Multi-objective NM-landscapes

Roberto Santana
Intelligent Systems Group
University of the Basque Country UPV/EHU
roberto.santana@ehu.eus

Alexander Mendiburu
Intelligent Systems Group
University of the Basque Country UPV/EHU
alexander.mendiburu@ehu.es

Jose A. Lozano
Intelligent Systems Group
University of the Basque Country UPV/EHU
ja.lozano@ehu.eus

ABSTRACT

In this paper we propose an extension of the NM-landscape to model multi-objective problems (MOPs). We illustrate the link between the introduced model and previous landscapes used to study MOPs. Empirical results are presented for a variety of configurations of the multi-objective NM-landscapes.

Categories and Subject Descriptors

H.4 [**Information Systems Applications**]: Miscellaneous; D.2.8 [**Software Engineering**]: Metrics—*complexity measures, performance measures*; I.2.8 [**Computing Methodologies**]: Artificial Intelligence—*Problem Solving, Control Methods, and Search*

Keywords

fitness-landscapes; MOEAs; Boltzmann distribution; Pareto approximations

1. NM-LANDSCAPE

Let $\mathbf{X} = (X_1, \dots, X_N)$ denote a vector of discrete variables. We will use $\mathbf{x} = (x_1, \dots, x_N)$ to denote an assignment to the variables. S will denote a set of indices in $\{1, \dots, N\}$, and X_S (respectively x_S) a subset of the variables of $\mathbf{X}$ (respectively $\mathbf{x}$) determined by the indices in S.

A fitness landscape F can be defined for N variables using a general parametric interaction model of the form [2]:

$$F(\mathbf{x}) = \sum_{k=1}^{l} \beta_{U_k} \prod_{i \in U_k} x_i \tag{1}$$

where l is the number of terms, and each of the l coefficients $\beta_{U_k} \in \mathbb{R}$. For $k = 1, \dots, l$, $U_k \subseteq \{1, 2, \dots, N\}$, where U_k is a set of indices of the variables in the kth term, and the length $|U_k|$ is the order of the interaction. By convention [2], it is assumed that when $U_k = \emptyset$, $\prod_{j \in U_k} x_j \equiv 1$. Also by convention, we assume the model is defined for binary variables represented as $x_i \in \{-1, 1\}$.

The NM models [2] comprise the set of all general interactions models specified by Equation 1, with the following constraints: 1) All coefficients β_{U_k} are non-negative. 2) Each feature value x_i ranges from negative to positive values. 3) The absolute value of the lower bound of the range is lower or equal than the upper bound of the range of x_i. We will focus on NM-models defined on the binary alphabet. In this case, the NM-landscape has a global maximum that is reached at $\mathbf{x} = (1, \dots, 1)$ [2].

A number of multi-objective landscapes have been proposed to study the behavior of MOEAs [1, 3, 4]. In the next section we introduce a multi-objective NM-landscape model.

2. MULTI-OBJECTIVE NM-LANDSCAPES

The multi-objective NM-landscape model (mNM-landscape) is defined as a vector function mapping binary vectors of solutions into m real numbers $\mathbf{f}(.) = (f_1(.), f_2(.), \dots, f_m(.)) : \mathcal{B}^N \to \mathcal{R}^m$, where N is the number of variables, m is the number of objectives, $f_i(.)$ is the i-th objective function, and $\mathcal{B} = \{-1, 1\}$. $\mathbf{M} = \{M_1, \dots, M_m\}$ is a set of integers where M_i is the maximum order of the interaction in the i-th landscape. Each $f_i(\mathbf{x})$ is defined similarly to Equation (1) as:

$$f_i(\mathbf{x}) = \sum_{k=1}^{l_i} \beta_{U_{k_i}} \prod_{j \in U_{k_i}} x_j, \tag{2}$$

where l_i is the number of terms in objective i, and each of the l_i coefficients $\beta_{U_{k_i}} \in \mathcal{R}$. For $k = 1, \dots, l_i$, $U_{k_i} \subseteq \{1, 2, \dots, N\}$, where U_{k_i} is a set of indices of the features in the kth term, and the length $|U_{k_i}|$ is the order of the interaction.

The mNM fitness landscape model allows that each objective may have a different maximum order of interactions. We will focus on bi-objective mNM-landscapes. The following transformation is applied to the two objectives:

$$f_1(\mathbf{y}) : y_i = -2x_i + 1 \tag{3}$$
$$f_2(\mathbf{z}) : z_i = 2x_i - 1 \tag{4}$$

where $\mathbf{y} = (y_1, \dots, y_N) \in \{-1, 1\}$ and $\mathbf{z} = (z_1, \dots, z_N) \in \{-1, 1\}$ are the new variables after the corresponding transformation have been applied to $\mathbf{x} = (x_1, \dots, x_N) \in \{0, 1\}$.

The transformation guarantees that the optimal solutions will be respectively reached at points $(0, \dots, 0)$ and $(1, \dots, 1)$ for objectives f_1 and f_2. Another constraint we set in some

GECCO '15 July 11-15, 2015, Madrid, Spain

ACM ISBN 978-1-4503-3488-4/15/07.
DOI: http://dx.doi.org/10.1145/2739482.2764704

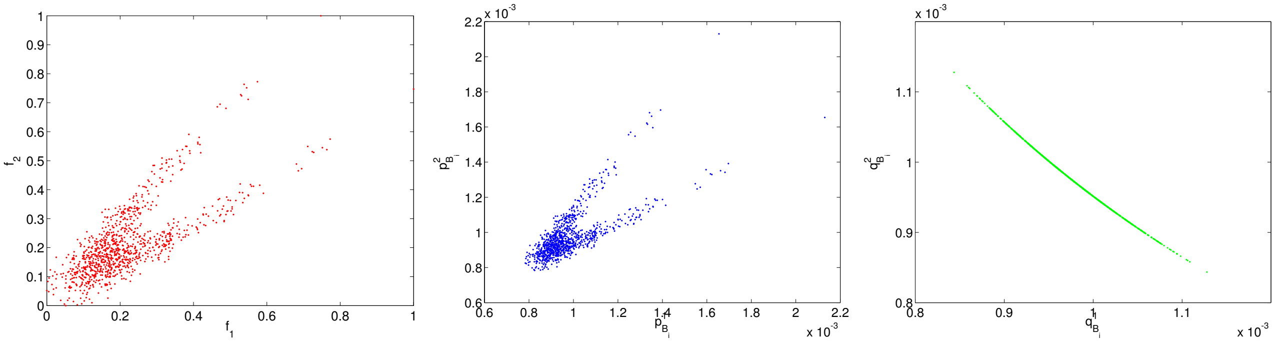

Figure 1: L) mNM landscape solutions, M) Boltzmann distributions, R) Product of the univariate marginals.

of the experiments is that, if $M_1 < M_2$ then, $\beta_{U_{k_1}} = \beta_{U_{k_2}}$ for all $|U_{k_i}| \leq M_1$. This means that all interactions contained in f_1 are also contained in f_2, but f_2 may also contains higher order interactions.

3. EXPERIMENTS

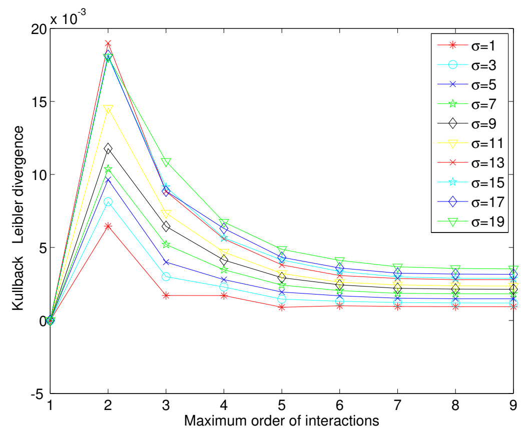

Figure 2: Influence of the maximum order of the interactions and σ in the Kullback-Leibler divergence between the Boltzmann distribution and its univariate factorization.

Figure 1 Left) shows the evaluation of the 2^{10} solutions that are part of the search space of a bi-objective mNM model. Figure 1 Middle) shows the Boltzmann probabilities associated to each point, i.e., $(p^1_{B_i}(\mathbf{x}^i), p^2_{B_i}(\mathbf{x}^i))$. Finally, Figure 1 Right) shows approximations of the Boltzmann distributions for the two objectives, each approximation computed using the corresponding product of the univariate marginals, i.e., $(q^1_{B_i}(\mathbf{x}^i), q^2_{B_i}(\mathbf{x}^i))$.

Since factorized approximations can be essential for feasible modeling of the search space, one relevant question is: *Under which conditions can factorized approximations respect the composition of the Pareto set?*. It seems that a sufficient condition is that the approximations keeps the ranking of the original functions for all the objectives, but this condition may not be necessary.

Figure 2 shows the values of the KL divergence for the combinations of the maximum order of the interactions and σ. When the maximum order of the interactions is 1, the approximation given by the univariate factorization is exact, therefore, the KL distance between the variables is 0 for all values of σ.

We summarize some of the findings from the experiments: 1) The mNM landscape can be used to create test problems with varying order of interactions. 2) Univariate factorizations are poor approximations for mNM models of maximum order two and higher. 3) The mutual information between the variables of the NM landscape is maximized for problems with maximum order of interaction 2. 4) The parity of the maximum order of the interactions of the mNM landscape influences the number of solutions in the Pareto fronts.

4. CONCLUSIONS

We have introduced the mNM-landscape as an extension of the NM-landscape to the multi-objective domain. Using the introduced model and the Boltzmann distribution, we have investigated the effect that interactions between the variables have in the shapes of the fronts, in the correlations between the objectives, and in the emergence of dependencies between the variables.

Acknowledgment

This work has been partially supported by IT-609-13 program (Basque Government) and TIN2013-41272P (Spanish Ministry of Science and Innovation).

5. REFERENCES

[1] H. Aguirre and K. Tanaka. Insights and properties of multiobjective MNK-landscapes. In *Proceedings of the 2004 Congress on Evolutionary Computation CEC-2004*, pages 196–203, 2004.

[2] N. Manukyan, M. J. Eppstein, and J. S. Buzas. Tunably rugged landscapes with known maximum and minimum. *arXiv.org*, arXiv:1409.1143, 2014.

[3] R. Santana, A. Mendiburu, and J. A. Lozano. Evolving MNK-landscapes with structural constraints. In *Proceedings of the 2015 Congress on Evolutionary Computation CEC-2012*, Sendai,Japan, 2015. IEEE Press. Accepted for publication.

[4] S. Verel, G. Ochoa, and M. Tomassini. Local optima networks of NK landscapes with neutrality. *Evolutionary Computation, IEEE Transactions on*, 15(6):783–797, 2011.