A Comparative Study Use of OTL for Many-objective Optimization

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ABSTRACT

This study exhaustively compares the abilities to solve many-objective problems of eight representative algorithms from four different classes (i.e., Pareto-, aggregation-, indicator-, and diversity-based EMO algorithms). The eight compared algorithms are tested on four types of well-defined continuous, discontinuous and combinatorial problems, through three performance metrics as well as a visual observation in the decision space. We can conclude from the experimental results that the performance of the eight algorithms differ not only on the dimensionality of the problems, but also on the shape and features of the Pareto front. From this it suggests an appropriate choice for researchers and practitioners when solving many-objective problems.

Categories and Subject Descriptors

I.2.8 [**Computing Methodologies**]: Problem Solving, Control Methods, and Search

1. TESTED ALGORITHMS AND PROBLEMS

Eight algorithms are selected from the four classes of many-objective EMO algorithms. CDAS [1] is from Pareto-based algorithms. MOEA/D+PBI (MOEA/D with PBI) [2], NSGA-III [3] and MSOPS [4] are from aggregation-based algorithms. IBEA [5], SMS-EMOA [6] and HypE [7] are from indicator-based algorithms. and SPEA2+SDE [8] is from diversity-based algorithms.

We consider four groups of test functions (DTLZ [9], WFG [10], TSP [11], and Rectangle problems [12]), and two performance metrics(IGD [13] and HV [14]).

All of the experiments are executed on the optimization template library (OTL).

2. RESULTS AND ANALYSIS

Specially here, we propose a new approach to generate any number of weight vectors. The main idea is using two EMO algorithms to generate well-spread weight vectors. Firstly, initialize many points. Secondly, we use NSGA-II to optimize DTLZ1 or DTLZ2 without distance function [10] and iterate it many times. Lastly, the truncation approach of SPEA2 is employed to truncate these points into any number of points required. Thus, we can obtain any number of uniformly distributed vectors for MOEA/D and NSGA-III.

Table 1 gives the IGD values of the eight EMO algorithms on DTLZ test suite, Table 2 and Table 3 represent the HV values on WFG and TSP problems, respectively. The values in each unit are the mean (above dividing line) and standard deviation (below dividing line). The dark and light gray represent the best and the second ranked EMO algorithms, respectively. Figure 1 and Figure 2 represent the final optimal solutions of the eight algorithms on 20-objective DTLZ3 (shown by parallel coordinates) and the four-objective rectangle test problem, respectively.

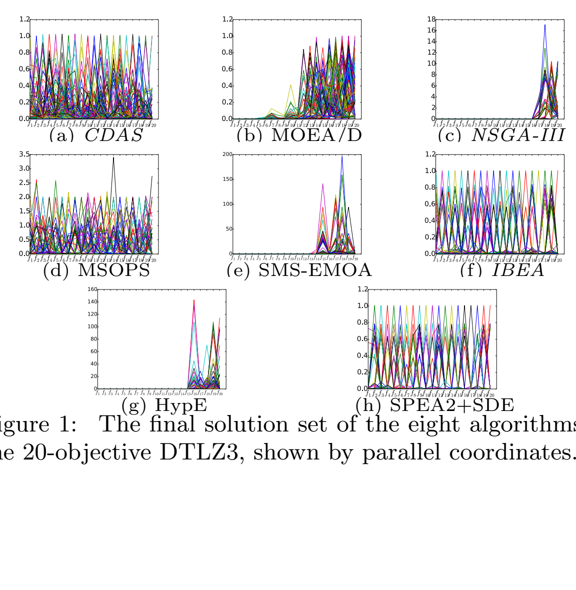


Figure 1: The final solution set of the eight algorithms on the 20-objective DTLZ3, shown by parallel coordinates.

GECCO '15 July 11-15, 2015, Madrid, Spain

ACM ISBN 978-1-4503-3488-4/15/07.
DOI: http://dx.doi.org/10.1145/2739482.2764707

Table 1: IGD comparison of the eight EMO algorithms on DTLZ problem suite

Problem	Obj.	CDAS	MOEA/D	NSGA-III	MSOPS	SMS-EMOA	IBEA	HypE	SPEA2+SDE
DTLZ1	10	1.22E-01 2.32E-03	1.66E+00 1.44E+00	1.17E-01 7.97E-04	2.06E-01 2.36E-02	2.37E-01 1.47E-02	1.46E-01 9.77E-04	8.08E-01 5.27E-01	1.15E-01 2.63E-03
	20	1.63E-01 5.34E-03	3.34E-01 1.29E-02	2.16E-01 3.38E-03	2.62E-01 5.63E-03	3.12E-01 1.25E-02	1.79E-01 3.49E-03	2.69E+00 1.95E+00	1.63E-01 4.46E-03
DTLZ2	10	4.63E-01 4.80E-03	7.83E-01 7.70E-03	5.38E-01 8.64E-04	7.93E-01 4.89E-02	6.24E-01 1.54E-02	4.97E-01 2.22E-03	9.64E-01 5.78E-02	5.58E-01 6.31E-03
	20	6.82E-01 7.01E-03	9.80E-01 2.49E-02	8.64E-01 1.08E-02	9.41E-01 3.96E-02	9.67E-01 3.19E-02	7.12E-01 8.34E-03	1.15E+00 2.71E-02	7.50E-01 1.69E-02
DTLZ3	10	4.60E-01 4.84E-03	4.50E+01 1.15E+01	8.29E-01 2.76E-01	8.12E-01 4.56E-02	4.32E+01 1.85E+01	1.12E+00 6.23E-01	5.72E+01 5.86E+01	5.86E-01 1.35E-02
	20	7.03E-01 9.80E-02	1.42E+01 5.83E+00	9.11E-01 8.26E-02	9.63E-01 2.56E-02	1.56E+01 1.19E+01	1.77E+00 1.28E+00	2.58E+01 1.62E+01	8.09E-01 3.44E-02
DTLZ4	10	4.71E-01 6.06E-03	7.52E-01 2.16E-02	7.80E-01 1.49E-01	7.40E-01 6.08E-02	6.40E-01 4.05E-02	5.16E-01 3.13E-03	1.23E+00 6.78E-02	5.65E-01 2.14E-02
	20	7.09E-01 5.10E-03	8.70E-01 1.71E-02	1.18E+00 1.34E-01	1.10E+00 5.14E-02	8.71E-01 1.16E-02	7.54E-01 3.16E-03	1.51E+00 1.30E-01	7.57E-01 7.64E-03
DTLZ5	10	5.47E-02 6.63E-03	5.33E-01 6.71E-02	3.19E-02 4.37E-05	7.02E-02 4.60E-03	4.08E-01 9.99E-02	3.03E-01 3.92E-02	1.15E+00 2.02E-01	1.44E-01 2.71E-02
	20	7.82E-02 8.63E-03	7.21E-01 4.74E-02	1.26E-02 6.77E-04	1.96E-02 1.13E-03	7.59E-01 1.15E-01	3.41E-01 5.85E-02	2.26E+00 1.52E-01	1.59E-01 3.61E-02
DTLZ6	10	2.81E-01 7.80E-03	2.11E+00 1.34E-01	9.11E-02 2.02E-02	1.50E-01 2.72E-02	2.07E+00 1.21E-01	3.22E+00 4.85E-01	8.15E+00 1.09E+00	2.55E-01 3.24E-02
	20	3.24E-01 9.44E-03	4.01E+00 3.51E-01	6.18E-02 1.32E-02	9.11E-02 1.91E-02	5.83E+00 3.33E-01	4.45E+00 5.35E-01	1.01E+01 2.77E-01	2.98E-01 8.70E-02
DTLZ7	10	9.63E-01 1.87E-02	1.16E+00 9.58E-02	3.38E+00 8.48E-01	1.11E+00 1.27E-01	3.29E+00 9.42E-01	4.58E+00 5.64E-01	3.09E+01 4.65E+00	9.97E-01 1.51E-02
	20	1.72E+00 3.54E-02	7.54E+00 1.34E+00	8.85E+00 2.25E+00	6.40E+00 1.63E+00	9.80E+00 2.85E-01	4.31E+00 5.97E-01	8.43E+01 1.02E+01	2.04E+00 1.18E-01

Table 2: HV comparison of the eight EMO algorithms on WFG problem suite

Problem	Obj.	CDAS	HypE	IBEA	MOEA/D	SMS-EMOA	MSOPS	NSGA-III	SPEA2+SDE
WFG1	10	5.91E+09 3.69E+08	5.24E+09 9.18E+07	5.06E+09 5.04E+07	4.09E+09 1.06E+08	5.25E+09 1.97E+08	4.01E+09 4.08E+07	3.50E+09 9.28E+07	5.54E+09 2.27E+08
	20	5.52E+24 3.78E+23	4.45E+24 8.29E+22	4.31E+24 4.22E+22	2.91E+24 4.50E+22	4.12E+24 4.07E+23	2.84E+24 3.38E+22	2.48E+24 3.65E+22	5.29E+24 3.58E+23
WFG8	10	6.50E+09 3.47E+08	9.95E+09 1.82E+08	8.46E+09 3.45E+08	2.23E+09 1.41E+09	8.94E+09 5.00E+08	6.08E+09 4.22E+08	6.79E+09 5.56E+08	9.75E+09 1.76E+08
	20	2.55E+24 5.83E+23	8.09E+24 5.07E+23	6.24E+24 1.08E+24	6.16E+24 4.17E+23	6.61E+24 7.40E+23	2.42E+24 5.18E+23	4.43E+24 7.19E+23	9.15E+24 4.42E+23
WFG9	10	6.85E+09 2.64E+08	9.48E+09 4.08E+08	7.97E+09 4.36E+08	4.08E+09 1.35E+09	8.00E+09 2.58E+08	6.19E+09 7.64E+08	7.15E+09 4.21E+08	9.07E+09 3.21E+08
	20	3.73E+24 4.03E+23	7.77E+24 4.37E+23	5.27E+24 5.65E+23	4.19E+24 4.40E+23	6.55E+24 5.36E+23	2.69E+24 5.59E+23	5.49E+24 3.20E+23	6.95E+24 6.11E+23

2.1 Summary

Based on the experimental results of eight algorithms above, the summary of performance observation can be given as follows.

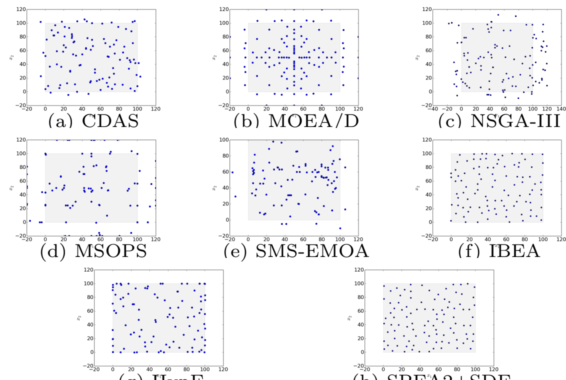
- The Pareto-based algorithm, CDAS performs the best on DTLZ2, DTLZ3, DTLZ4, DTLZ7, WFG1, and TSP of three instances and behaves well on DTLZ1, DTLZ5 and rectangle problem, but encounters great difficulties on WFG8 and WFG9.
- Among three aggregation-based algorithms, MOEA/D performs the worst on almost all DTLZ test problems except for biased DTLZ4. NSGA-III performs the best on degenerated DTLZ5 and DTLZ6, and MSOPS ranks the second on them. However, they all cannot perform well on WFG and TSP test problems.
- Among the three indicator-based algorithms, IBEA achieves the best on almost all DTLZ problems except for 10-objective DTLZ6, DTLZ7, but it encounters difficulties on WFG1,WFG8, WFG9 and TSP problems. SMS-EMOA only works well on 10-objective DTLZ6, DTLZ7. HypE works the best on WFG1, WFG8 and WFG9, TSP with two instances. This class of algorithms shows good performance on rectangle problem.
- The diversity-based algorithm, SPEA2+SDE performs well on DTLZ1 and DTLZ7, and it appears to be more competitive on more difficult problems like WFG and TSP test suite, rectangle problem.

3. CONCLUSIONS

Our study has revealed that none of the algorithms is able to solve problems with all different properties. It suggests an appropriate choice for engineering application. Subsequent work is to further discuss the computational budget of EMO algorithms.

Table 3: HV comparison of the eight EMO algorithms on TSP test problems

Problem	Obj.	CDAS	HypE	IBEA	MOEA/D	SMS-EMOA	MSOPS	NSGA-III	SPEA2+SDE
$TSP_{-0.2}$	15	9.73E+15 1.19E+15	7.25E+15 9.69E+14	2.50E+15 4.12E+14	4.95E+15 4.43E+14	6.71E+15 1.41E+15	1.41E+15 4.17E+14	6.14E+13 3.90E+13	7.56E+15 7.90E+14
TSP_0	15	8.37E+15 9.07E+14	7.84E+15 9.07E+14	3.57E+15 4.60E+14	3.94E+15 4.32E+14	7.33E+15 9.37E+14	1.73E+15 3.75E+14	7.18E+13 3.02E+13	7.35E+15 6.15E+14
$TSP_{0.2}$	15	7.42E+15 5.09E+14	7.34E+15 5.81E+14	4.06E+15 5.94E+14	3.03E+15 3.72E+14	6.93E+15 6.22E+14	2.12E+15 3.79E+14	1.24E+14 7.14E+13	6.76E+15 4.60E+14



(a) CDAS (b) MOEA/D (c) NSGA-III (d) MSOPS (e) SMS-EMOA (f) IBEA (g) HypE (h) SPEA2+SDE

Figure 2: The final solution set of the eight algorithms on the 4-objective rectangle test problem in two dimensional decision space.

Acknowledgment

The authors wish to thank the support of the National Natural Science Foundation of China (Grant Nos. 61379062, 61372049, 61403326), the Science Research Project of the Education Office of Hunan Province (Grant Nos. 12A135, 12C0378), the Hunan Province Natural Science Foundation (Grant Nos. 14JJ2072, 13JJ8006), the Science and Technology Project of Hunan Province (Grant No. 2014GK3027), the Construct Program of the Key Discipline in Hunan Province, and the Hunan Provincial Innovation Foundation For Postgraduate (Grant No. CX2013A011).

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