A Dimension-Decreasing Particle Swarm Optimization Method for Portfolio Optimization

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ABSTRACT
Portfolio optimization problems are challenging as they contain different kinds of constrains and their complexity becomes very high when the number of assets grows. In this paper, we develop a dimension-decreasing particle swarm optimization (DDPSO) for solving multi-constrained portfolio optimization problems. DDPSO improves the efficiency of PSO for solving portfolio optimization problems with a lot of asset and it can easily handle the cardinality constraint in portfolio optimization. To improve search diversity, the dimension-decreasing method is coupled with the comprehensive learning particle swarm optimization (CLPSO) algorithm. The proposed method is tested on benchmark problems from the OR library. Experimental results show that the proposed algorithm performs well.

Categories and Subject Descriptors
• Mathematics of computing~Mathematical optimization • Applied computing~Economics

General Terms
Algorithms, Economics.

Keywords
Portfolio optimization, particle swarm optimization, cardinality constraint, dimension-decreasing.

1. INTRODUCTION
Portfolio means investors investing tradable assets into different securities and it can reduce risks. Portfolio optimization tries to find the best combination of assets according to investors' needs [1]. In order to increase returns and reduce risks as much as possible, portfolio optimization has become an attractive topic in the modern portfolio theory (MPT) [2]. Markowitz [3] presented a mean-variance (MV) model of portfolio, which laid the theoretical foundation of MTP. There are two main challenges in portfolio optimization, i.e., the large number of assets and different kinds of constrains.

This paper proposed a dimension-decreasing particle swarm optimization (DDPSO), for multi-constrained portfolio optimization. The proposed DDPSO adopts a comprehensive learning particle swarm optimization (CLPSO) [4] to prevent premature convergence. The most important part of the DDPSO is cutting dimensions dynamically during evolution process. Cutting dimensions means reducing complexity, which is a huge advantage when solving a large-scale portfolio optimization problem. Moreover, by decreasing the number of selected assets gradually, DDPSO can easily handle the cardinality constraint in portfolio optimization.

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2. MATHEMATICAL FORMULATION
Assuming that there are N assets, μ_i is the expected return rate of the ith asset and the covariance between the returns of the ith and the jth assets can be denoted by σ_{ij}. The portfolio optimization problem is a bi-objective problem [5], i.e., minimizing risk and maximizing return. Some researchers [1] have defined a parameter λ, which is called risk-aversion parameter, to make the portfolio optimization problem a single-target one. The more risks an investor can afford, the nearer to 1 its value will be. Then the portfolio optimization model [1] is as follows:

$$Min(\lambda\sum_{i=1}^{N}\sum_{j=1}^{N}W_iW_j\sigma_{ij}-(1-\lambda)\sum_{i=1}^{N}W_i\mu_i) \tag{1}$$

Subject to

$$\lambda\in[0,1] \tag{2}$$

$$0\le W_i\le 1 \tag{3}$$

$$\sum_{i=1}^{N}W_i=1 \tag{4}$$

$$0\le W_i\le\delta_i\le 1 \tag{5}$$

$$\sum_{i=1}^{N}P_i=K\quad P_i=\begin{cases}1 & \text{if } Wi>0\\ 0 & \text{otherwise}\end{cases} \tag{6}$$

(2)(3)(4) are basic constraints. (5) is bounding constraint and δ_i is the upper bound of W_i. (6) is cardinality constraint, which means the investor just wants to invest in K out of N assets.

3. A DDPSO OPTIMIZATION PROCESS
CLPSO is a revised PSO [6], [7]. If the ith particle is denoted by $X_i(x_{i1},x_{i2},\ldots,x_{iD})$, its velocity is denoted by $V_i(v_{i1},v_{i2},\ldots,v_{iD})$. The update rule of a particle's velocity and position is as (7) (8).

$$v_{id}=w\times v_{id}+c\times rand\times(e_{id}-x_{id}) \tag{7}$$

$$x_{id}=x_{id}+v_{id} \tag{8}$$

In (7), e_{id} is the best position in dth dimension of the ***PBest*** position of a special particle.

```
procedure DDPSO
    initialization, counter=0;
    while terminal condition not met
        if (counter = S)
            counter=0;
            cut C worst dimensions for the population;
        else
            counter++;
        end if
        for each particle i (i = 1, 2, … , NP)
            velocity updating;
            position updating;
        end for
    end while
end procedure
```

Figure 1. Structure of the DDPSO

The most important part of DDPSO is cutting dimensions. If the total number of iterations is T, the dimensions a particle has at the beginning is N, and we need to maintain K dimensions at last, then we can cut C dimensions after every S iterations. And the relationship among these numbers should be (9).

$$N - \left[\frac{T}{S}\right] \times C = K \tag{9}$$

Here are the DDPSO for portfolio optimization. Assuming that there are N assets in total and an investor is going to invest in K assets. We can consider a vector $W = (W_1, W_2, W_3, \ldots, W_N)$ as a particle, which represents the proportions of every asset to invest in a portfolio. Then generate NP different W randomly as a population. The rest of work is using DDPSO to find the best particle after a number of iterations. And we use a similar method in paper [1] to handle (3) and (4).

4. EXPERIMENTAL TEST

This experiment tests 8 test cases from the OR library (available: http://people.brunel.ac.uk/~mastjjb/jeb/orlib/portrebinfo.html).

We compare the DDPSO with a k-means cluster analysis [1] method. Table 1 shows the comparative result.

5. CONCLUSIONS

A dimension-decreasing particle swarm optimization (DDPSO) algorithm has been proposed to solve the problem of portfolio optimization. This algorithm can deal with three kinds of constraints of the portfolio optimization problem and it performs well in dealing with cardinality constraint. At the same time, this algorithm is based on the comprehensive learning PSO (CLPSO), which is simple, efficient and won't fall into local optimum. Furthermore, the DDPSO can solve a relatively large-scale portfolio optimization which the traditional methods can't solve. Experimental results showed that the DDPSO is promising.

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Table 1. Comparison of DDPSO and k-means cluster analysis

instance	algorithm	mean	best	std	t-test
Port 1	DDPSO	-0.00335	-0.00336	5.51E-06	-33.371*
	k-means	-0.00329	-0.00329	8.07E-06	
Port 2	DDPSO	-0.00401	-0.00404	5.75E-05	-70.982*
	k-means	-0.00348	-0.00348	4.28E-06	
Port 3	DDPSO	-0.00285	-0.00287	1.66E-05	-106.072*
	k-means	-0.00244	-0.00244	3.63E-06	
Port 4	DDPSO	-0.00359	-0.00364	2.18E-05	-128.625*
	k-means	-0.00264	-0.00264	2.50E-06	
Port 5	DDPSO	-0.00135	-0.00143	4.63E-05	-33.281*
	k-means	-0.00105	-0.00106	9.30E-06	
Port 6	DDPSO	0.01435	0.01353	4.54E-04	-957.976*
	k-means	0.09377	0.09377	0	
Port 7	DDPSO	0.001338	0.000843	2.71E-04	-818.792*
	k-means	0.04184	0.04183	5.28E-07	
SG	DDPSO	-0.52782	-0.53858	0.00314	-11.572*
	k-means	-0.5109	-0.52537	0.00522	

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