GECCO 2015
Tutorial on
Evolutionary Multiobjective Optimization

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Inria INVENTORS FOR THE DIGITAL WORLD

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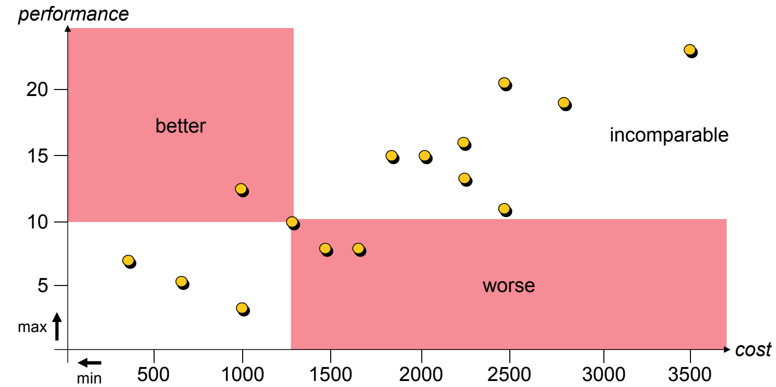
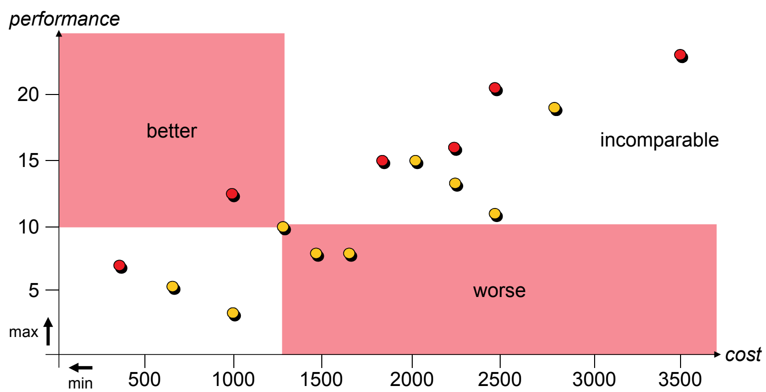
A Brief Introduction to Multiobjective Optimization

Multiobjective Optimization
Multiple objectives that have to be optimized simultaneously



A Brief Introduction to Multiobjective Optimization

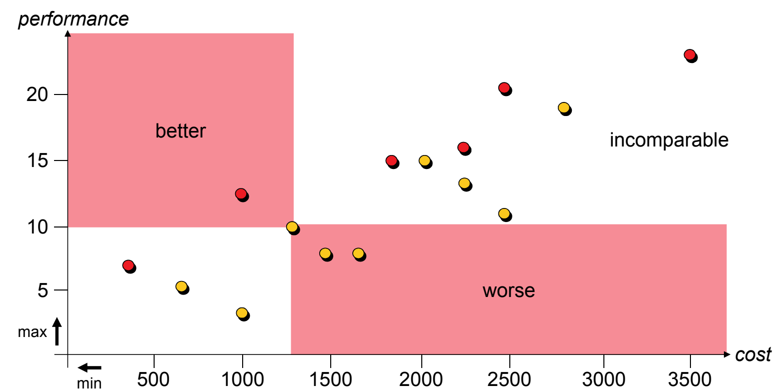
Observations: ❶ there is no single optimal solution, but
❷ some solutions (●) are better than others (●)



A Brief Introduction to Multiobjective Optimization

u *weakly Pareto dominates* v $(u \preccurlyeq_{par} v)$: $\forall 1 \leq i \leq k : f_i(u) \leq f_i(v)$

u *Pareto dominates* v $(u \prec_{par} v)$: $u \preccurlyeq_{par} v \ \wedge \ v \not\preccurlyeq_{par} u$



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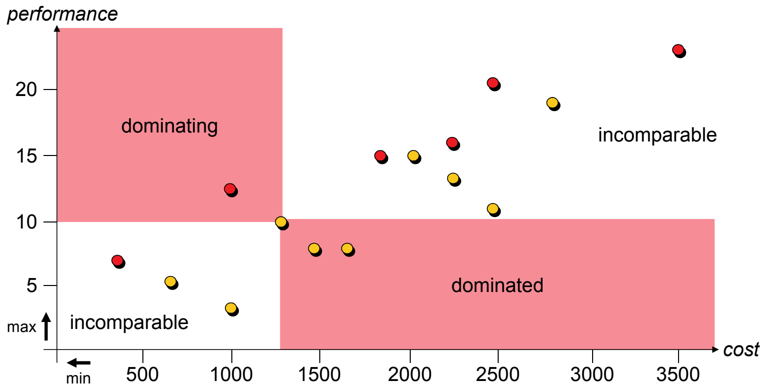
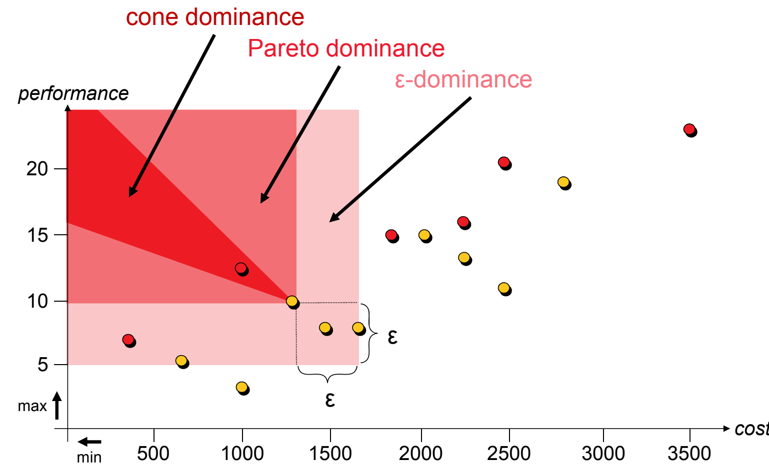
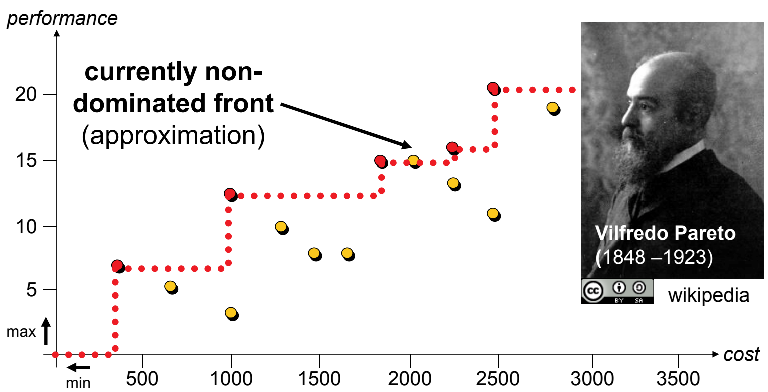


A Brief Introduction to Multiobjective Optimization



A Brief Introduction to Multiobjective Optimization

Pareto set: set of all non-dominated solutions (decision space)
Pareto front: its image in the objective space



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Pareto set: set of all non-dominated solutions (decision space)
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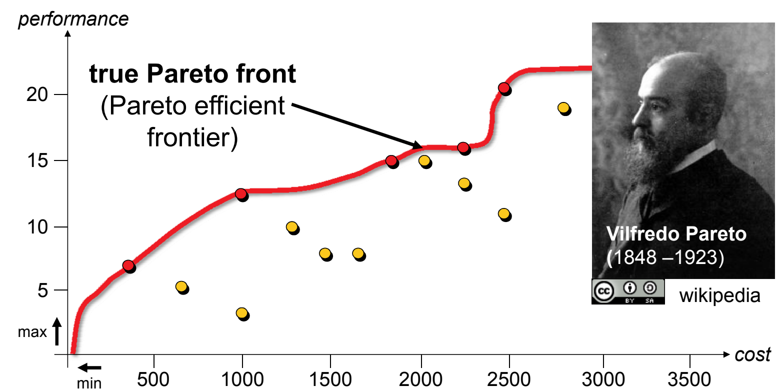

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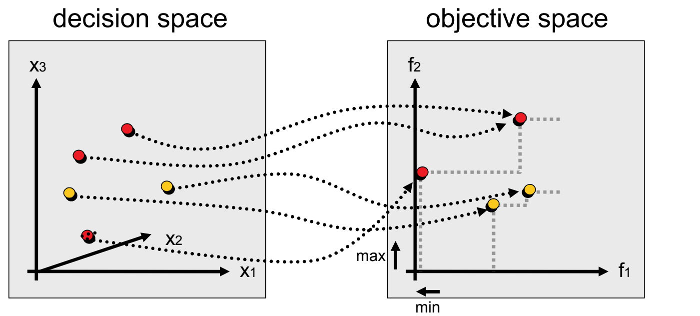
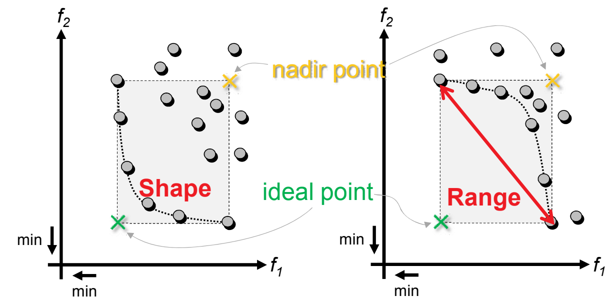


solution of Pareto-optimal set — vector of Pareto-optimal front
non-optimal decision vector — non-optimal objective vector

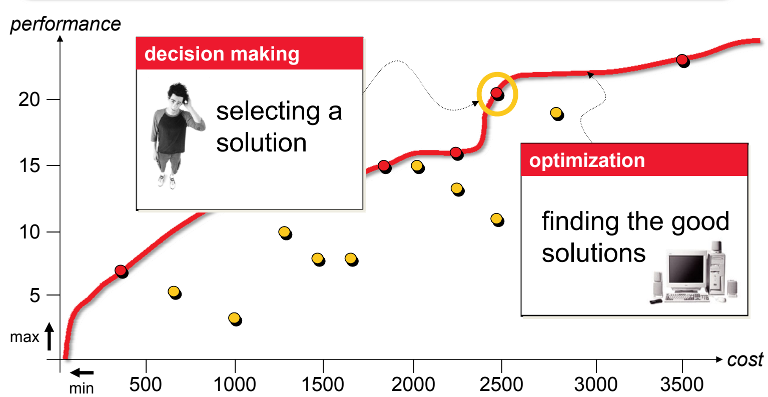
A Brief Introduction to Multiobjective Optimization



ideal point: best values
nadir point: worst values } obtained for *Pareto-optimal* points

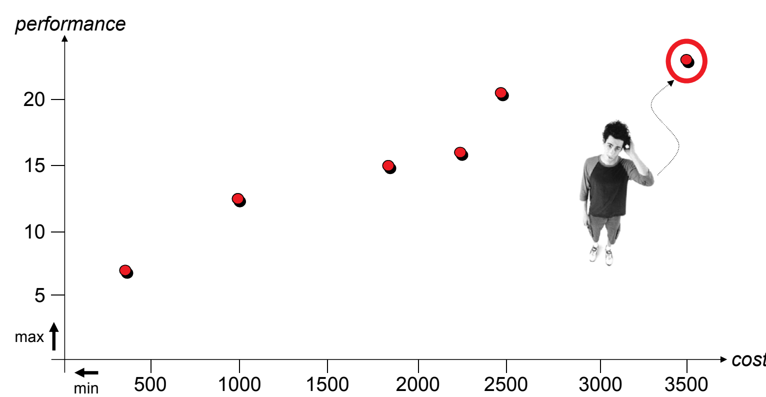
Optimization vs. Decision Making

Multiobjective Optimization
combination of optimization of a set and a decision for a solution



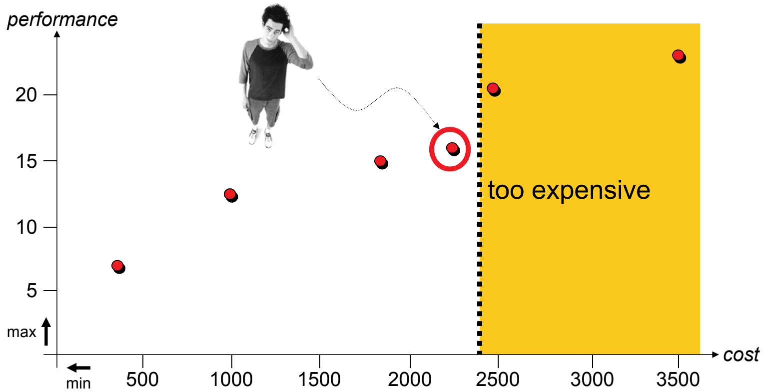
Selecting a Solution: Examples

Possible Approaches: ❶ **ranking:** performance more important than cost



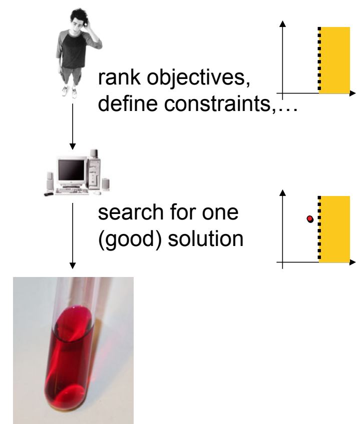
Selecting a Solution: Examples

Possible Approaches:
❶ ranking: performance more important than cost
❷ **constraints:** cost must not exceed 2400



When to Make the Decision

Before Optimization:



When to Make the Decision

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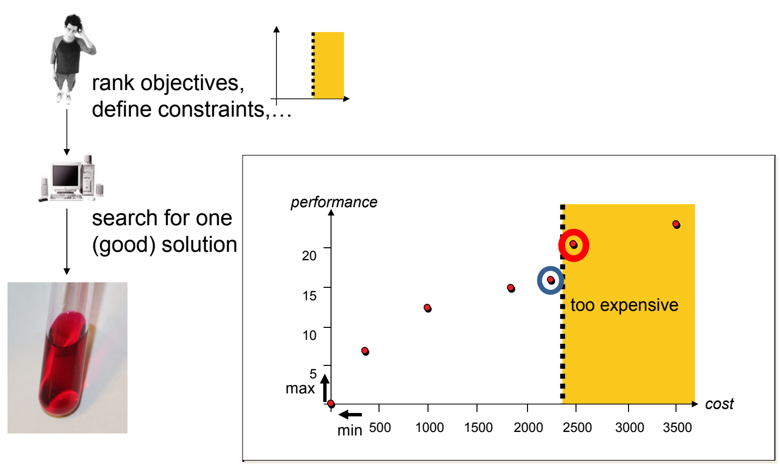
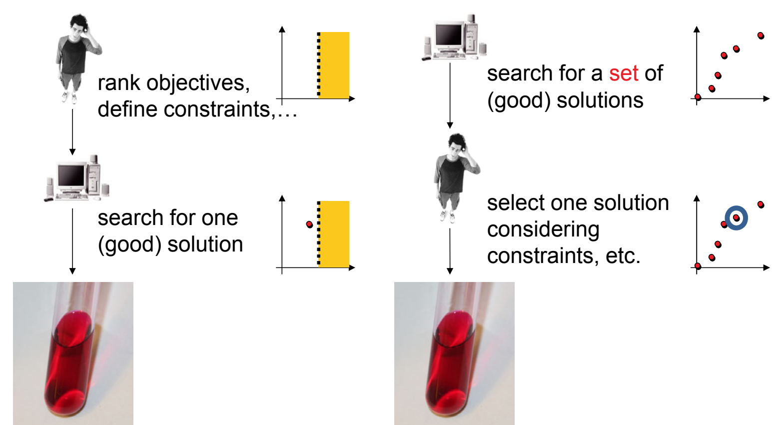


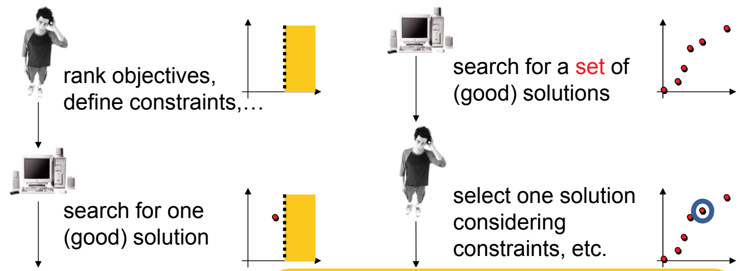
When to Make the Decision

Before Optimization: **After Optimization:**



When to Make the Decision

Before Optimization: / **After Optimization:**



Focus: learning about a problem

- trade-off surface
- interactions among criteria
- structural information
- also: interactive optimization

Two Communities...

MCDM International Society on Multiple Criteria Decision Making

- established field (beginning in 1950s/1960s)
- bi-annual conferences since 1975
- background in economics, math, management and social sciences
- focus on optimization and decision making

EMO

- quite young field (first papers in mid 1980s)
- bi-annual conference since 2001
- background in computer science, applied math and engineering
- focus on optimization algorithms

...Slowly Merge Into One

MCDM International Society on Multiple Criteria Decision Making EMO

- MCDM track at EMO conference since 2009
- special sessions on EMO at the MCDM conference since 2008
- joint Dagstuhl seminars since 2004

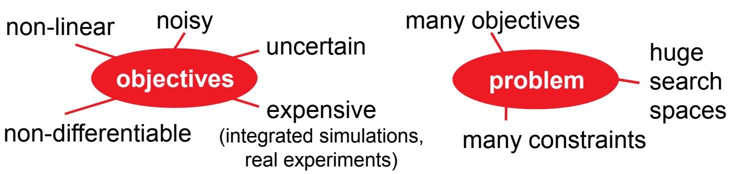
One of the Main Differences

Blackbox optimization

$$x \in X \longrightarrow \boxed{f} \longrightarrow (f_1(x), \ldots, f_k(x))$$

only mild assumptions

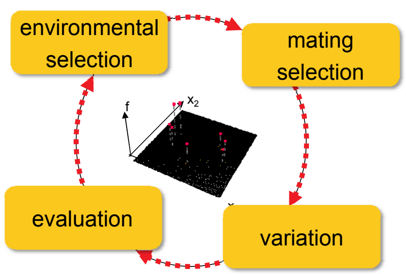
→ EMO therefore well-suited for real-world engineering problems

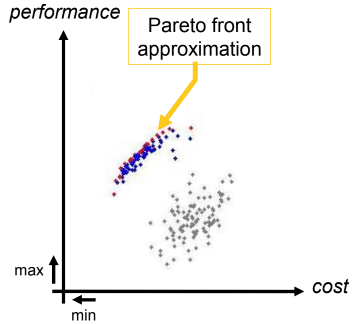


The Other Main Difference

Evolutionary Multiobjective Optimization

- set-based algorithms
- therefore possible to approximate the Pareto front in one run

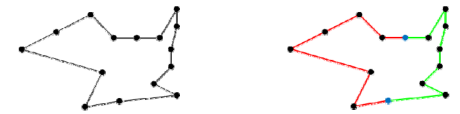




Multiobjectivization

Some problems are easier to solve in a multiobjective scenario

example: TSP
[Knowles et al. 2001]



$\pi \in S_n \rightarrow f(\pi)$ $\pi \in S_n \rightarrow (f_1(\pi, a, b), f_2(\pi, a, b))$

Multiobjectivization

by addition of new "helper objectives" [Jensen 2004]

job-shop scheduling [Jensen 2004], frame structural design [Greiner et al. 2007], VRP [Watanabe and Sakakibara 2007], ...

by decomposition of the single objective

TSP [Knowles et al. 2001], minimum spanning trees [Neumann and Wegener 2006], protein structure prediction [Handl et al. 2008a], ...

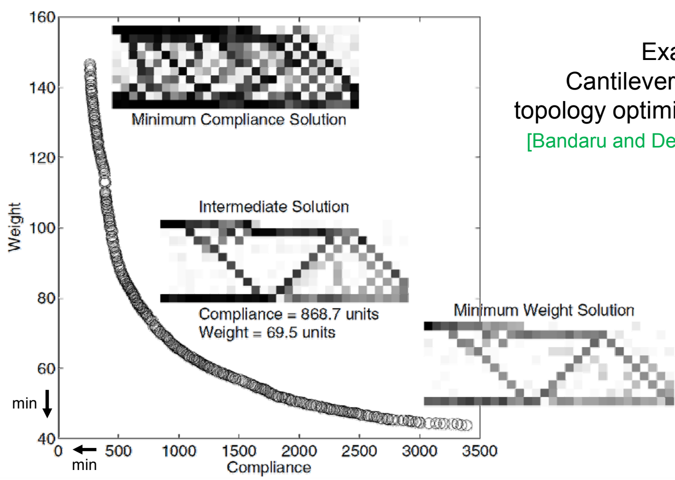
also backed up by theory e.g. [Brockhoff et al. 2009, Handl et al. 2008b]

related to *constrained* and *multimodal* single-objective optimization

see also this recent overview: [Segura et al. 2013]

Innovization

Often innovative design principles among solutions are found

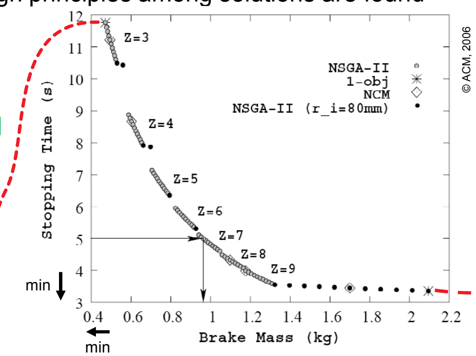


Example:
Cantilever beam topology optimization
[Bandaru and Deb 2015]

Innovization

Often innovative design principles among solutions are found

Example:
Clutch brake design
[Deb and Srinivasan 2006]



Solution	x_1	x_2	x_3	x_4	x_5	f_1	f_2
Min. f_1	70	90	1.5	1000	3	0.4704	11.7617
Min. f_2	80	110	1.5	1000	9	2.0948	3.3505

Innovization

Often innovative design principles among solutions are found

Innovization [Deb and Srinivasan 2006]

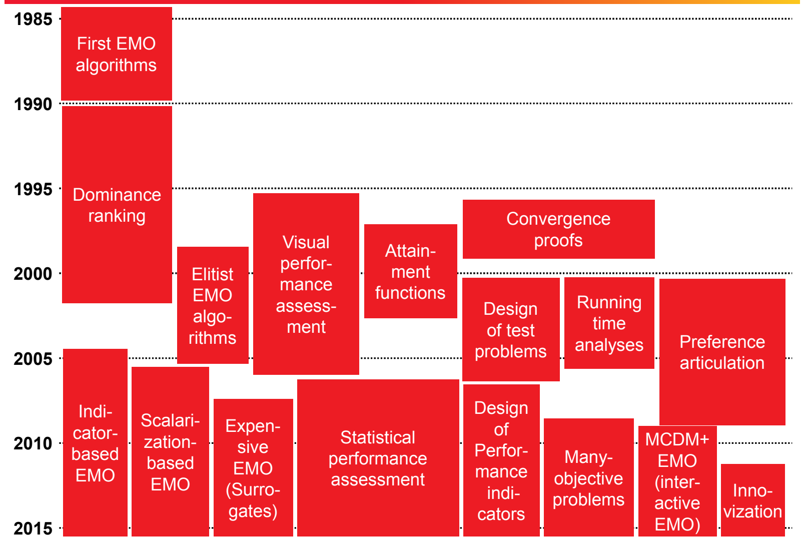
= using machine learning techniques to find new and innovative design principles among solution sets

= learning from/about a multi-objective optimization problem

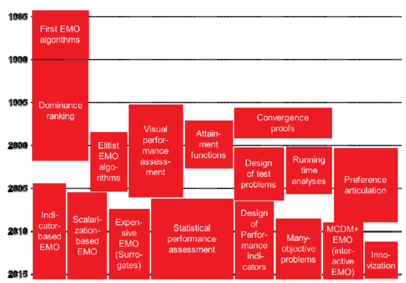
Other examples:

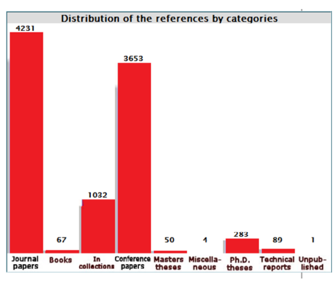
- SOM for supersonic wing design [Obayashi and Sasaki 2003]
- Biclustering for processor design and knapsack [Ulrich et al. 2007]
- Successful case studies in engineering (noise barrier design, polymer extrusion, friction stir welding) [Deb et al. 2014]

The History of EMO At A Glance

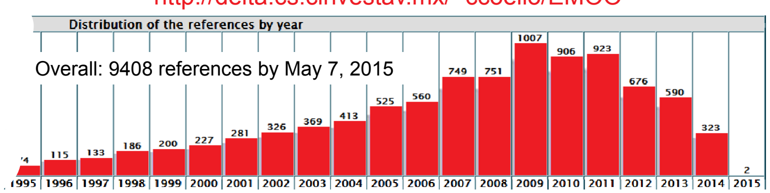


The History of EMO At A Glance

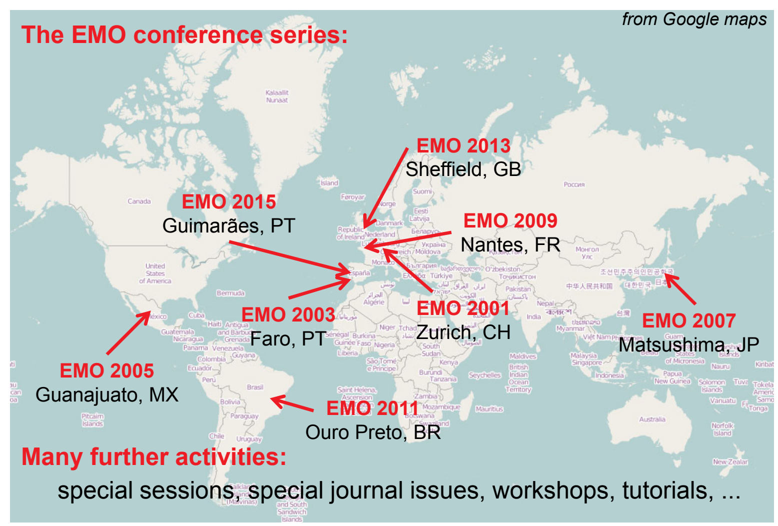




http://delta.cs.cinvestav.mx/~ccoello/EMOO



The EMO Community



Overview

The Big Picture

Basic Principles of Multiobjective Optimization

- algorithm design principles and concepts
- performance assessment

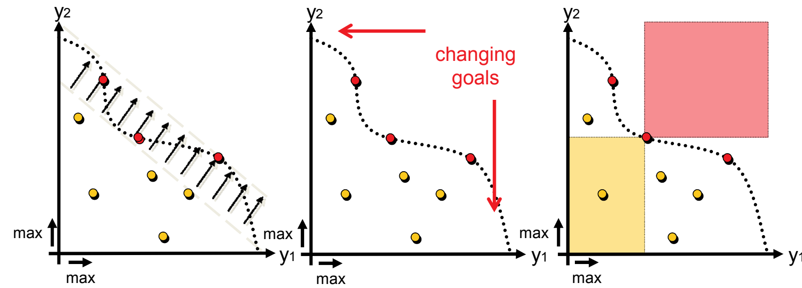
Selected Advanced Concepts

- preference articulation
- surrogate-based EMO

A Few Examples From Practice

Fitness Assignment: Principal Approaches

aggregation-based	criterion-based	dominance-based
problem decomposition (multiple single-objective optimization problems)	*VEGA*	*SPEA2, NSGA-II "modern" EMOA*



solution-oriented scaling-dependent ◄····► set-oriented scaling-independent

Solution-Oriented Problem Transformations

multiple objectives $(f_1(x), f_2(x), \dots, f_k(x))$ → [transformation] (parameters ↓) → single objective s(x)

A scalarizing function s is a function $s : Z \to \mathbb{R}$ that maps each objective vector $u = (u_1, \dots, u_n) \in Z$ to a real value $s(u) \in \mathbb{R}$

Solution-Oriented Problem Transformations

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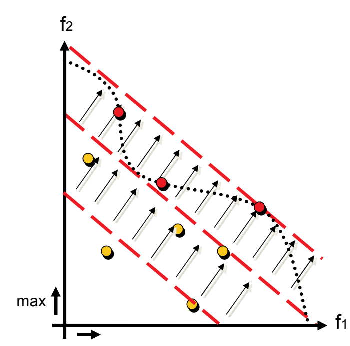


Example 1: weighted sum approach

$(w_1, w_2, \dots, w_k)$ ↓

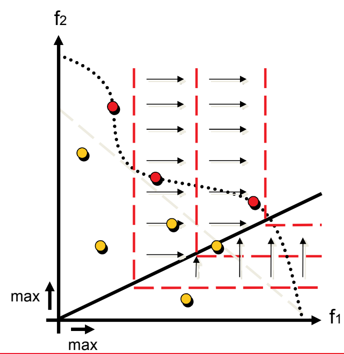
→ $y = w_1 y_1 + \dots + w_k y_k$ →

Disadvantage: not all Pareto-optimal solutions can be found if the front is not convex

Solution-Oriented Problem Transformations

multiple objectives $(f_1(x), f_2(x), \ldots, f_k(x))$ → transformation → single objective $s(x)$

parameters ↓ transformation



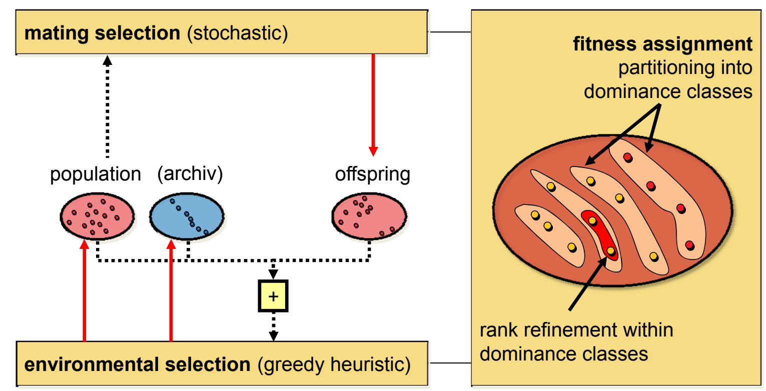
Example 2: weighted Tchebycheff

$(\lambda_1, \lambda_2, \ldots, \lambda_k)$

$$y = \max_i \mid \lambda_i(u_i - z_i) \mid$$

Several other scalarizing functions are known, see e.g. [Miettenen 1999]

General Scheme of Most Set-Oriented EMO



Ranking of the Population Using Dominance

... goes back to a proposal by David Goldberg in 1989.
... is based on pairwise comparisons of the individuals only.

- **dominance rank:** by how many individuals is an individual dominated?
  MOGA, NPGA
- **dominance count:** how many individuals does an individual dominate?
  SPEA, SPEA2
- **dominance depth:** at which front is an individual located?
  NSGA, NSGA-II, most of the recently proposed algorithms

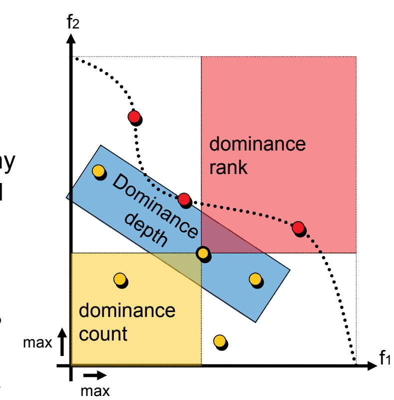
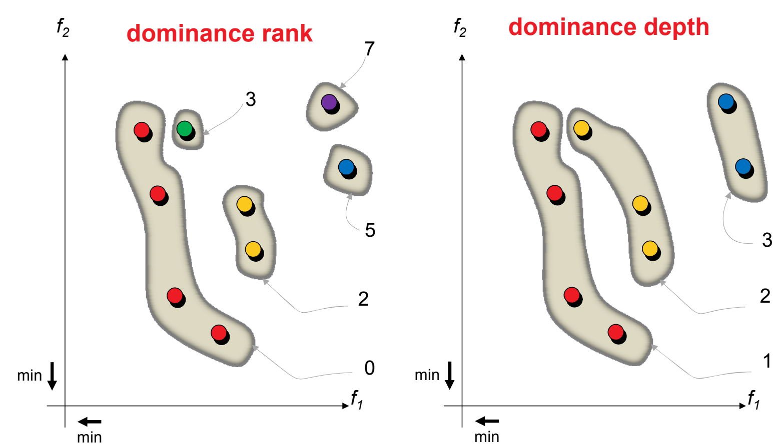


Illustration of Dominance-Based Partitioning

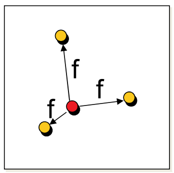


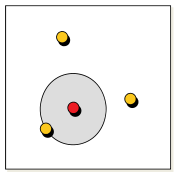
Refinement of Dominance Rankings

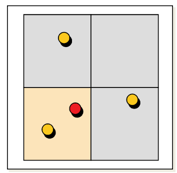
Goal: rank incomparable solutions within a dominance class

❶ Diversity information

Kernel method	**k-th nearest neighbor**	**Histogram method**
diversity = function of the distances	diversity = function of distance to k-th nearest neighbor	diversity = number of elements within box(es)





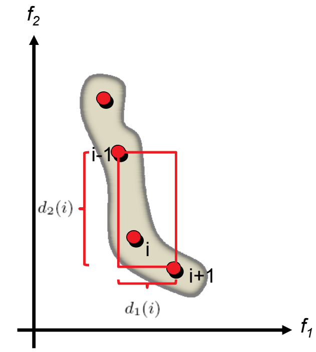


❷ (Contribution to a) quality indicator

Example: NSGA-II Diversity Preservation

Crowding Distance (CD)

- sort solutions with regard to each objective
- assign CD maximum value to extremal objective vectors
- compute CD based on the distance to the neighbors in each objective

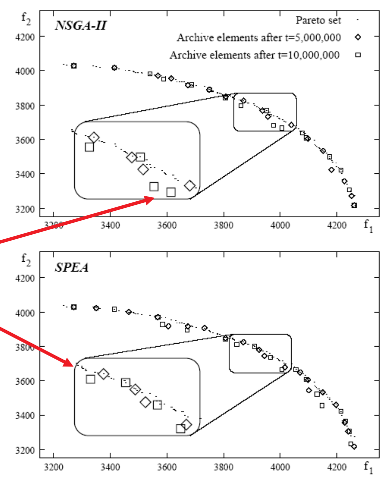


$$\mathrm{CD}(i) = \frac{d_1(i)}{f_{1,\max} - f_{1,\min}} + \cdots + \frac{d_m(i)}{f_{m,\max} - f_{m,\min}}$$

SPEA2 and NSGA-II: Deteriorative Cycles

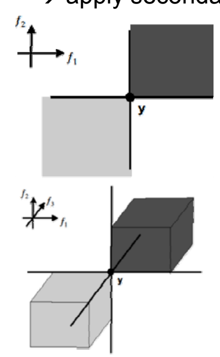
Selection in SPEA2 and NSGA-II can result in *deteriorative* cycles

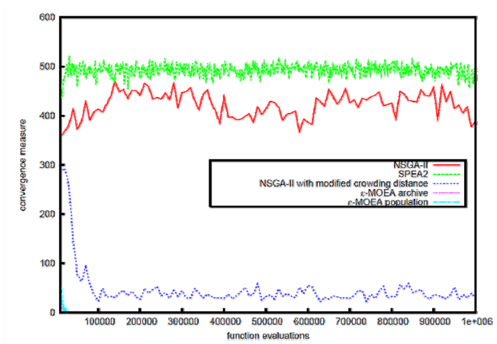
non-dominated solutions already found can be lost



Remark: Many-Objective Optimization

- high number of objectives
  - → percentage of non-dominated solutions within a random sample quickly approaches 100 %
  - → optimization is mainly guided by diversity criterion
  - → apply secondary criterion compliant with dominance relation





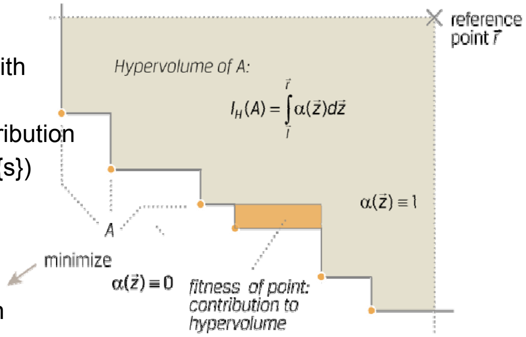
Hypervolume-Based Selection

Latest Approach (SMS-EMOA, MO-CMA-ES, HypE, …)

use hypervolume indicator to guide the search: refines dominance

Main idea

Delete solutions with the smallest hypervolume contribution $d(s) = I_H(P)-I_H(P / \{s\})$ iteratively



But: can also result in cycles if reference point is not constant [Judt et al. 2011]

and is expensive to compute exactly [Bringmann and Friedrich 2009]

Indicator-Based Selection

- Concept can be generalized to any quality indicator

A (unary) quality indicator I is a function $I : \Psi = 2^X \mapsto \mathbb{R}$ that assigns a Pareto set approximation a real value.

Multiobjective Problem $\xrightarrow{\text{Indicator}}$ Single-objective Problem

- for example: R2-indicator [Brockhoff et al. 2012], [Trautmann et al. 2013], [Díaz-Manríquez et al. 2013]

- Generalizable also to contribution to larger sets
  HypE [Bader and Zitzler 2011]: Hypervolume sampling + contribution if more than 1 (random) solution deleted

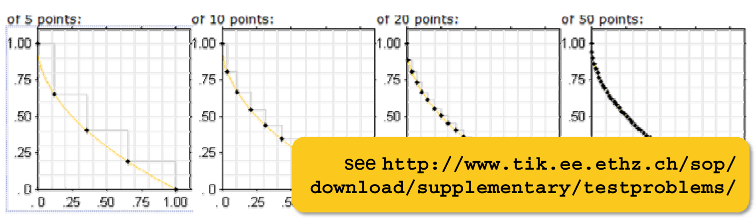
The Optimization Goal in Indicator-Based EMO

When the goal is to maximize a unary indicator…

- we have a single-objective problem on sets
- but what is the optimum?
- important: population size μ plays a role!

Optimal μ-Distribution:

A set of μ solutions that maximizes a certain unary indicator I among all sets of μ solutions is called optimal μ-distribution for I. [Auger et al. 2009a]



see http://www.tik.ee.ethz.ch/sop/download/supplementary/testproblems/

Optimal μ-Distributions for the Hypervolume

Hypervolume indicator refines dominance relation

$\Longrightarrow$ most results on optimal μ-distributions for hypervolume

Optimal μ-Distributions (example results)

[Auger et al. 2009a]:

- contain equally spaced points iff front is linear
- density of points $\propto \sqrt{-f'(x)}$ with f' the slope of the front

[Friedrich et al. 2011]:

optimal μ-distributions for the hypervolume correspond to ε-approximations of the front

OPT	$1+\frac{\log(\min\{A/a,\,B/b\})}{n}$
HYP	$1+\frac{\sqrt{A/a}+\sqrt{B/b}}{n-4}$
logHYP	$1+\frac{\sqrt{\log(A/a)\,\log(B/b)}}{n-2}$

! (probably) does not hold for > 2 objectives

Indicator-Based EMO

Open Questions:

- How do the optimal μ-distributions look like for >2 objectives?
- how to compute certain indicators quickly in practice?
  - several recent improvements for the hypervolume indicator [Yildiz and Suri 2012], [Bringmann 2012], [Bringmann 2013]
- how to do indicator-based subset selection quickly?
  - also here several recent improvements [Kuhn et al. 2014], [Bringmann et al. 2014], [Guerreiro et al. 2015]
- what is the best strategy for the subset selection?

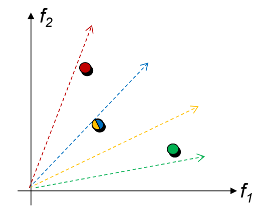
further open questions on indicator-based EMO available at
`http://simco.gforge.inria.fr/doku.php?id=openproblems`

Decomposition-Based Selection: MOEA/D

MOEA/D: Multiobjective Evolutionary Algorithm Based on Decomposition [Zhang and Li 2007]

Ideas:

- optimize N scalarizing functions in parallel
- use best solutions of neighbor subproblems for mating
- keep the best solution for each scalarizing function
- update neighbors
- use external archive for non-dominated solutions
- several variants and enhancements



Remark: Variation in EMO

- at first sight not different from single-objective optimization
- most research on selection mechanisms (until now)
- but: convergence to a set ≠ convergence to a point

Open Question:

- how to achieve fast convergence to a *set*?

Related work:

- set-based gradient of the HV [Emmerich et al. 2007]
- multiobjective CMA-ES [Igel et al. 2007] [Voß et al. 2010]
- RM-MEDA [Zhang et al. 2008]
- set-based variation [Bader et al. 2009]
- set-based fitness landscapes [Verel et al. 2011]
- offline and online configuration based on libraries of variation operators [Bezerra et al. 2015] [Hadka and Reed 2013]

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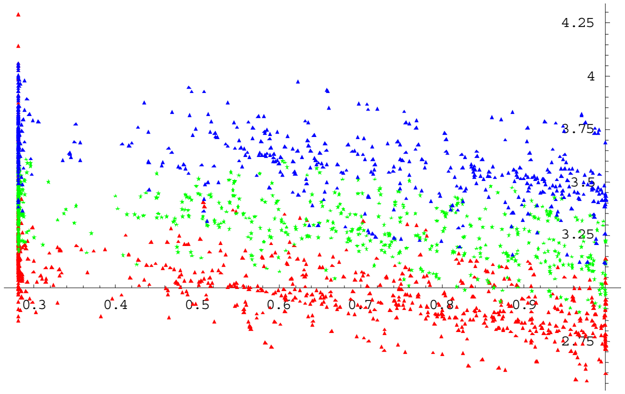
A Few Examples From Practice

Once Upon a Time...

... multiobjective EAs were mainly compared visually:



ZDT6 benchmark problem: IBEA, SPEA2, NSGA-II

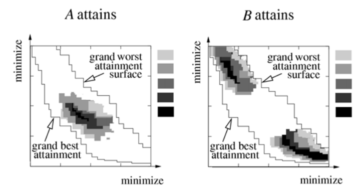
Two Approaches for Empirical Studies

Attainment function approach

- applies statistical tests directly to the approximation set
- detailed information about how and where performance differences occur

Quality indicator approach

- reduces each approximation set to a single quality value
- applies statistical tests to the quality values

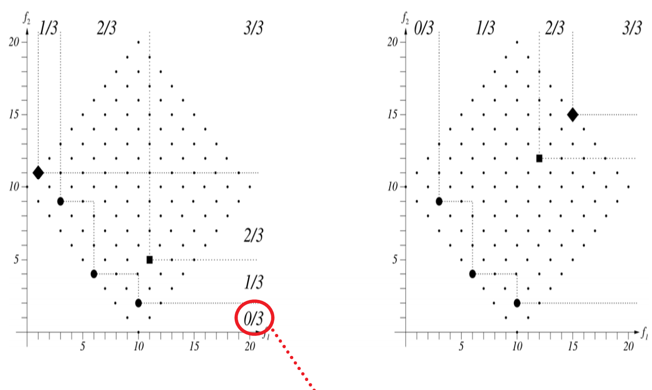


Indicator	A	B
Hypervolume indicator	6.3431	7.1924
ϵ-indicator	1.2090	0.12722
R_2 indicator	0.2434	0.1643
R_3 indicator	0.6454	0.3475

see e.g. [Zitzler et al. 2003]

Empirical Attainment Functions

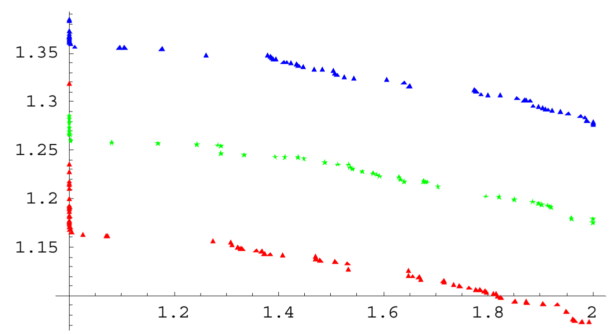
three runs of two multiobjective optimizers



frequency of attaining regions

Attainment Plots

50% attainment surface for IBEA, SPEA2, NSGA2 (ZDT6)



latest implementation online at
http://eden.dei.uc.pt/~cmfonsec/software.html
see [Fonseca et al. 2011]

Quality Indicator Approach

Idea:

- transfer multiobjective problem into a set problem
- define an objective function ("quality indicator") on sets
- use the resulting total (pre-)order (on the quality values)

Question:

Can any total (pre-)order be used or are there any requirements concerning the resulting preference relation?

$\Rightarrow$ Underlying dominance relation should be reflected!

$$A \preceq B :\Leftrightarrow \forall_{y \in B} \exists_{x \in A} x \preceq_{par} y$$

Refinements and Weak Refinements

❶ $\overset{\text{ref}}{\preccurlyeq}$ **refines** a preference relation $\preccurlyeq$ iff

$$A \preccurlyeq B \wedge B \not\preccurlyeq A \Rightarrow A \overset{\text{ref}}{\preccurlyeq} B \wedge B \overset{\text{ref}}{\not\preccurlyeq} A \qquad \textit{(better} \Rightarrow \textit{better)}$$

$\Rightarrow$ fulfills requirement

❷ $\overset{\text{ref}}{\preceq}$ **weakly refines** a preference relation $\preceq$ iff

$$A \preceq B \wedge B \not\preceq A \Rightarrow A \overset{\text{ref}}{\preceq} B \qquad \textit{(better} \Rightarrow \textit{weakly better)}$$

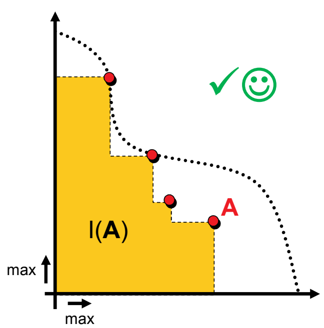
$\Rightarrow$ does not fulfill requirement, but $\overset{\text{ref}}{\preceq}$ does not contradict $\preccurlyeq$

! sought are total refinements… [Zitzler et al. 2010]

Example: Refinements Using Indicators

$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A) \geq I(B)$

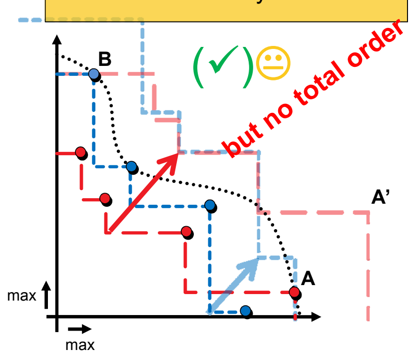
I(A) = volume of the weakly dominated area in objective space



unary hypervolume indicator

$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A,B) \leq I(B,A)$

I(A,B) = how much needs A to be moved to weakly dominate B

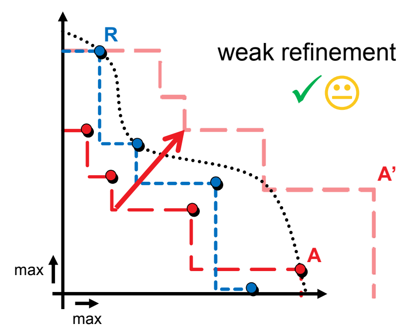


binary epsilon indicator

Example: Weak Refinement / No Refinement

$A \overset{\text{ref}}{\preceq} B :\Leftrightarrow I(A,R) \leq I(B,R)$

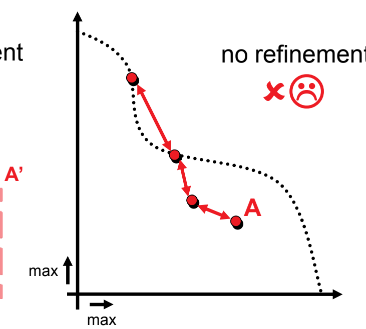
I(A,R) = how much needs A to be moved to weakly dominate R



unary epsilon indicator

$A \overset{\text{ref}}{\preceq} B :\Leftrightarrow I(A) \leq I(B)$

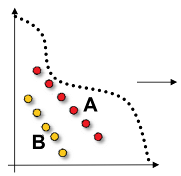
I(A) = variance of pairwise distances



unary diversity indicator

Quality Indicator Approach

Goal: compare two Pareto set approximations A and B



	A	B
hypervolume	*432.34*	*420.13*
distance	*0.3308*	*0.4532*
diversity	*0.3637*	*0.3463*
spread	*0.3622*	*0.3601*
cardinality	*6*	*5*

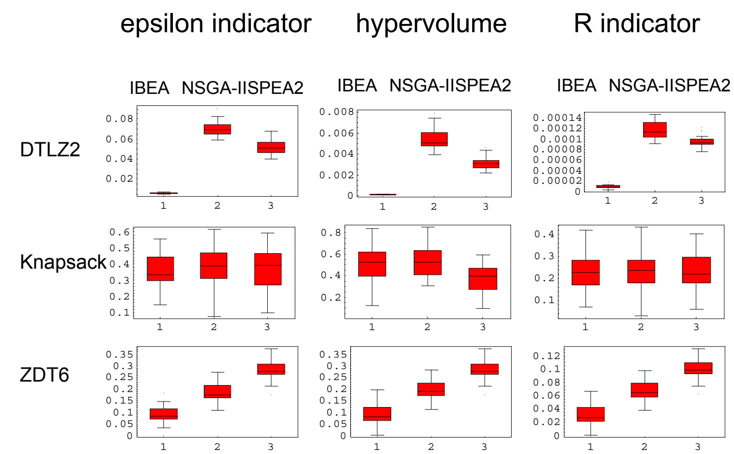
→ "**A** better"

Comparison method C = quality measure(s) + Boolean function

A, B $\xrightarrow{\text{quality measure}}$ $\mathbb{R}^n$ $\xrightarrow{\text{Boolean function}}$ statement

reduction **interpretation**

Example: Box Plots



Statistical Assessment (Kruskal Test)

ZDT6
Epsilon

is better than

	IBEA	NSGA2	SPEA2
IBEA		~0 ☺	~0 ☺
NSGA2	1		~0 ☺
SPEA2	1	1	

Overall p-value = 6.22079e-17.
Null hypothesis rejected (alpha 0.05)

DTLZ2
R

is better than

	IBEA	NSGA2	SPEA2
IBEA		~0 ☺	~0 ☺
NSGA2	1		1
SPEA2	1	~0 ☺	

Overall p-value = 7.86834e-17.
Null hypothesis rejected (alpha 0.05)

Knapsack/Hypervolume: H_0 = No significance of any differences

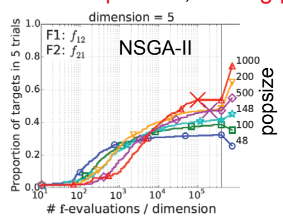
Set Quality Indicators

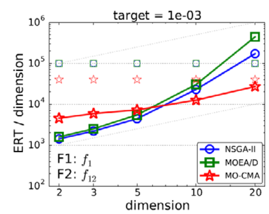
Open Questions:

- are there other unary indicators that are (weak) refinements?
- how to compute indicators efficiently (enough for practice)?
  - especially for >3 objective functions
- how to achieve good indicator values?

Automated Benchmarking

- State-of-the-art in single-objective optimization: Blackbox Optimization Benchmarking (BBOB) with Coco platform
- Multiobjective BBOB on the way to release
- Focus on target-based runlengths
  - gives (nearly) anytime, interpretable results
  - defines problem=(test function instance, single-objective goal e.g. min. indicator difference to reference set, target precision)
  - reports Expected Runtimes (ERT) to reach target precision
- provides data profiles, scaling plots, etc.





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A Few Examples From Practice

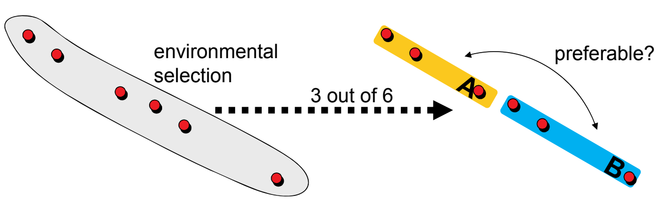
Articulating User Preferences During Search

What we thought: EMO is preference-less

> **Search before decision making:** Optimization is performed without any preference information given. The result of the search process is a set of (ideally Pareto-optimal) candidate solutions from which the final choice is made by the DM.

[Zitzler 1999]

What we learnt: EMO just uses weaker preference information

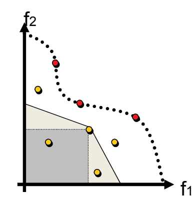


Incorporation of Preferences During Search

Nevertheless...
- the more (known) preferences incorporated the better
- in particular if search space is large

[Branke and Deb 2004] [Branke 2008] [Bechikh et al. 2015]

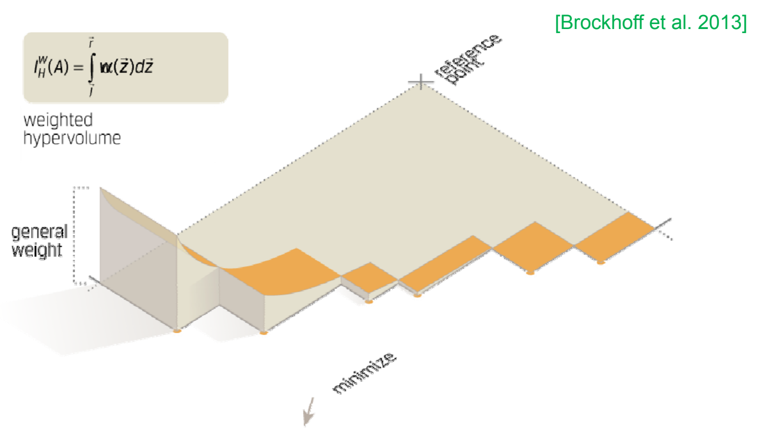
❶ **Refine/modify dominance relation, e.g.:**
- using goals, priorities, constraints [Fonseca and Fleming 1998a,b]
- using different types of dominance cones [Branke and Deb 2004]



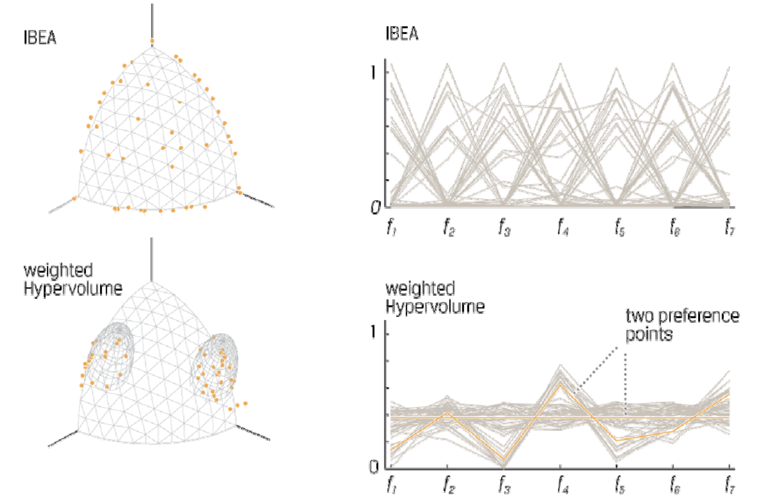
❷ **Use quality indicators, e.g.:**
- based on reference points and directions [Deb and Sundar 2006, Deb and Kumar 2007]
- based on the hypervolume indicator [Brockhoff et al. 2013] [Wagner and Trautmann 2010]
- based on the R2 indicator [Trautmann et al. 2013]

Example: Weighted Hypervolume Indicator

[Brockhoff et al. 2013]



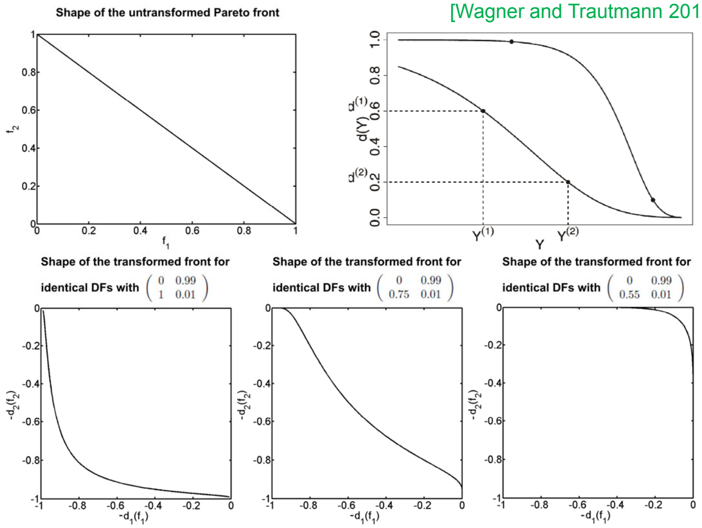
Weighted Hypervolume in Practice



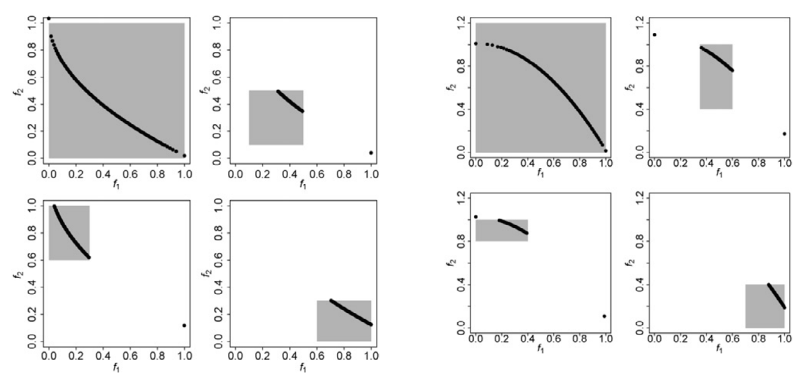
[Auger et al. 2009b]

Example: Desirability Function (DF)-SMS-EMOA

[Wagner and Trautmann 2010]



DF-SMS-EMOA in Practice



Example: R2-EMOA

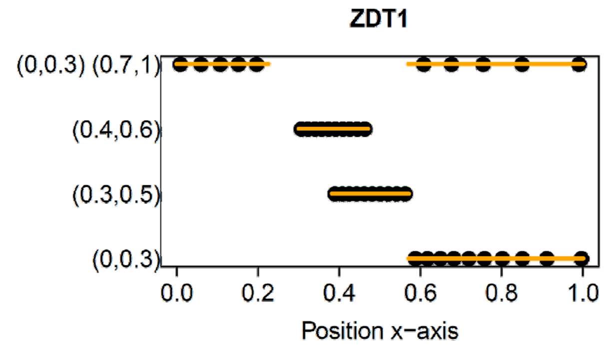
Concept
Integration of preferences by varying the scalarizing functions

Position of ideal point

Example: R2-EMOA

Concept
Integration of preferences by varying the scalarizing functions

Restriction of the weight space



Interactive Approaches

Successive Preference Articulation = Interactive EMO

- recent interest of both EMO and MCDM community
- important in practice

Examples
- first interactive EMO: [Tanino et al. 1993]
- good overview: [Jaszkiewicz and Branke 2008]
- more recent work: [Brockhoff et al. 2014] [Branke et al. 2014]

Issues/Open Questions
- realistic scenarios/ value functions
- evaluation of interactive algorithms [López-Ibáñez and Knowles 2015]

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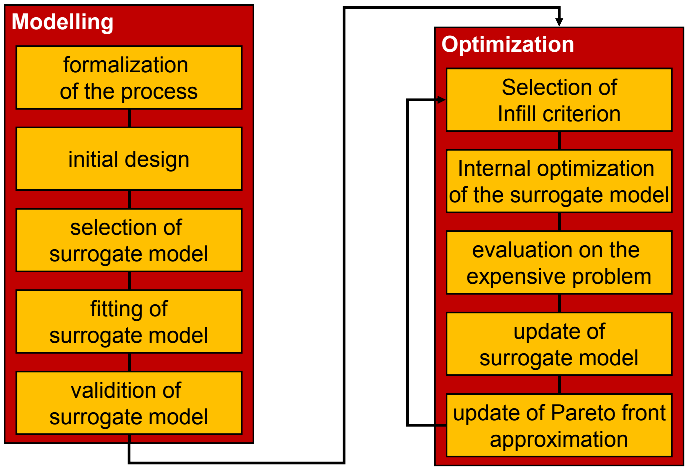
Surrogate-Based EMO

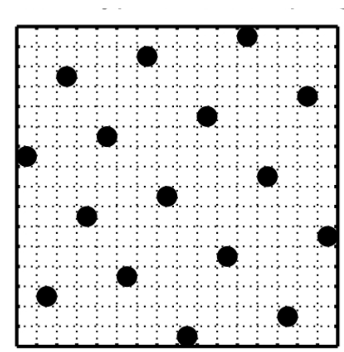
EMO + modeling and sequential experimental design



Initial design

Latin Hypercube Sampling (LHS)

- Space-filling coverage of the decision space
- Maximum resolution for each parameter

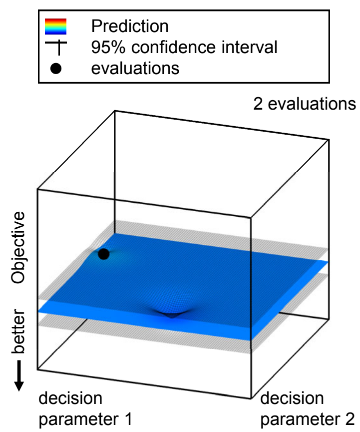


Selection of Surrogate Model

Design and Analysis of Computer Experiments (DACE)

[Sacks et al. 1989]

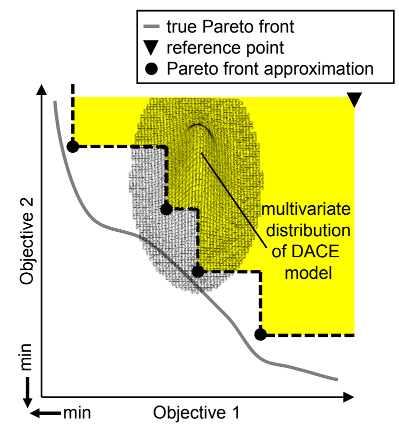
- based on kriging models
- local modeling of the available evaluations
- possibility of adaptive refinement



Selection of infill criterion

Aim

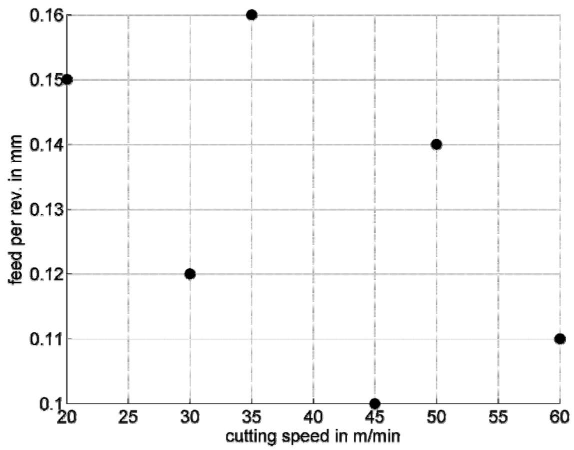
- refinement of the Pareto front approximation
- maximization of the dominated hypervolume
- improvements through experiments in the currently nondominated area
- see [Wagner et al. 2010] for a survey and first theoretical results



Practical application (drilling of Inconel 708)

Initial design

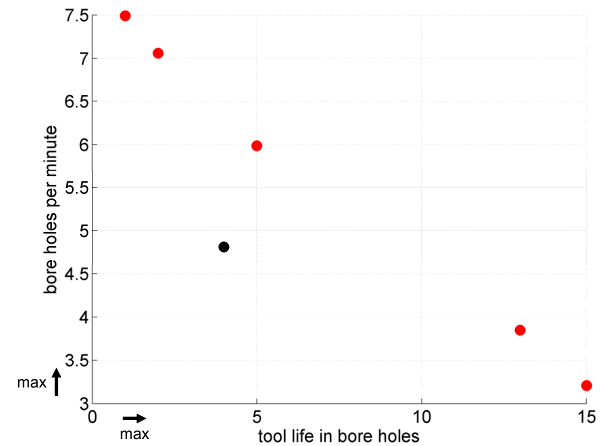
[Zhang et al. 2012]



Practical application (drilling of Inconel 708)

Initial Pareto front approximation

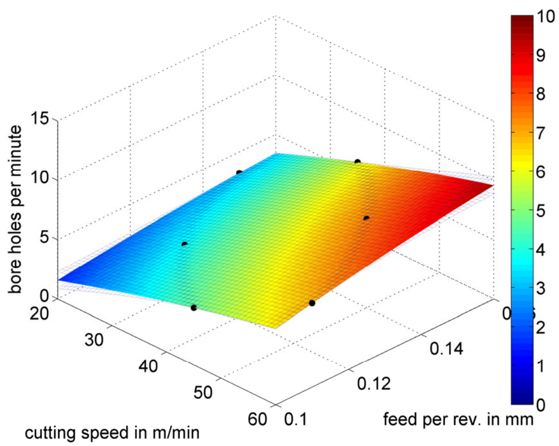
[Zhang et al. 2012]



Practical application (drilling of Inconel 708)

Initial model for objective 1 (productivity)

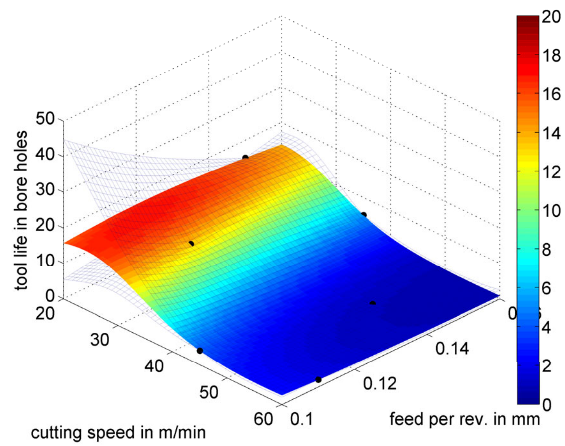
[Zhang et al. 2012]



Practical application (drilling of Inconel 708)

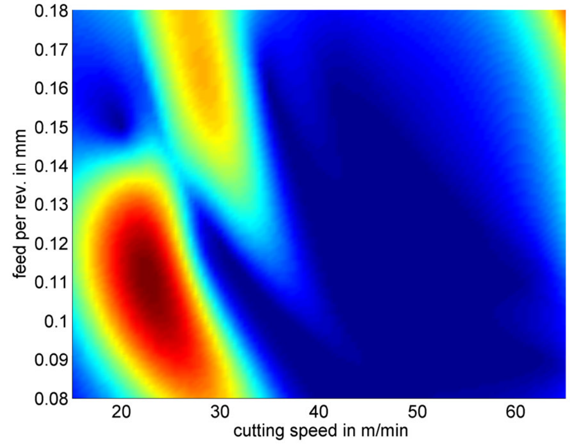
Initial model for objective 2 (tool life)

[Zhang et al. 2012]



Practical application (drilling of Inconel 708)

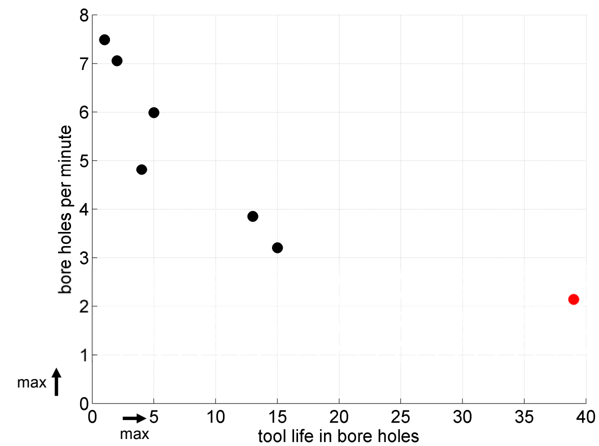
Optimization of Infill criterion



Practical application (drilling of Inconel 708)

Updated Pareto front approximation

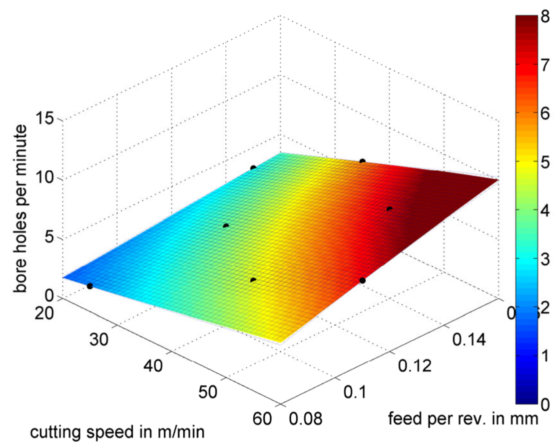
[Zhang et al. 2012]



Practical application (drilling of Inconel 708)

Updated model for objective 1 (productivity)

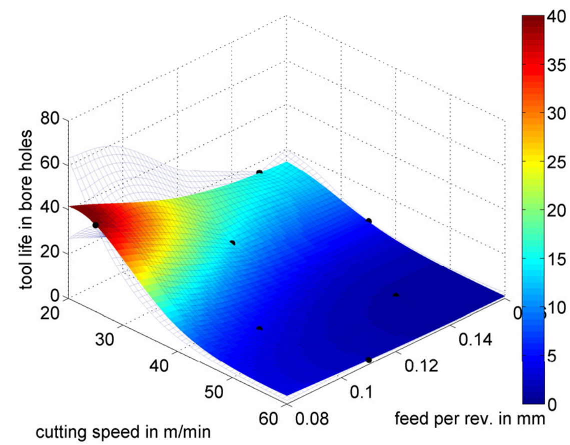
[Zhang et al. 2012]



Practical application (drilling of Inconel 708)

Updated model for objective 2 (tool life)

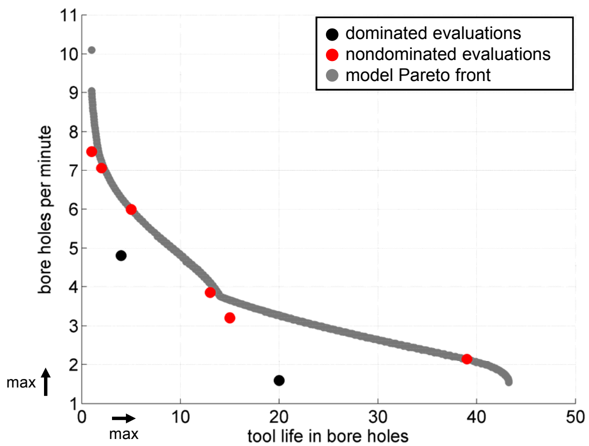
[Zhang et al. 2012]



Practical application (drilling of Inconel 708)

Final Pareto front approximation

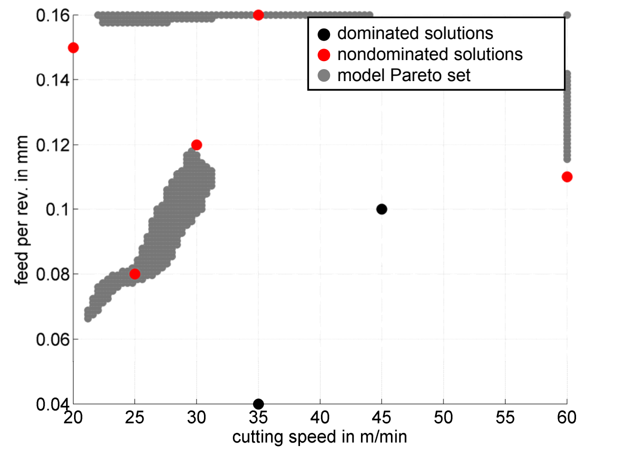
[Zhang et al. 2012]



Practical application (drilling of Inconel 708)

Final Pareto set approximation (Innovization)

[Zhang et al. 2012]



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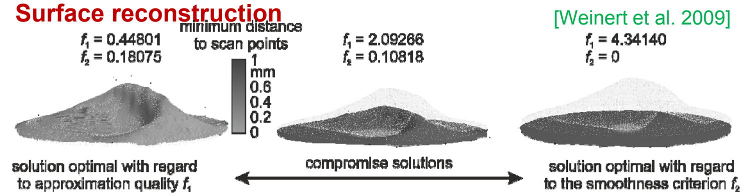
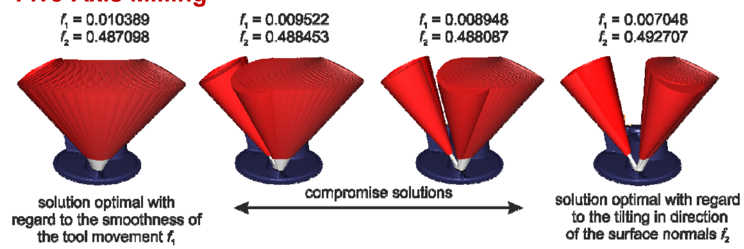
Applications of EMO

Surface reconstruction

[Weinert et al. 2009]



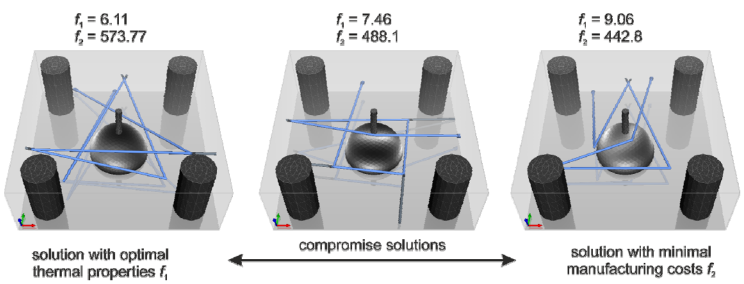
Five-Axis Milling



Applications of EMO

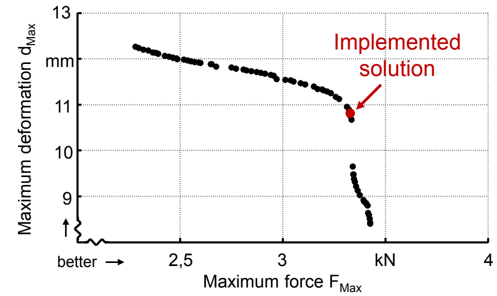
Mold Temperature Cooling Systems

[Weinert et al. 2009]



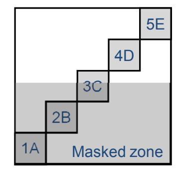
Applications of EMO

Hot Compaction of Thermoplastic Composites



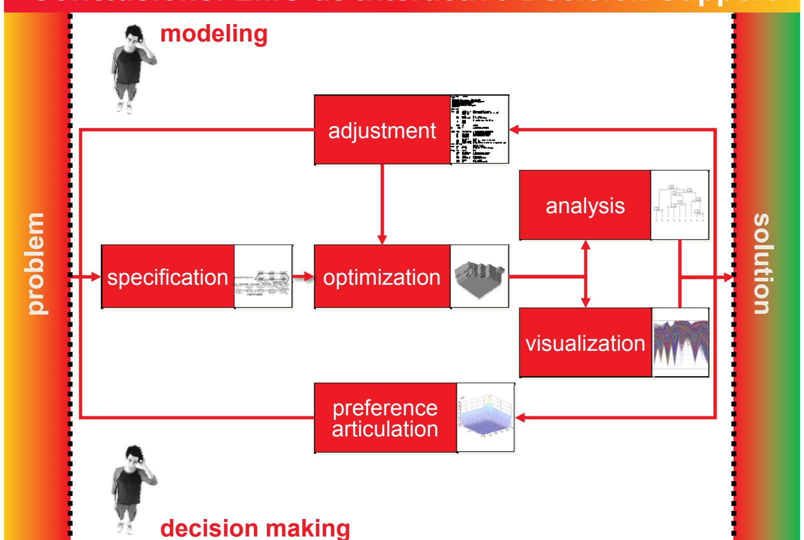
[Wagner et al. 2008]

T_{PH}^* = 146 °C,
t_{PH}^* = 174 s,
t_M^* = 170 s,
p_P^* = 1.1 MPa,
T_p^* = 169 °C,
t_P^* = 180 s



Position	T_M [°C]	F_{Max} [kN]	d_{Max} [mm]
1A	54	2.4	10.7
2B	84	2.8	9.8
3C	121	3.3	8.8
4D	147	3.3	8.3
5E	100	3.2	9.3

Conclusions: EMO as Interactive Decision Support



The EMO Community

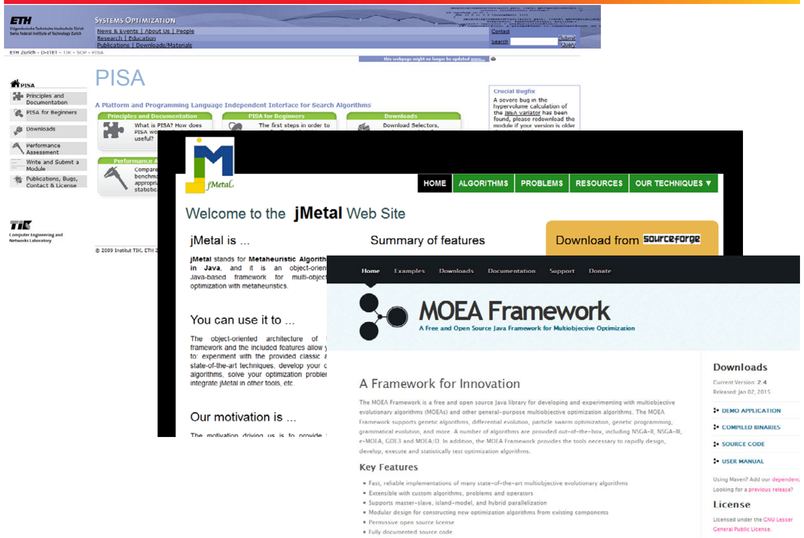
Links:

- EMO mailing list: *https://lists.dei.uc.pt/mailman/listinfo/emo-list*
- MCDM mailing list: *http://lists.jyu.fi/mailman/listinfo/mcdm-discussion*
- EMO bibliography: *http://www.lania.mx/~ccoello/EMOO/*
- EMO conference series: *http://www.dep.uminho.pt/EMO2015/*

Books:

- ***Multi-Objective Optimization using Evolutionary Algorithms*** Kalyanmoy Deb, Wiley, 2001
- ***Evolutionary Algorithms for Solving Multi Evolutionary Algorithms for Solving Multi-Objective Problems Objective Problems***, Carlos A. Coello Coello, David A. Van Veldhuizen & Gary B. Lamont, Kluwer, 2nd Ed. 2007
- **Multiobjective Optimization—Interactive and Evolutionary Approaches**, J. Branke, K. Deb, K. Miettinen, and R. Slowinski, editors, volume 5252 of *LNCS*. Springer, 2008 [(still) many open questions!]
- and more…

Software



Perspectives

Challenging Open (Research) Directions

- from algorithms to toolkits
  - libraries of modules for each task (selection, variation, etc.)
  - problem-specific algorithm configuration/ parameter tuning
- benchmarking
  - comparison with classical approaches
  - design/selection of practically relevant problems
  - Algorithm/toolkit recommendations for practice
- integration of EMO and MCDM into one field
- interactive preference articulation and learning
- interactive problem design
- integration of problem-specific knowledge

Questions?

Additional Slides

Instructor Biography: Dimo Brockhoff

Dimo Brockhoff

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DOLPHIN team
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France

After obtaining his diploma in computer science (Dipl.-Inform.) from University of Dortmund, Germany in 2005, Dimo Brockhoff received his PhD (Dr. sc. ETH) from ETH Zurich, Switzerland in 2009. Between June 2009 and October 2011 he held postdoctoral research positions---first at INRIA Saclay Ile-de-France in Orsay and then at Ecole Polytechnique in Palaiseau, both in France. Since November 2011 he has been a junior researcher (now CR1) at INRIA Lille - Nord Europe in Villeneuve d'Ascq, France. His most recent research interests are focused on evolutionary multiobjective optimization (EMO) and other (single-objective) blackbox optimization techniques, in particular with respect to benchmarking, theoretical aspects, and expensive optimization.

Instructor Biography: Tobias Wagner

Tobias Wagner

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