pareto dominance* between $\boldsymbol{x}$ and $\boldsymbol{y}$ is define as follows: If

$$\forall i : f_i(\boldsymbol{x}) \geq f_i(\boldsymbol{y}) \ \wedge \ \exists i : f_i(\boldsymbol{x}) > f_i(\boldsymbol{y}) \ \ (i = 1, 2, \ldots, m) \quad (3)$$

is satisfied $\boldsymbol{x}$ dominates $\boldsymbol{y}$ on objective function values, which is denoted by $\boldsymbol{x} \succ_f \boldsymbol{y}$ in the following. Also, a feasible solution $\boldsymbol{x}$ not dominated by any other feasible solution is said to be a non-dominated solution. The set of non-dominated solutions in the solution space is called Pareto optimal solutions (POS), and the trade-off among objective functions represented by POS in the objective space is called Pareto front.

To solve CMOPs by using evolutionary algorithms, we need to employ a constraint-handling mechanism which determines a way to treat infeasible solutions in the optimization process. In this work, we focus on a MOEA using the two-stage non-dominated sorting and the directed mating (TNSDM) [3]. TNSDM employs an approach to evolve infeasible solutions into feasible ones during the solution search such as Constrained NSGA-II (CNSGA-II) [4]. The previous work [3] showed that TNSDM achieved higher search performance than CNSGA-II in several benchmark problems.

3. TNSDM

TNSDM is designed based on the framework of NSGA-II [4]. The entire population $\mathcal{R}$ consists of the parent population $\mathcal{P}$ and the offspring population $\mathcal{Q}$, i.e. $\mathcal{R} = \mathcal{P} \cup \mathcal{Q}$.

3.1 Two-Stage Non-Dominated Sorting [3]

To select the parent population $\mathcal{P}$ from the entire population $\mathcal{R}$, TNSDM classifie $\mathcal{R}$ into several fronts $\mathcal{F}_1^f, \mathcal{F}_2^f, \ldots$ by using the two-stage non-dominated sorting based on constraint violation values v_j $(j = 1, 2, \ldots, k)$ and objective function values f_i $(i = 1, 2, \ldots, m)$. As the result, upper front $\mathcal{F}_i^f$ with small index i includes solutions having lower constraint violation values and higher objective function values. The half of $\mathcal{R}$ is selected as the parent population $\mathcal{P}$ from upper fronts while simultaneously considering the crowding distance (CD) [4].

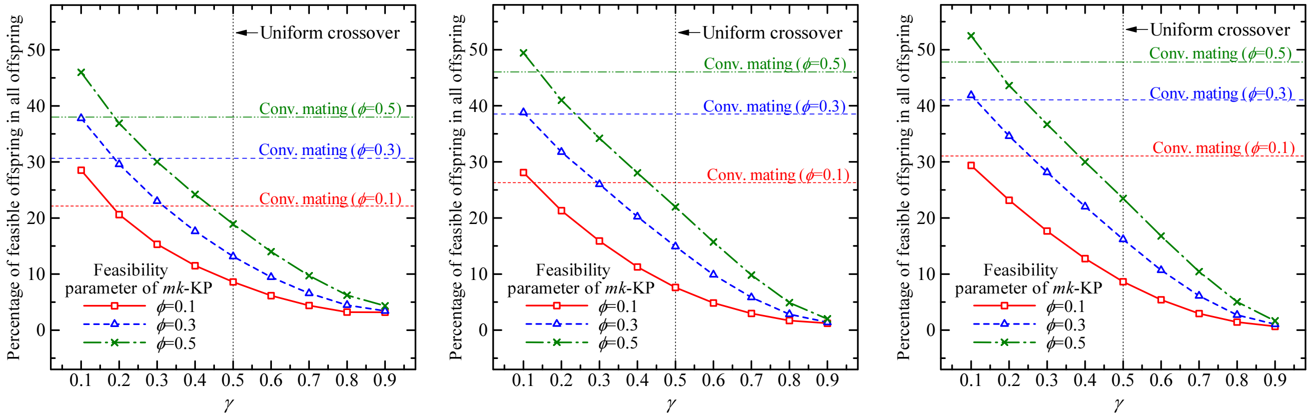
3.2 Directed Mating [3]

To improve the convergence of solutions toward Pareto front, TNSDM introduces the directed mating which utilizes useful infeasible solutions. **Fig. 1** shows a conceptual figur of the directed mating. In this figure all solutions in the entire population $\mathcal{R}$ are distributed in the objective space, and feasible solutions belonging to $\mathcal{F}_1^f$ are the parent population $\mathcal{P}$. First, a primary parent $\boldsymbol{p_a}$ is selected from the parent population $\mathcal{P}$ by using the crowded tournament selection introduced in [4]. In the tournament, two solutions are randomly chosen from $\mathcal{P}$, and the solution belonging to the upper front (with a lower front index number) becomes parent $\boldsymbol{p_a}$. If both of them belong to the same front, the solution having a larger CD becomes parent $\boldsymbol{p_a}$. Next, we pick a set of candidate solutions $\mathcal{M}$ $(= \{\boldsymbol{x} \in \mathcal{R} \mid \boldsymbol{x} \succ_f \boldsymbol{p_a}\})$ dominating $\boldsymbol{p_a}$ in the objective space from the entire population $\mathcal{R}$ including infeasible solutions. If the primary parent $\boldsymbol{p_a}$ is feasible and the number of solutions in $\mathcal{M}$ is more than or equal to two ($|\mathcal{M}| \geq 2$), the directed mating is performed. Otherwise, the conventional mating is performed.

In the conventional mating, a secondary parent $\boldsymbol{p_b}$ is selected also from $\mathcal{P}$ by using the crowded tournament. That is, feasible solutions are preferred rather than infeasible ones for both parents.

In the directed mating, a secondary parent $\boldsymbol{p_b}$ is selected from $\mathcal{M}$ dominating the primary parent $\boldsymbol{p_a}$. To select $\boldsymbol{p_b}$ from $\mathcal{M}$, first two solutions are randomly chosen from $\mathcal{M}$, and the solution belonging to the upper front becomes $\boldsymbol{p_b}$. If the two solutions belong to the same front, the solution with the larger CD [4] becomes $\boldsymbol{p_b}$. In the example of **Fig. 1**, two solutions belonging to $\mathcal{F}_4^f$ and $\mathcal{F}_5^f$ are randomly chosen from $\mathcal{M}$, and the solution belonging to $\mathcal{F}_4^f$ becomes $\boldsymbol{p_b}$ to mate with $\boldsymbol{p_a}$.

In the most of MOEAs for CMOPs such as CNSGA-II [4], feasible solutions have a high priority to become parents after feasible solutions are found in the population. On the other hand, in the directed mating, all primary parents are selected from $\mathcal{P}$ but secondary parents are selected even from infeasible solutions discarded in the selection of $\mathcal{P}$ if they dominate their primary parents in the objective space. As shown in **Fig. 1**, although secondary parent $\boldsymbol{p_b}$ is infeasible, there is a possibility that $\boldsymbol{p_b}$ has valuable genetic information to enhance the convergence of primary $\boldsymbol{p_a}$ toward Pareto front since $\boldsymbol{p_b}$ dominates $\boldsymbol{p_a}$ in the objective space.



(a) $m = 2$ objectives (b) $m = 3$ objectives (c) $m = 4$ objectives

Figure 3: The percentage of feasible offspring in all generated offspring by varying γ

3.3 A Problem in Directed Mating: Low Feasibility of Generated Offspring

In our previous study [3], we showed that TNSDM using the directed mating achieved higher search performance than CNSGA-II [4] using the conventional mating in several benchmark problems. However, since the directed mating utilizes infeasible solutions, generated offspring tend to become infeasible solutions compared with the conventional mating which avoids using infeasible solutions. **Fig. 2** shows the percentage of feasible offspring in all offspring generated by the conventional mating and the directed mating in m-objective k-knapsack problems (mk-KPs) [9] with $m = 2$ objectives, and feasibility parameters $\phi = \{0.1, 0.3, 0.5\}$. In this analysis, we employ the same parameters used in [3]. As mentioned before, the conventional mating uses feasible solutions if they exist in the population and avoids using infeasible solutions, and the directed mating utilizes useful infeasible solutions having better objective values than feasible solutions.

From the results in **Fig. 2**, we can see that the percentage of feasible solutions generated by the directed mating is about half of the one of feasible solutions generated by the conventional mating. Although TNDSM using the directed mating achieves better search performance than one using the conventional mating [3], the feasibility of offspring generated by the directed mating is lower than the conventional mating because the directed mating actively utilizes infeasible solutions. If the feasibility of offspring generated by the directed mating can be improved, we can expect to further improve the effectiveness of the directed mating in evolutionary constrained multi-objective optimization.

4. PROPOSAL: CONTROL OF CROSSED GENES RATIO IN DIRECTED MATING

To further improve the effectiveness of the directed mating by improving the feasibility of generated offspring, in this work we focus on a constrained combinatorial optimization problem and a crossover operator used with the directed mating.

For solving mk-KP, we have used the uniform crossover [7] to generate offspring after two parents are selected by the directed mating [3]. Generally, the uniform crossover swaps each gene of two parents with the probability $\gamma = 0.5$ [7]. That is, almost half genes of an offspring are copied from a primary parent $\boldsymbol{p_a}$, and other genes are copied from a secondary parent $\boldsymbol{p_b}$. In the case of the directed mating, secondary parents $\boldsymbol{p_b}$ are infeasible. If many genes of a secondary parent $\boldsymbol{p_b}$ are copied to an offspring, it would become an infeasible solution. Therefore, to increase the feasibility of offspring generated by the directed mating, in this work we introduce a concept of the controlling the number of crossed genes (CCG) [8] and control the amount of genes copied from infeasible secondary parents to offspring in the directed mating.

For a primary parent $\boldsymbol{p_a}$ and a secondary $\boldsymbol{p_b}$, their genes (variables) are presented by $\boldsymbol{x}^{p_a} = (x_1^{p_a}, x_2^{p_a}, \ldots, x_n^{p_a})$ and $\boldsymbol{x}^{p_b} = (x_1^{p_b}, x_2^{p_b}, \ldots, x_n^{p_b})$, respectively. The genes of their offspring $\boldsymbol{x}^o = (x_1^o, x_2^o, \ldots, x_n^o)$ is obtained by the following equation.

$$x_i^o = \begin{cases} x_i^{pb}, & \text{if } rand\,[0,1] < \gamma \\ x_i^{pa}, & \text{otherwise} \end{cases} \quad (i = 1, 2, \ldots, n), \qquad (4)$$

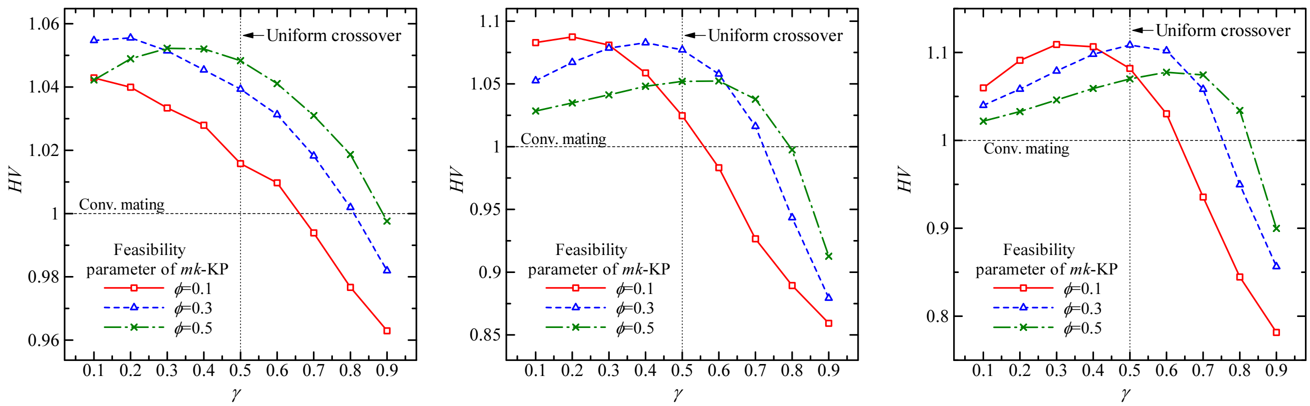
where, $rand$ generates a random real number in the range $[0, 1]$, γ is the user-define parameter to control the crossed ratio of genes in the range $[0, 1]$. The CCG crossover with $\gamma = 0.5$ is equivalent to the conventional uniform crossover. The amount of crossed genes copied from infeasible secondary parents is decreased by decreasing γ, then we can expect to increase the feasibility of generated offspring because the copy of genes from infeasible secondary parents is restricted. In contrast, the amount of crossed genes copied from infeasible secondary parents is increased by increasing γ.

The above method is similar to the CCG crossover [8]. Actually, the CCG crossover was originally used to avoid too destructive variation in unconstrained many-objective optimization problems with more than three objectives because the genetic (variable) diversity in the population is significantl increased in many-objective problems. Therefore, both parents are feasible, and each of them exchange a small number of genes with a small γ. On the other hand, in this work, to restrict crossed genes from infeasible secondary parents and increase the feasibility of generated offspring in the directed mating, we utilize the concept of the CCG in evolutionary constrained multi-objective optimization.

5. EXPERIMENTAL SETUP

In this work, we use mk-KP [9] with $n = 500$ items (bits), $m = \{2, 3, 4\}$ objectives, $k = 6$ knapsacks (constraints) and feasibility parameters $\phi = \{0.1, 0.3, 0.5\}$. As genetic parameters, we use the CCG crossover with crossover ratio $P_c = 1.0$, bit-fli mutation with mutation ratio $P_m = 1/n$, and the population size is set to $|\mathcal{R}| = 200$ ($|\mathcal{P}| = |\mathcal{Q}| = 100$). The total number of generations is set to $T = 10^4$ for each run. In the following experiments, we show average (mean) results of 50 runs.

To evaluate the obtained non-dominated set of solutions, we use Hypervolume (HV). HV measures m-dimensional volume covered by obtained non-dominated set and a reference point $\boldsymbol{r} = \{0, 0, \ldots, 0\}$ in the objective space. Higher HV values denote



(a) $m = 2$ objectives (b) $m = 3$ objectives (c) $m = 4$ objectives

Figure 4: Results of HV by varying γ

better search performance in term of both the convergence and the diversity of obtained solutions toward Pareto front.

6. RESULTS AND DISCUSSION

6.1 Feasibility of Generated Offspring

First, we verify effects of the CCG crossover on the feasibility of generated offspring in the directed mating. **Fig. 3** shows the percentage of feasible offspring in all generated offspring as the parameter γ is varied. For each mk-KP, the result obtained by the conventional mating with the conventional uniform crossover ($\gamma = 0.5$) is also plotted as the horizontal line. From the results, in the case of the conventional uniform crossover with $\gamma = 0.5$, we can see that the feasibilities of offspring generated by the directed mating are lower than the one generated by the conventional mating in all mk-KPs. However, we can see that the feasibility of offspring generated by the directed mating is increased by decreasing γ.

These results reveal that feasibility of offspring generated by the directed mating can be improved by restricting genes crossed from infeasible secondary parents with small γ.

6.2 Results of HV

Fig. 4 shows the results of HV achieved by TNSDM using the directed mating as the parameter γ is varied. For each mk-KP, all the results are normalized by the result obtained by the conventional mating with the conventional uniform crossover ($\gamma = 0.5$), and its value is shown as the horizontal line.

From the results, firs we can see that there is the optimal γ^* to maximize HV in each mk-KP. When γ is decreased from γ^*, although the feasibility of generated offspring is improved as shown in **Fig. 3**, HV is deteriorated. This is because useful genetic (variable) information of infeasible secondary parents to enhance the solution search is decreased by decreasing γ even generated offspring become feasible solutions. Also when γ is increased from γ^*, HV is deteriorated. This is because crossed genes from infeasible secondary parents are increased by increasing γ, and generated offspring tend to be infeasible as shown in **Fig. 3**.

Furthermore, we can see that γ^* to maximize HV depends on each mk-KP. In each of problems with a high feasibility parameter $\phi = 0.5$ and $m = \{3, 4\}$ objectives, the highest HV is surprisingly achieved by $\gamma^* > 0.5$ which copies more than half genes of infeasible secondary parents. In each of these problem, since the feasibility of offspring generated by the directed mating is relatively high as shown in **Fig. 3**, a high HV is achieved by crossing many genes from infeasible secondary parents which have useful genetic information to enhance the solution search.

7. CONCLUSIONS

To further improve the effectiveness of the directed mating in evolutionary constrained multi-objective optimization, in this work we proposed the control of crossed genes ratio in the directed mating. The experimental results using mk-KPs showed that the feasibility of offspring generated by the directed mating is improved by decreasing the amount of genes crossed from infeasible secondary parents to offspring. Also, we showed that the search performance of the directed mating is further improved by controlling crossed ratio of genes.

Since the concept of the CCG crossover used in this work can be extend to some crossovers for continuous optimization problems, as a future work we will verify the proposed approach on constrained continuous multi-objective optimization problems.

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