Selection Hyper-Heuristic Using a Portfolio of Derivative Heuristics

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ABSTRACT

Generally, we distinguish two classes of hyper-heuristic approaches, heuristic selection and heuristic generation. The former one works with existing heuristics and tries to find their optimal order for solving the instance. The later approach automatically generates new heuristic. Here, these two approaches are combined so that, first, a number of various heuristics are derived from a limited set of pre-existing heuristics for the selected optimization problem with regard to the diversity among the heuristics. Then, the heuristic selection approach is used to find the optimal sequence of heuristics leading to the best solution. Proof-of-concept experiments on the Capacitated Vehicle Routing Problem were carried out with the well-known Clarke-Wright, Mole-Jameson and Kilby constructive heuristics. Results show that the derived heuristics produce consistently better results than the original ones.

Categories and Subject Descriptors

G.1.1 [**Mathematics of computing**]: Discrete mathematics Combinatorics[Combinatorial optimization]
; K.3.4 [**Computing methodologies**]: Artificial intelligence Search methodologies[Heuristic function construction]

Keywords

Hyper-heuristics, Capacitated Vehicle Routing Problem, Genetic Programming, Genetic Algorithms

1. INTRODUCTION

This work presents an approach that combines two main hyper-heuristics (HH) [2] classes – the selection and the generative HH. First, a number of various low level heuristics (LLHs) are derived from the set of existing LLHs using a genetic programming focusing on their diversity, i.e. each individual heuristic dominates on a different type of problem instances. These derivatives are used in the heuristic selection approach based on a genetic algorithm (GA). GA tries to to find the optimal sequence of heuristics leading to the best solution. As a case study for the proof-of-concept experiments the Capacitated Vehicle Routing Problem (CVRP) [4] was chosen.

2. PROPOSED APPROACH

There are three phases of the proposed approach – a GP-based derivation of new heuristics based on the existing ones, a selection of the most valuable heuristics to the portfolio of heuristics and the selection HH realized by a genetic algorithm.

2.1 Derivation of New Heuristics

Firstly, we derived new low-level heuristics from existing constructive LLHs, namely Clarke Wright [3], Mole-Jameson [6] and Kilby [5]. In particular, a tree-based GP is used to evolve new mathematical functions that are used in the LLHs to order the nodes to be added to the constructed solution. A standard GP with generational strategy, elitism, *homologous one-point crossover* and *subtree mutation* was implemented.

For learning of the new low-level heursitic functions, data from Augerat [1], freely downloadable at[1], were used. There are seven benchmark sets A, B, E, F, G, M and P with instances ranging in size from 22 to 262 cities. The learning algorithm was run multiple times, each time using a different pair of benchmark sets in order to support a diversity of the evolved LLHs.

2.2 Portfolio Composition

Out of the set of LLHs derived in the previous step from the basis constructive LLHs (Clarke-Wright, Mole-Jameson and Kilby), the most valuable LLHs were chosen to the final portfolio of LLHs. The LLHs were tested on the set of test instances and the top three ones from each group were chosen. As a measure of the candidate LLH's quality the number of test instances on which the particular LLH is superior to the other candidates was used. Thus, the final extended portfolio consists of these nine modified LLHs plus the three basis LLHs.

[1]http://www.branchandcut.org/VRP/data/

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2.3 Selective Hyper-Heuristic

A GA-based selective HH is used to find the best parameterized sequence of LLHs' to construct the solution.

Individuals in the population, i.e. sequences of LLHs, are represented as dynamic array of genes with a variable length between 2 and 20 genes. Every gene is represented by a tuple (h, n), where h stands for an identifier of the LLH and n stands for the number of cities i.e. steps, which will be processed by the heuristics h in order to extend the constructed solution.

One-point crossover which switches parts of the parental chromosomes of the same size is implemented together with 5 mutation operators: *add LLH*, *remove LLH*, *replace LLH with another LLH*, *redistribute steps*, and *move LLH*. All these operators hold the total sum of the steps equal to the instance size. *Fitness value* of each individual is defined as a sum of the routes length of the created solution in actual instance. This sum is minimized.

The GA uses generational replacement strategy together with elitism and the tournament selection with 3 contestants.

3. EXPERIMENTS AND RESULTS

This work deals with the basic variant of the symmetric and metric Vehicle Routing Problem (VRP) with one depot, one type of goods and all vehicles of the same capacity. Two experiments were carried out: one with portfolio composed of the original basis LLHs only (i.e Original portfolio), the second one with portfolio extended by derived LLHs (Extended portfolio). Augerat's benchmarks and Tailard's instances were used as test set. Together it contains 106 instances with the sizes from 22 to 385 cities.

Result of the best single LLH, results obtained with the Original Portfolio and results obtained with the Extended portfolio were compared for each instance. Each experiment was replicated 30 times. Statistical Wilcoxon's rank-sum test with $\alpha = 1$ % was carried out on the obtained results to confirm the observed differences.

First observation is that the selection HH using the original portfolio consistently outperforms the single heuristics. This is in agreement with the intuition that the selection HH should be more robust than any of the original heuristics alone.

Second observation is that the Extended portfolio clearly outperforms the Original one. The Extended portfolio produced significantly better (as confirmed by the statistical test) results in 91 cases out of the 106 instances. On 14 instances the distributions of results obtained with Original and Extended portfolio were equal. And only on a single instance the Original portfolio produced better set of results than the Extended portfolio.

Table 1: Results obtained with single heuristics and selection hyper-heuristic using Original and Extended portfolio of heuristics. Median values shown in bold indicate that the difference between the distribution of 30 results achieved with the Original portfolio and the distribution obtained with the Extended portfolio was confirmed by the Wilcoxon rank-sum test at the $\alpha = 1$ % level.

		Original portfolio		Extended portfolio	
Inst.	Single heuristic	median	best	median	best
t75a	1645.5	1631.9	1627.3	**1630.3**	1626.1
t75b	1356.6	1356.6	1350.4	1350.4	1350.0
t75c	1334.8	1324.1	1322.7	1324.1	1294.6
t75d	1428.5	1411.3	1399.4	**1406.5**	1396.0
t100a	2166.0	2121.9	2098.0	**2098.0**	2090.6
t100b	2034.3	1979.0	1969.6	1984.5	1951.1
t100c	1434.1	1427.5	1417.7	**1422.9**	1415.9
t100d	1678.0	1635.4	1620.8	**1606.9**	1598.6
t150a	3388.6	3244.9	3183.7	**3201.2**	3143.0
t150b	2890.4	2799.8	2786.1	**2776.6**	2754.9
t150c	2457.2	2430.1	2405.4	**2399.5**	2377.9
t150d	2788.2	2722.6	2701.6	**2696.1**	2683.0
t385	25343.0	25200.8	25079.2	**25157.0**	24976.0

4. CONCLUSIONS

This work presents a new HH framework that combines generative and selection HH. The main contribution of this work is the combination of selective and constructive HHs resulting in a robust HH approach.

The future work will focus on automating the process of generation of a diverse portfolio of well-performing heuristics for the given problem derived from a few existing heuristics. Besides the discrete combinatorial optimization problems, the real-valued optimizations might be a possible application domain for this approach as well. We believe there is a great potential in developing the generative HHs for generating new local search algorithms that would combine features of existing local search methods.

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