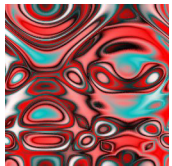


# Cartesian Genetic Programming

Evolved picture



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Evolved picture



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GECCO'15 Companion, July 11–15, 2015, Madrid, Spain.  
ACM 978-1-4503-3488-4/15/07.  
<http://dx.doi.org/10.1145/2739482.2756571>



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## Abstract

- ❖ Cartesian Genetic Programming (CGP) is a well-known form of Genetic Programming developed by Julian Miller in 1999-2000. In its classic form, it uses a very simple integer address-based genetic representation of a program in the form of a directed graph. Graphs are very useful program representations and can be applied to many domains (e.g. electronic circuits, neural networks). It can handle cyclic or acyclic graphs.
- ❖ In a number of studies, CGP has been shown to be comparatively efficient to other GP techniques. It is also very simple to program. The classical form of CGP has undergone a number of developments which have made it more useful, efficient and flexible in various ways. These include self-modifying CGP (SMCGP), cyclic connections (recurrent-CGP), encoding artificial neural networks and automatically defined functions (modular CGP).
- ❖ SMCGP uses functions that cause the evolved programs to change themselves as a function of time. This makes it possible to find general solutions to classes of problems and mathematical algorithms (e.g. arbitrary parity, n-bit binary addition, sequences that provably compute pi and e to arbitrary precision, and so on).
- ❖ Recurrent-CGP allows evolution to create programs which contain cyclic, as well as acyclic, connections. This enables application to tasks which require internal states or memory. It also allows CGP to create recursive equations.
- ❖ CGP encoded artificial neural networks represent a powerful training method for neural networks. This is because CGP is able to simultaneously evolve the networks connections weights, topology and neuron transfer functions. It is also compatible with Recurrent-CGP enabling the evolution of recurrent neural networks.
- ❖ The tutorial will cover the basic technique, advanced developments and applications to a variety of problem domains. It will present a live demo of how the open source cgplibary can be used.

## Genetic Programming

- ❖ The automatic evolution of computer programs
  - Tree-based, Koza 1992
  - Stack-based, Perkis 1994, Spector 1996 onwards (push-pop GP)
  - Linear GP, Nordin and Banzhaf 1996
  - **Cartesian GP**, Miller 1999
  - Parallel Distributed GP, Poli 1996 (related to CGP)
  - Grammatical Evolution, Ryan 1998
  - Lots of others...

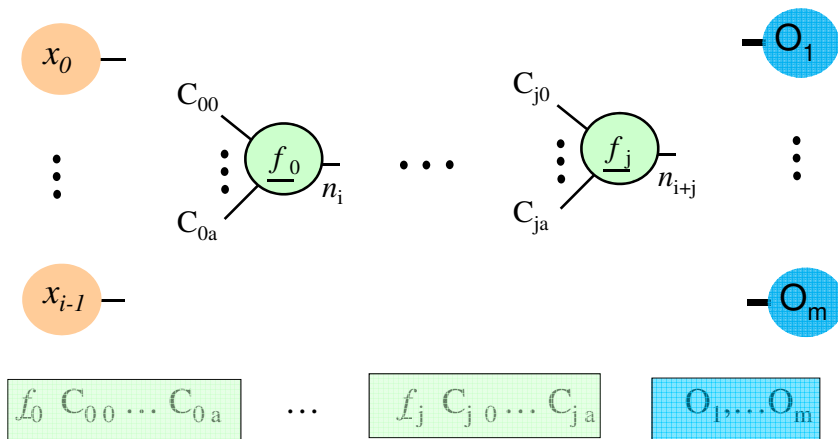
## Origins

- ❖ Grew out of work in the evolution of digital circuits, Miller and Thomson 1997.
  - First actual mention of the term *Cartesian Genetic Programming* appeared at GECCO in 1999.
- ❖ Originally, represents programs or circuits as a two dimensional grid of program primitives.
  - Hence *Cartesian* GP
- ❖ This is loosely inspired by the architecture of digital circuits called FPGAs (field programmable gate arrays)

## Why use CGP?

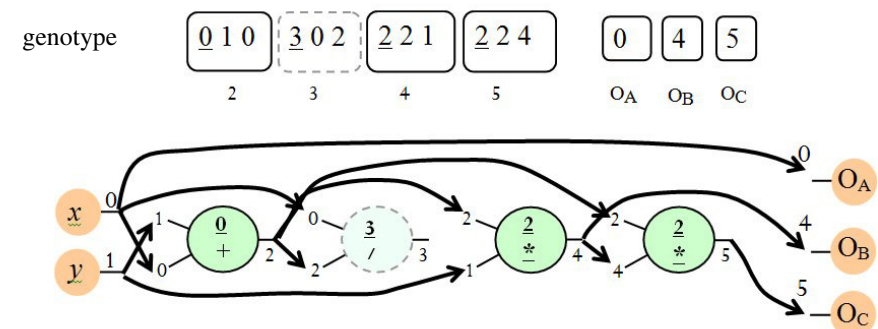
- ❖ Its performance is highly competitive with other forms of GP
- ❖ It is naturally suited to multiple input multiple-output (MIMO) tasks
- ❖ It is simple and only requires a simple evolutionary algorithm
- ❖ It encodes cyclic or acyclic graphs which are highly flexible for many applications
  - It allows internally calculated values to be reused multiple times
  - It can naturally encode artificial neural networks
- ❖ It benefits from explicit neutral genetic drift
- ❖ It does not suffer from program bloat. In fact, it naturally compresses solutions.

## General form of CGP



All functions have as many inputs as the *maximum* function arity  
Unused connections are ignored

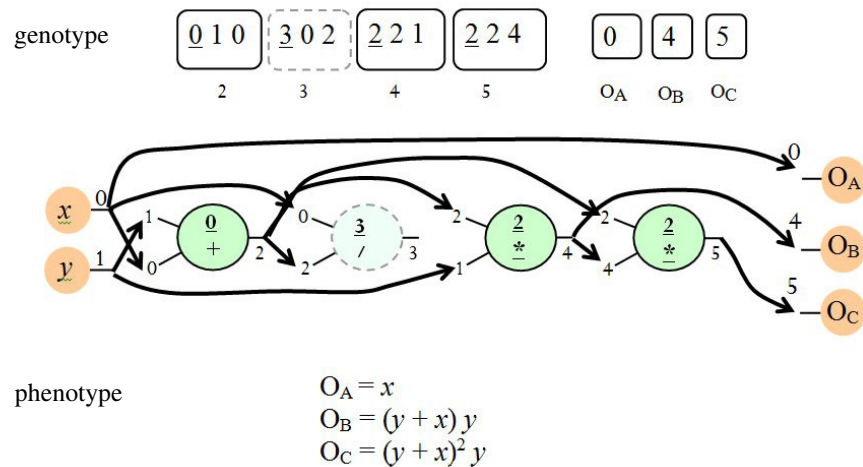
## Example



Function genes (are addresses in user defined lookup table)

0	+ Add 1	- Subtract
2	* Multiply 3	/ Divide (protected)

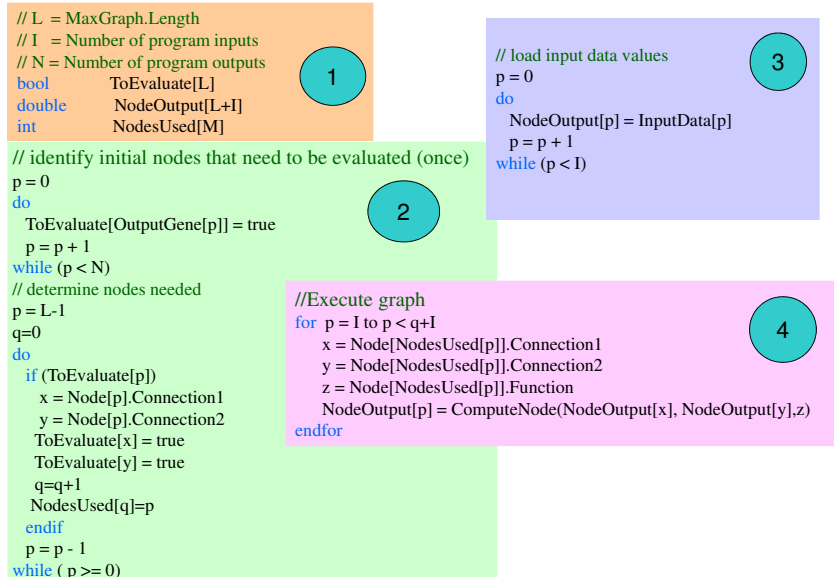
## So what does the graph represent?



## The CGP genotype-phenotype map

- ❖ When you decode a CGP genotype many nodes and their genes can be ignored because they are not referenced in the path from inputs to outputs
- ❖ These genes can be altered and make no difference to the *phenotype*, they are non-coding
- ❖ Clearly there is a many-to-one genotype to phenotype map
- ❖ How redundant is the mapping?
  - Massively redundant.
  - For example, a nine node genotype has **118,124** possible 3 node phenotypes (assuming one program input and all nodes have arity equal to one).

## Decoding CGP chromosomes is *easy and fast*



## Why decoding the CGP genotype map is extremely fast

- ❖ Some people wrongly think that CGP is slow compared to other forms of GP because it has a genotype-phenotype mapping stage.
- ❖ First one determines which nodes are used. This is one pass and is only done once.
  - Time is insignificant compared with fitness evaluation
- ❖ To determine the output from CGP only requires looking up a few entries in an array
- ❖ Phenotypes in CGP are usually a fraction of the genotype size

## Point mutation

- ❖ Most CGP implementations only use mutation.
- ❖ Carrying out mutation is very *simple*. It consists of the following steps. The genes must be chosen to be valid alleles
  - Of course, it can be also be done probabilistically

```
//Decide how many genes to change:num_mutations
while (mutation_counter < num_mutations)
{
    get random gene to change
    if (gene is a function gene)
        change gene to randomly chosen new valid function
    else if (gene is a connection gene)
        change gene to a randomly chosen new valid connection
    else
        change gene to a new valid output connection
}
```

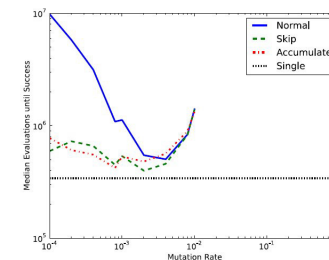
## Advice on CGP parameters and experiments

- ❖ Only allow a small number of mutations
- ❖ Choose a fairly large number of nodes (100 to 1000)
- ❖ Use a 1+ 4 evolutionary strategy
- ❖ Only calculate fitness of genotypes that have different phenotypes
  - Check whether an offspring has exactly the same active nodes as the parent, if so do not calculate its fitness and do not increment the evaluation counter (Skip method – see next slide)
- ❖ Only evaluate active nodes
- ❖ When making comparisons with other GP methods either give them the same budget in processed nodes or the same number of fitness evaluations.

## Single, a new mutation method?

- ❖ *Single*: Keep mutating until an active gene is mutated (Goldman and Punch 2013)
- ❖ Exactly one active gene is mutated for all offspring.
- ❖ It looks like no mutation rate is required, however its effective mutation rate is determined by the length of the genotype
  - Remains to be investigated

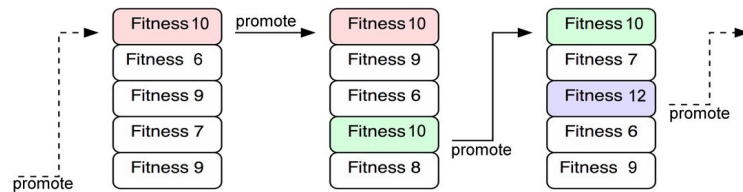
## Single active gene mutation strategy: results (Goldman and Punch 2013)



- ❖ 3bit parallel multiplier
  - Multiplies two three-bit numbers in parallel
  - Hard problem
- ❖ On other problems the best performance of *single* is slightly worse than other mutation schemes
- ❖ *Normal* = standard CGP
- ❖ *Skip*: set offspring's fitness to parent if identical
- ❖ *Single*: mutate until one active gene is changed
  - 29% less computation than Normal!

## Evolutionary Strategy

- ❖ CGP often uses a variant of a simple algorithm called (1 + 4) Evolutionary Strategy
  - However, an offspring is always chosen if it *is equally as fit* or has better fitness than the parent



## CGP Library

- ❖ **Andrew Turner** – developer/maintainer
- ❖ [cgplibrary.co.uk](http://cgplibrary.co.uk)
- ❖ **Cross Platform** – Linux, Windows, Mac
- ❖ **Open Source** – GNU LGPL
- ❖ **Fully Documented API**
- ❖ **Many Tutorials** – beginner and advance
- ❖ **Very Simple**
- ❖ **Written in C** – fast, no dependencies
- ❖ **Language Bindings** – via SWIG



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## CGP Library – Overview

- ❖ **Cartesian Genetic Programing (CGP)**
- ❖ **Recurrent CGP**
- ❖ **CGP Artificial Neural Networks**
- ❖ **Recurrent CGP Artificial Neural Networks**
- ❖ **What more could you want ;)**
- ❖ **Designed for:**
  - Teaching
  - Research
  - Applications

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## Live Demo

- ❖ **Tutorials:** [cgplibrary.co.uk](http://cgplibrary.co.uk)
- ❖ [andrew.turner@york.ac.uk](mailto:andrew.turner@york.ac.uk)
- ❖ **Please ask questions!**
- ❖ **Or talk to me later :)**

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## Additional Features

- ❖ Save, store and load chromosomes
- ❖ Embed CGP in other applications
- ❖ Highly Customisable
  - Fitness functions
  - Node Functions
  - Selection Scheme
  - Mutation Scheme
  - Reproduction Scheme
- ❖ Additional Language bindings

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## Final Words

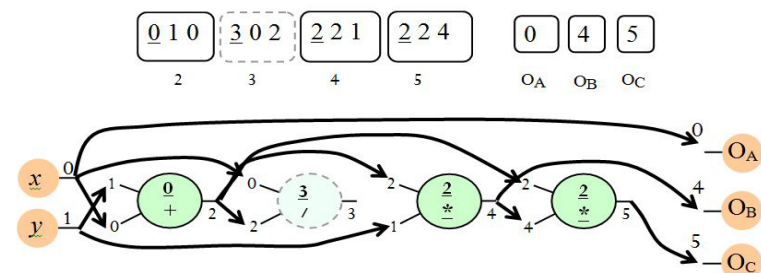
- ❖ Much more info @ [cgplibrary.co.uk](http://cgplibrary.co.uk)
- ❖ Hosted with GitHub:  
[Github.com/AndrewJamesTurner/CGP-Library](https://github.com/AndrewJamesTurner/CGP-Library)
- ❖ Citable using:  
A. J. Turner, J. F. Miller, *Introducing a cross platform open source Cartesian Genetic Programming library*, GPEM, 2014, 83-91

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## Crossover or not?

- ❖ Recombination doesn't seem to add anything (Miller 1999, "An empirical study...")
- ❖ However if there are multiple chromosomes with independent fitness assessment then it helps a LOT (Walker, Miller, Cavill 2006, Walker, Völk, Smith, Miller, 2009)
- ❖ Some work using a floating point representation of CGP shows that crossover is useful for symbolic regression (Clegg, Walker, Miller 2007, Meier, Gonter, Kruse 2013)
- ❖ Cone-based crossover (Kaufmann and Platzner 2007, 2008)

## CGP samples the solution space in a very interesting way



One gene mutation can create a wide variety of offspring, since it can activate or deactivate large number of nodes



## Neutral search is fundamental to success of CGP

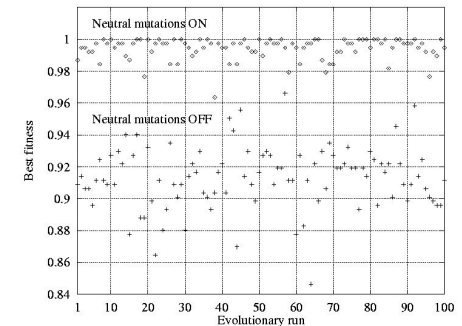
❖ A number of studies have been carried out to indicate the importance to neutral search

- *Miller and Thomson 2000, Vassilev and Miller 2000, Yu and Miller 2001, Miller and Smith 2006*

## Neutral search and the three bit multiplier problem (*Vassilev and Miller 2000*)

Importance of neutral search can be demonstrated by looking at the success rate in evolving a correct three-bit digital parallel multiplier circuit

Graph shows final fitness obtained in each of 100 runs of 10 million generations with neutral mutations enabled compared with disabled neutral mutations.



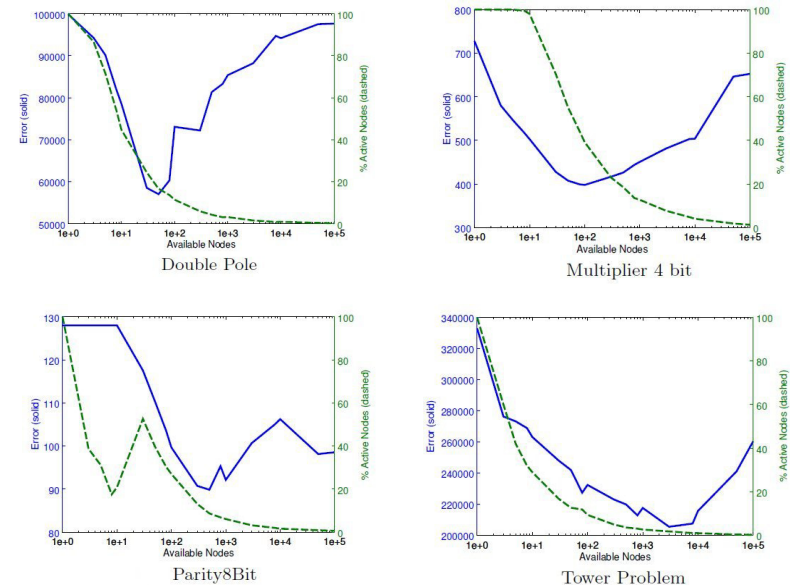
## How big should the genotype be?

❖ Miller and Smith investigated the efficiency of CGP search as a function of the number of available nodes and mutation rate on a couple of small problems (*Miller and Smith 2006*). It was found that

- Least number of evaluations required for success when 95% of the genes were inactive!
- Efficiency of the search appeared to continuously improve as the number of nodes increased...

❖ Turner and Miller have recently investigated neutral drift in a recent paper (*Turner and Miller in press*). Using 3% probabilistic mutation and up to 100,000 nodes, the following results were obtained:

## Efficiency of search v. max number of nodes



Modular/Embedded CGP (*Walker, Miller 2004, 2008, Kaufmann and Platzner 2007, 2008* )

- ❖ So far have described a form of CGP (classic) that does not have an equivalent of Automatically Defined Functions (ADFs)
- ❖ Modular CGP allows the use of modules (ADFs)
  - Modules are dynamically created and destroyed
  - Modules can be evolved
  - Modules can be re-used

## Creating, Destroying, Replicating and Evolving Modules

- ❖ Created by the **compress** operator
  - Randomly acquires sections of the genotype into a module
    - Sections cannot contain modules
- ❖ Destroyed by the **expand** operator
  - Converts the original module back into a section of the genotype
- ❖ Replicated by mutation operator
  - Mutation can change a primitive function node to a module, or change a module to a primitive function node
- ❖ Evolution of Module
  - All instances of a module in the genotype can be changed by a mutation to a module

## Module Survival

- ❖ Twice the probability of a module being destroyed than created
- ❖ Modules have to replicate to improve their chance of survival
  - This reduces the probability of their extinction
- ❖ Modules must also be associated with a high fitness genotype in order to survive
  - Offspring inherit the modules of the fittest parent

## Evolving Modules

- |  |   |
|--|---|
| <ul style="list-style-type: none"><li>❖ Structural mutation<ul style="list-style-type: none"><li>• Add input</li><li>• Remove input</li><li>• Add output</li><li>• Remove output</li></ul></li></ul> | <ul style="list-style-type: none"><li>❖ Module point-mutation operator<ul style="list-style-type: none"><li>• Restricted version of genotype point-mutation operator<ul style="list-style-type: none"><li>– Uses only primitive functions</li></ul></li></ul></li></ul> |
|--|---|



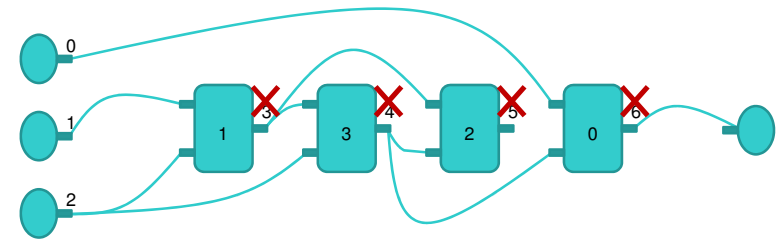
## Self-modifying Cartesian Genetic programming (Harding, Miller, Banzhaf 2007 onwards)

### ❖ A developmental form of CGP

- Includes self modification functions in addition to computational functions
- ‘General purpose’ GP system
- Phenotype can vary over time (with iteration)
- Can switch off its own self-modification

### ❖ Some representational changes from classic CGP...

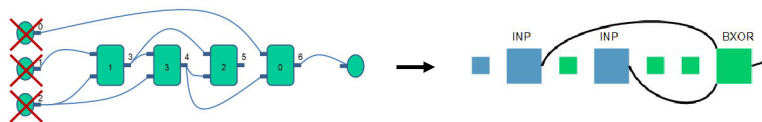
## Changes to CGP: relative addressing



### ❖ Replaced direct node addressing with relative addressing

- Always use 1 row (not rectangular)
- Connection genes say how many nodes back

## Changes to CGP: Inputs



### ❖ Replace input calls with a function.

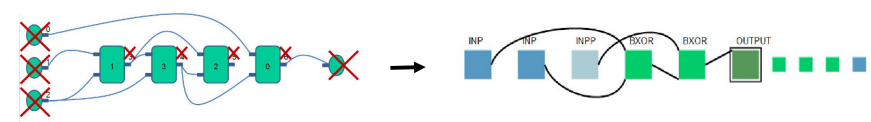
- We call these functions INP, INPP, SKIPINP

### ❖ Pointer keeps track of ‘current input’.

- Call to INP returns the current input, and moves the pointer to the next input.

### ❖ Connections beyond graph are assigned value 0.

## Changes to CGP: Outputs



### ❖ Removed output nodes.

### ❖ Genotype specifies which nodes are outputs.

### ❖ If no OUTPUT function then last active node is used

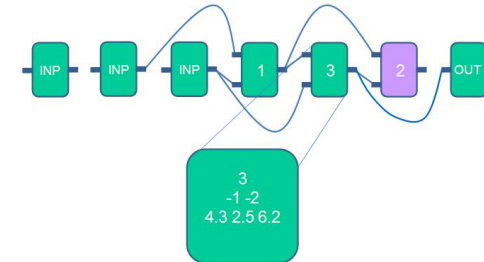
- Other defaults are used in situations where the number of outputs does not match the number required

## Changes to CGP: Arguments

- ❖ Nodes also contain a number of ‘arguments’.
  - 3 floating point numbers
  - Used in various self-modification instructions
  - Cast to integers when required

## SMCGP Nodes: summary

- ❖ Each node contains:
  - Function type
  - Connections as relative addresses
  - 3 floating point numbers



## SMCGP: Functions

- ❖ Two types of functions:
  - Computational
    - Usual GP computational functions
  - Self-modifying
    - Passive computational role (see later)

## Some Self-Modification Functions

Operator	Parameters: use node address and the three node arguments	Function
MOVE	Start, End, Insert	Moves each of the nodes between Start and End into the position specified by Insert
DUP	Start, End, Insert	Inserts copies of the nodes between Start and End into the position specified by Insert
DELETE	Start, End	Deletes the nodes between Start and End indexes
CHF	Node, New Function	Changes the function of a specified node to the specified function
CHC	Node, Connection1, Connection2	Changes the connections in the specified node

## SMCGP Execution

### ❖ Important first step:

- Genotype is duplicated to phenotype.
- Phenotypes are executed:
  - Self modifications are only made to the phenotype.

## Self Modification Process: The To Do list

- ❖ Programs are iterated.
- ❖ If *triggered*, self modification instruction is added to a To Do list.
- ❖ At the end of each iteration, the instructions on this list are processed.
- ❖ The maximum size of the To Do list can be predetermined

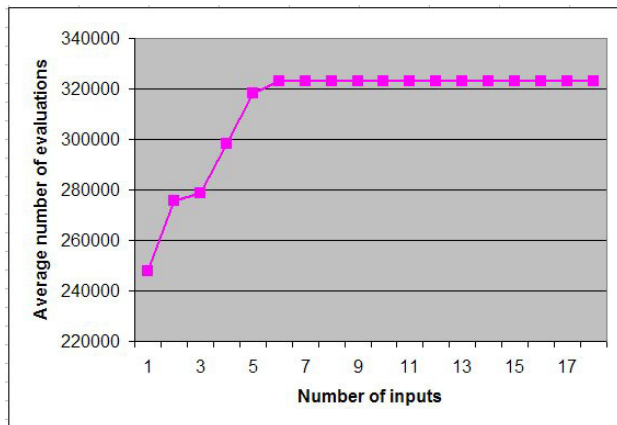
## Computation of a SM node

- ❖ Functions can be appended to the To Do list under a variety of conditions (triggered)
  - If active
  - If  $\text{value}(\text{first input}) > \text{value}(\text{the second input})$ 
    - Data dependent self-modification
- ❖ And:
  - The To Do list isn't too big.

## Example: Evolving Even-Parity

- ❖ Each iteration of program should produce the next parity circuit.
  - On the first iteration the program has to solve 2 bit parity. On the next iteration, 3 bit ... up to 22 parity
  - Fitness is the cumulative sum of *incorrect* bits
- ❖ Aim to find *general* solution
  - Solutions can be proved to general
    - See GPEM 2010 paper
- ❖ CGP or GP cannot solve this problem as they have a finite set of inputs (terminals)

## Scaling behaviour of SMCGP on even-parity



## Evolving pi

- ❖ Iterate a maximum of 10 times
- ❖ If program output does not get closer to pi at the next iteration, the program is stopped and large fitness penalty applied
- ❖ Fitness at iteration,  $i$ , is absolute difference of output at iteration  $i$  and pi
- ❖ One input: the numeric constant 1.

## Evolving pi: an evolved solution

- ❖ An evolved solution

$$f(i) = \begin{cases} \cos(\sin(\cos(\sin(0)))) & i = 0 \\ f(i-1) + \sin(f(i-1)) & i > 0 \end{cases}$$

- ❖  $f(10)$  is correct to the first 2048 digits of pi
- ❖ It can be **proved** that  $f(i)$  rapidly converges to pi in the limit as  $i$  tends to infinity

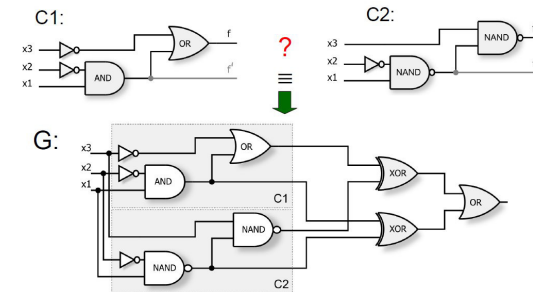
## Further results

- ❖ Other mathematically provable results found so far:
  - Evolved a program that can carry out the bitwise addition of an arbitrary number of inputs
  - Evolved a sequence that converges to e
- ❖ Other results
  - Evolved a sequence function that generates the first 10 Fibonacci numbers (probably general)
  - Evolved a power function  $x^n$
  - Bioinformatics classification problem (finite inputs)
    - SMCGP performed no worse than CGP

## Application 1: Digital circuit synthesis with CGP

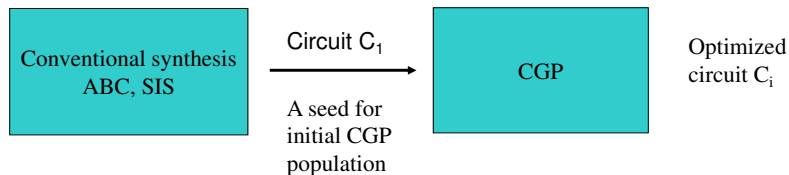
- ❖ Digital Circuits with **hundreds** of variables can be optimized using CGP (*Vassicek and Sekanina 2011*)
  - Won the \$3000 silver award in human competitive workshop at GECCO 2011
- ❖ The method employs a SAT solver to identify whether two circuits are logically equivalent
  - In many cases this can be done in polynomial time

## Circuit equivalence checking and SAT



- ❖ If C1 (reference) and C2 (evolved) are not functionally equivalent then there is at least one assignment of the inputs for which the output of G is 1.

## CGP for optimizing conventionally synthesized circuits



The seed for CGP is provided by using the logic synthesis package, ABC (<http://www.eecs.berkeley.edu/~alanmi/abc/>)

The fitness function is as follows:

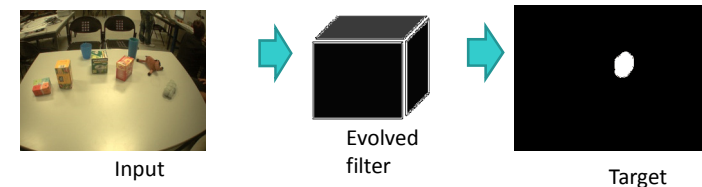
- ❖ Use a **SAT solver** to decide whether candidate circuit  $C_i$  and reference circuit  $C_1$  are functionally equivalent.
  - If so, then  $\text{fitness}(C_i) = \text{number of nodes} - \text{number of gates in } C_i$ ;
  - Otherwise:  $\text{fitness}(C_i) = 0$ .
- ❖ The method is now much faster by using a circuit simulator prior to SAT solver to disprove the equivalence between a candidate solution and its parent (*Vassicek 2015*)

## Application 2: Evolving Image Filters with CGP

(*Harding, Leitner, Schmidhuber 2013*)

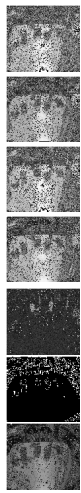


- ❖ Detecting/locating objects with the iCub cameras
- ❖ Done by evolving image filters that take a camera image, and return only the objects of interest

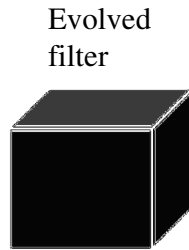


## Input data

Image from camera

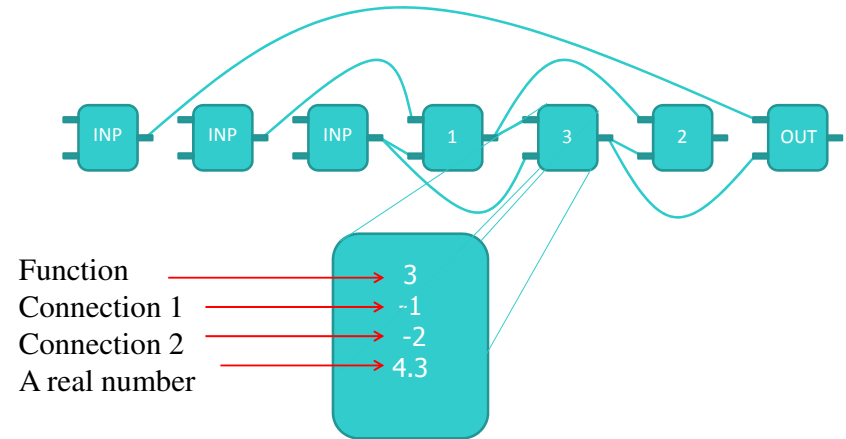


Grey  
Red  
Green  
Blue  
Hue  
Saturation  
Luminosity



Split colour image is used as inputs

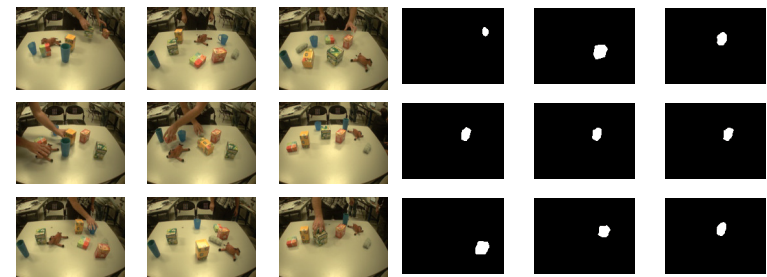
## Genotype representation (like SMCGP but no SM functions)



## Large Function Set

NOP	LOG	<i>TRIANGLES</i>
INP	MAX	<i>LINES</i>
INPP	MIN	SHIFTDOWN
SKIP	EQ	SHIFTUP
ADD	<i>GAMMA</i>	SHIFTLEFT
SUB	GAUSS	SHIFTRIGHT
CONST	SOBELX	<i>SIFTa</i>
MUL	SOBELY	GABOR
ADDC	AVG	NORMALIZE
SUBC	UNSHARPEN	RESCALE
MULC	THRESHOLD	<i>GRAB CUT</i>
ABSDIFF	THRESHOLDBW	MINVALUE
CANNY	SMOOTHMEDIAN	MAXVALUE
DILATE	<i>GOODFEATURESTOTRACK</i>	AVGVALUE
ERODE	<i>SQUARES</i>	RESCALE
LAPLACE	<i>CIRCLES</i>	RESIZETHENGABOR

## Fitness



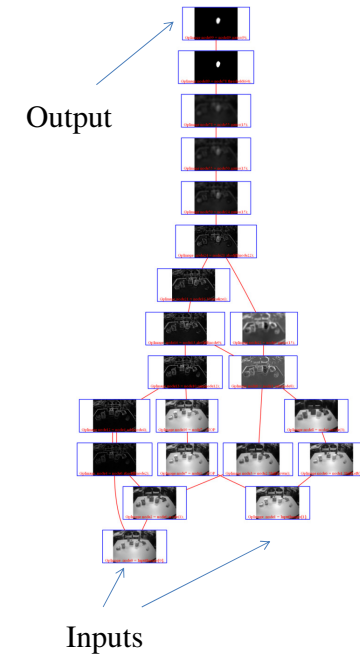
- Fitness = sum of mean square error of pixel values between each input/target



## Evolved Filter code

```
public class MyEvolvedFilter : GpImageFilterRunner
{
    public override GpImage RunFilter() {
        GpImage node0 = InputImages[0];
        GpImage node1 = InputImages[1];
        GpImage node2 = node0.erode(1);
        GpImage node3 = node2.ShiftDown();
        GpImage node4 = node0.absdiff(node2);
        GpImage node5 = node0.avg(node2);
        GpImage node7 = node5; //NOP
        GpImage node8 = node1.erode(3);
        GpImage node9 = node3.sub(node8);
        GpImage node12 = node4.add(node4);
        GpImage node13 = node7.min(node12);
        GpImage node16 = node13.absdiff(node9);
        GpImage node24 = node16.sub(node9);
        GpImage node50 = node24.gauss(15);
        GpImage node78 = node50.gauss(15);
        GpImage node89 = node78.threshold(64);
        GpImage node99 = node89.gauss(13);
        return node99;
    }
    public override void SetUsedInputs() {
        this.UsedInputs.Add(1);
        this.UsedInputs.Add(0);
    }
}
```

## Evolved Filter Dataflow



## Tea-box filter: demonstration



## Application 3: CGP encoded Artificial Neural Networks (CGPANN)

- ❖ CGP has been used to encode both feed-forward ANNs and recursive ANNs. The nodes genes consist of:
  - Connection genes (as usual)
  - Function genes
    - Sigmoid, hyperbolic tangent, Gaussian
  - Weights
    - Each connection gene carries a real-numbered weight
- ❖ Pole balancing, Arm Throwing, Classification
  - Very competitive results with other TWEANN methods (*Khan, Khan and Miller 2010, Turner and Miller 2013*)
- ❖ Breast cancer detection (*Ahmad et al 2012, Turner and Miller 2013*)

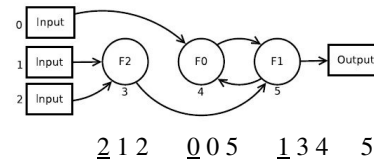
## Cyclic CGP

- ❖ When outputs are allowed to connect to inputs through a clocked delay (flip-flop) it is possible to allow CGP to include feedback.
- ❖ By feeding back outputs generated by CGP to an input, it is possible to get CGP to generate sequences
  - In this way iteration is possible
- ❖ There are a couple of publications using iteration in CGP (*Khan, Khan and Miller 2010, Walker, Liu, Tempesti, Tyrrell 2010, Minarik, Sekanina 2011*)

## Recurrent CGP

- ❖ By allowing nodes to receive inputs from the right CGP can be easily extended to encode recursive computational structures
- ❖ Recurrent CGP Artificial Neural Networks can be explored in this framework
- ❖ Only just begun to be explored (*Turner and Miller 2014*)

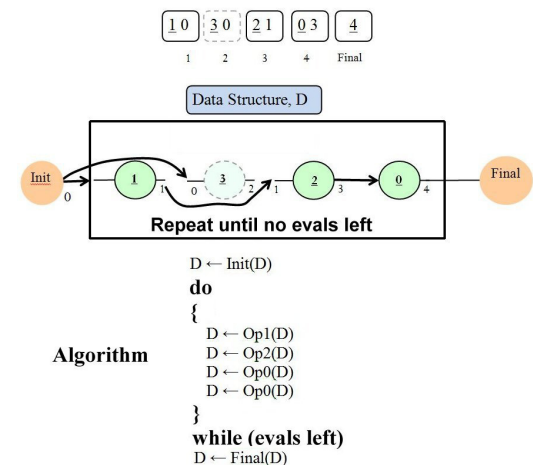
## Recurrent CGP: Details



- ❖ Probability of recursive links controlled by a user-defined parameter recurrent connection probability (rcp)
- ❖ Decoding
  1. set all active nodes to output zero
  2. apply the next set of program inputs
  3. update **all** active nodes **once** from program inputs to program outputs
  4. read the program outputs
  5. repeat from 2 until all program input sets have been applied

## CGP encoded algorithms (Ryser-Welch, Miller, Asta 2015)

- ❖ Algorithms can be encoded in CGP
- ❖ Here is one way
  1. Assume all nodes have an arity of one
  2. Assume a single input
  3. Assume no data passed through graph
  4. CGP defines a variable length ordered set of instructions



## CGP acceleration (*Vassicek and Slany 2012*)

- ❖ CGP decoding step is replaced with native machine code that directly calculates response for a single training vector.
- ❖ Requires little knowledge of assembly language or target machine code.
- ❖ Integration of the machine code compiler requires modifying only a few lines of code
- ❖ Achieves 5 times speedup over standard implementation

## Some Applications of CGP (incomplete)

- ❖ Circuit Design
  - ALU, parallel multipliers, digital filters, analogue circuits, circuit synthesis and optimization
- ❖ Machine Learning
  - Classification
- ❖ Mathematical functions
  - Prime generating polynomials
- ❖ Control systems
  - Maintaining control with faulty sensors, helicopter control, general control, simulated robot controller
- ❖ Image processing
  - Image filters, Mammary Tumour classification, object recognition
- ❖ Robotics
  - gait
- ❖ Bio-informatics
  - Molecular Post-docking filters
- ❖ Artificial Neural Networks
- ❖ Developmental Neural Architectures
  - Wumpus world, checkers, maze solving
- ❖ Evolutionary Art
- ❖ Artificial Life
  - Regenerating 'organisms'
- ❖ Optimization problems
  - Applying CGP to solve GA problems

## CGP Resources I:

<http://www.cartesiangp.co.uk>

- ❖ **Julian Miller:** C implementations of CGP and SMCGP available at

<http://www.cartesiangp.co.uk>

- ❖ **Andrew Turner:** Easy to use, highly extendable, C implementation that includes CGPANNS

<http://www.cgplibrary.co.uk/>

- ❖ **Eduardo Pedroni:** Java implementation with GUI

<https://bitbucket.org/epedroni/jcgp/downloads>

- ❖ **Zdenek Vassicek:** Highly optimised C/Machine Code implementation

<http://www.fit.vutbr.cz/~vasicek/cgp/>

- ❖ Cartesian Genetic Programming book

- Published in 2011 by Springer



## CGP Resources II:

- ❖ **David Oranchak** has implemented CGP in Java.

Documentation is available at

<http://oranchak.com/cgp/doc/>

- ❖ **Brian Goldman** has implemented CGP in Python

<https://github.com/brianwgoldman/ReducingWastedEvaluationsCGP>

- ❖ **Jordan Pollack** has implemented symbolic regression in CGP with Matlab

- See CGP web site

- ❖ **Lawrence Ashmore** has implemented a Java evolutionary art package using CGP

- See CGP web site

## Conclusions

- ❖ Cartesian Genetic Programming is a graph based GP method capable of representing many computational structures
  - programs, circuits, neural networks, systems of equations, algorithms...
- ❖ Genetic encoding is compact, fast, simple and easy to implement and can handle multiple outputs easily.
- ❖ The unique form of genetic redundancy in CGP makes mutational search highly effective
- ❖ The effectiveness of CGP has been compared with many other GP methods and it is highly competitive

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