Complete Multi-Objective Coverage with PaCcET

Logan Yliniemi
Oregon State University
Corvallis, OR, USA
logan.yliniemi@engr.orst.edu

Kagan Tumer
Oregon State University
Corvallis, OR, USA
kagan.tumer@oregonstate.edu

ABSTRACT

The Pareto Concavity Elimination Transformation (PaCcET) is a promising new development in multi-objective optimization. It transforms the objective space so that a computationally-cheap linear combination of objectives can attain (even concave) Pareto-optimal points. In this work we propose a simple extension to the PaCcET framework, which biases the optimization process toward less-covered areas of the Pareto front.

Categories and Subject Descriptors

I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search—*Heuristic methods*

1. INTRODUCTION

In multi-objective optimization, one common goal is to discover the "Pareto optimal set" [2]. It represents the set of optimal tradeoffs between objectives, where no change can improve one objective without harming another. The Pareto optimal set can be very difficult to find. Multi-Objective Evolutionary Algorithms (MOEAs) such as NSGA-II are accomplished at approximating these sets in many different types of problems. Their populations represent an approximation (P_I^*) of the Pareto front. However, these calculations can be expensive, and the population must be very large to give good coverage over the entire Pareto front.

The Pareto Concavity Elimination Transformation (PaCcET), achieves the same outcome using a fundamentally different mechanism [6]. Instead of using population members to compare against themselves, it keeps a constantly-improving upper bound estimate of the Pareto frontier (P_I^*), and transforms the objective space such that all solutions not dominated by this upper bound lie in the area of the search space that will be discovered by a computationally cheap linear combination of objectives.

In this work we provide a simple extension to PaCcET, The Complete Coverage ($\mathcal{CC}$) Extension, which manipulates a copy of P_I^* in such a way that it forces the underlying optimizer away from areas of the space that already have a dense coverage of solutions.

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2. BACKGROUND: PACCET INTUITION

The core functionality of PaCcET is to transforms the objective space such that the Pareto front will be non-concave. This implementation of PaCcET has been shown to approximate the Pareto front faster on a per-member-evaluation basis than NSGA-II and SPEA2, with comparable final solution quality [6].

Intuitively, the purpose of the PaCcET transformation is to make the current Pareto approximate set equally valuable, as we are indifferent between these solutions [3]. It registers the Pareto approximation (P_I^*) on to points on the normalized utopia hyperplane [5]. This can be achieved through a non-rigid registration [1], which forms the core of the transformation.

PaCcET generalizes to k objectives, but for the intuition, consider two objectives. PaCcET can be seen as radially expanding or contracting the scaling of the space, centered on the approximate utopia point (See Figure 1). In locations where P_I^* is concave, that scaling factor will be < 1, contracting the space along that vector until that point in P_I^* is on the utopia hyperplane. Where P_I^* is convex, that scaling factor will be > 1, expanding the space so that the P_I^* point is on the utopia hyperplane. The normalized utopia hyperplane is equally valuable to an unweighted linear combination of objectives, guaranteeing that all points in P_I^* have the same linear combination evaluation.

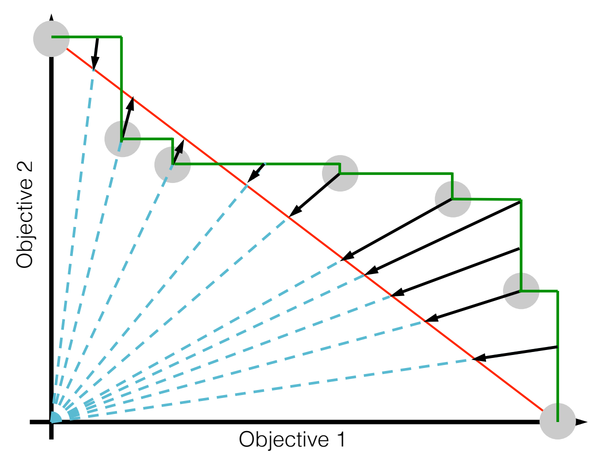

Figure 1: Intuition for the PaCcET process in a two-objective problem (PaCcET generalizes to k objectives). Grey points in P_I^* are scaled radially (centered on the approximate utopia point) away if they are in front of the utopia hyperplane, and scaled radially closer if they are behind the utopia hyperplane. All points in P_I^* then have the same unweighted linear combination.

This means that points that are not dominated by P_I^* will have a linear combination of $< LC(P_I^*)$, while points that are dominated by P_I^* will have a linear combination of $> LC(P_I^*)$. Thus, non-dominated solutions are preferred, and will be discovered during optimization. Complete details are available in [6].

3. COMPLETE COVERAGE ($\mathcal{CC}$) EXTENSION

Algorithm 1 describes the process, and Figure 2 illustrates it. At each iteration I, the members of P_I^* are compared pairwise. Those that are sufficiently close ($< \delta$, a user-defined parameter) generate a surrogate (Line 10). If any of their objective values are too close ($< \epsilon$), a modified surrogate is generated (Line 12), so that the search is biased away from densely covered areas of the Pareto front. This set S is used instead of P_I^* during the calculation of $||v||_B$ in the PaCcET Transformation [6].

3.1 Result

The KUR problem is a two-objective benchmark problem with a discontinuous, locally concave Pareto front [4]. Figure 3 (Top) shows the density of solutions produced by PaCcET on KUR. In this implementation, the top curve is mostly ignored. With $\mathcal{CC}$ ($\delta = 0.1$) (Figure 3, Bottom), the entire Pareto front is more fairly covered with solutions.

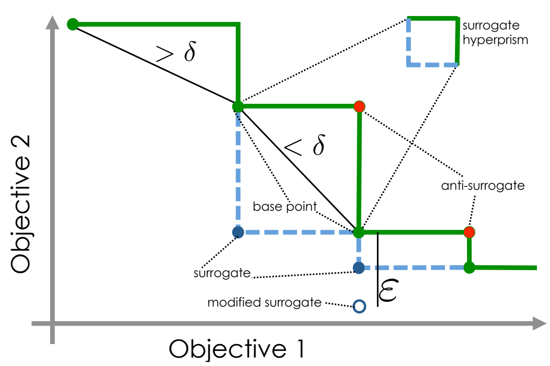

Figure 2: Diagram of terms included in $\mathcal{CC}$ module, and visualization of surrogate and modified surrogate process.

Algorithm 1 $\mathcal{CC}$: S calculation for Iteration I

Require: Pareto Approximate Set P_I^*; Empty S.
1: **for all** Members $p \in P_I^*$ **do**
2: **for all** Members $q \neq p, q \in P_I^*$ **do**
3: $z = size(S) = size(AS)$
4: $v_1 = P_{I,p}^*$; $v_2 = P_{I,q}^*$
5: **if** $||P_{I,p}^* - P_{I,q}^*|| > \delta$ **then** $S = S$
6: **end if**
7: **if** $||P_{I,p}^* - P_{I,q}^*|| < \delta$ **then**
8: $\forall c \in C : AS_{z+1}(c) = max(v_1(c), v_2(c))$ (Anti-Surrogate)
9: **if** $|P_{I,p}^*(c) - P_{I,q}^*(c)| > \epsilon$ **then**
10: $\forall c \in C : S_{z+1}(c) = min(v_1(c), v_2(c))$ (Surrogate)
11: **else**
12: $\forall c \in C : S_{z+1}(c) = max(v_1(c) - \epsilon, v_2(c) - \epsilon)$ (Modified Surrogate)
13: **end if**
14: **end if**
15: $S \leftarrow \texttt{Pareto\_Filter}(P_I^* \cup S)$
16: **end for**
17: **end for**

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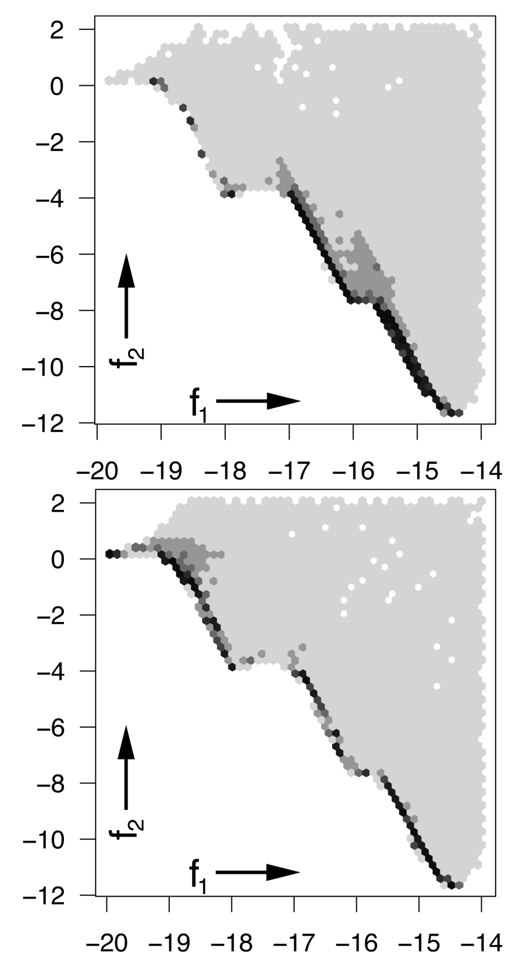

Figure 3: PaCcET solution densities in KUR without (Top) and with (Bottom) the $\mathcal{CC}$ Extension, which encourages a more even spread of solutions across the Pareto front.