Synergies between Evolutionary Algorithms and Reinforcement Learning

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1

Instructors

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2

Course agenda

- Reinforcement learning: short introduction
  - Multi-armed bandits
- Evolutionary Computation in Reinforcement Learning
  - Motivation
  - Multi-objective reinforcement learning
  - Multi-criteria decision making and reinforcement learning
- Reinforcement Learning in Evolutionary Computation
  - Online adaptive operator selection
  - Schemata bandits
- Examples
- Discussion
- Questions

3

Reinforcement Learning (RL): short introduction

[Sutton and Barto, 1998] [Wiering and van Otterlo, 2012]

- The most general on-line/off-line learning technique that includes a long-term versus a short term reward trade-off.
- Applications: game theory, robot control, control theory, operations research, etc.
- RL solves environments modelled as Markov decision processes (MDP) by rewarding good actions and punishing bad actions
- The best actions are identified by trying them out and evaluating their consequences including long term consequences which might only be apparent after a large number of other actions have been taken
- The exploration / exploitation dilemma is crucial for online RL
  - exploration refers to trying out actions of which the outcome is still uncertain
  - exploitation refers to selecting actions which have shown to be good in the past

4

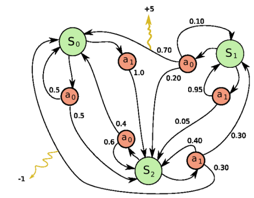
Markov decision processes (MDPs)

- *Markov decision processes* (MDP) a popular formalism to study decision making under uncertainty with the goal of maximising the long term reward intake
- An MDP is characterised with a tuple $< S, A, T, R >$
  - the *state – search space* $S = \{s_1, s_2, \ldots, s_N\}$, where $s_t \in S$
  - a set of actions $A = \{a_1, a_2, \ldots, a_N\}$ available to the agent in each state.
  - a *transition probability* $T(s, a, s')$ mapping state action pairs to a probability distribution over successor states
  - a *reward function* $R : S \times A \times S \rightarrow \mathbb{R}$ to denote the expected reward when the agent makes the transition from state s to state s' using action a.
  - r_t is the *immediate scalar reward* obtained at time t
- This process is Markovian → the distribution over the next states is independent on the past given the current state and action.

5

Markov decision processes (MDP)

- An example



- Policies can perform deterministic or stochastic action selection
- **Goal of MDPs**: to find the best policy which is the policy that receives the most rewards

6

MDPs paradigms: Dynamic programming (DP)

- Bellman equation (1957) states that the *expected value* of a state $V^{\pi}(s)$ is defined in terms of the immediate reward and the expected future rewards
  - the value of a state s under an optimal policy π^* is

$$V^*(s) = \max_{a \in A} \sum_{s' \in S} T(s, a, s') \left(R(s, a, s') + \gamma V^*(s') \right)$$

- Dynamic programming methods assume that
  - the transition model *P* is known → strong assumption
  - the reward function R is known → mild assumption
- DP breaks the problem up in smaller problems
- ***Model based techniques***
  - policy iteration
  - value iteration

7

Value vs policy iteration for solving MDPs

- Value iteration algorithms
  - Each iteration, the value function is updated and then a new optimal policy given the new value function is computed
  - A policy is optimal if it maximises the expected cumulative reward for any initial state
- Policy iteration algorithms
  - First fully evaluate a policy
  - Then improve the policy

8

Fundamental DP algorithms: policy iteration

- The model of the MDP is given meaning that it can be solved with dynamic programming
- Two phases:
  - policy evaluation: evaluate the value function V^{π} of a fixed policy π
    - Bellman equation is now an update rule by extending the planning horizon with one step
    - The current value function $V_k^{\pi}(s)$ is updated to $V_{k+1}^{\pi}(s)$

$$V_{k+1}^{\pi}(s) = \sum_{s' \in S} T(s, \pi(s), s') \left(R(s, \pi(s), s') + \gamma V_k^{\pi}(s)\right)$$

    - The sequence of $V_k^{\pi}(s)$ converges as k goes to infinity
  - policy improvement:
    - improves the quality of the policy π

$$\pi'(s) = argmax_{a \in A} \sum_{s' \in S} T(s, a, s') \left(R(s, a, s') + \gamma V^{\pi}(s')\right)$$

    - policy iteration

$$\pi_0 \to V^{\pi_0} \to \pi_1 \to V^{\pi_1} \to \ldots \to \pi_k \to V^{\pi_k} \to \ldots \to \pi^*$$

9

Fundamental DP algorithms: value iteration

- A simplified version of policy iteration that merges the policy evaluation and policy improvement in a single step
- The value function is updated on the fly

$$V_{k+1}(s) = \max_{a \in A} \sum_{s' \in S} T(s, a, s') \left(R(s, a, s') + \gamma V_k(s)\right)$$

- V_k converges to the optimal value V^*

$$V_0 \to V_1 \to \ldots \to V_k \to V_{k+1} \ldots \to V^*$$

- Very popular due its simplicity

Partially observable Markov decision processes (POMDPs)

- Standard MDPs → completely observable MDPs (CO-MDPs)
- POMDPs → do not have direct access to the current state

10

Online Reinforcement Learning

- ***Model free*** reinforcement learning algorithms
  - The transition model and the reward functions are not known a-priori
- Rewards classification
  - Immediate rewards returned by environment
    - Scalar values that can be stochastic
    - Reward good actions → positive rewards
    - Punish bad actions → negative rewards
  - Value function denotes cumulative rewards
  - The goal of RL is to optimise the long term reward intake $Q(s, a)$
  - Not known long-term reward → a large number of actions have to be taken in order to approximate its value since there are an infinite number of futures

11

Online RL paradigms: Q-learning

- Very popular model free RL algorithm [Watkins, 1989]
- Off-policy learning algorithm meaning that it learns a policy π while searching for the optimal policy π^*
- Q-values are estimated based on immediate rewards and taken actions
- Incrementally decreases the learning rate such that Q-values converge to the true values, exploring all actions an infinite number of times is necessary
- **ε** -greedy policy selects often the best action and with small probability another action
- Q-learning algorithm's inner loop
  - select an action a based on Q-values and an exploration strategy
  - perform action a
  - observe the new state s' and receive reward r_t

$$Q(s, a) \leftarrow Q(s, a) + \alpha \cdot \left(r_t + \gamma \cdot \max_{a' \in A(s')} Q(s', a') - Q(s, a)\right)$$

  - $s \leftarrow s'$
- Converges to the optimal policy π^*

12

Model free RL

- **SARSA**
  - On-policy algorithm that optimises the policy that is executed
    $$Q_{t+1}(s_t, a_t) = Q_t(s_t, a_t) + \alpha\left(r_t + \gamma Q_t(s_{t+1}, a_{t+1}) - Q_t(s_t, a_t)\right)$$
  - max operator in Q-learning is replaced with the action estimate according to the policy
  - converges in infinite time to the optimal policy when all states and actions were tried infinitely often and exploration decreases over time
- **QV learning**
  - On policy algorithm that uses also states values to speed up learning
    $$Q_{t+1}(s_t, a_t) = r_t + \gamma V_t(s_{t+1})$$
    $$V_{t+1}(s_t) = r_t + \gamma V_t(s_{t+1})$$
- **Temporal difference learning (TD)**
  - On policy method that learns the values of states based on estimates of other values, or *bootstrapping*

13

RL paradigms: Multi-armed bandits (MAB)

- Popular mathematical formalism used to study the convergence properties of RL with a single state
- A machine learning paradigm used to study and analyse resource allocation in stochastic and noisy environments.
- An example: a gambler faces a row of slot machines and decides
  - which machines to play,
  - how many times to play each machine
  - in which order to play them

- When played, each machine provides a reward generated from an unknown distribution specific to a machine.
- The goal of the gambler is to *maximise the sum of rewards* earned through a sequence of lever pulls.

14

Multi-armed bandits (MAB) algorithms

- Intuition on the MAB algorithms
  - An agent must choose between *N*-arms (= actions) such that the expected reward over time is maximised.
  - The algorithm starts by fairly exploring the *N*-arms, gradually focusing on the arm with the best performance.
  - The distribution of the stochastic payoff of the different arms is assumed to be unknown to the agent.
- ***Exploration / exploitation trade-off***
  - Explore the sub-optimal arms that might have been unlucky
  - Exploit the optimal arm as much as possible
- **Performance measures**
  - Cumulative regret is a measure of how much reward a strategy loses by playing the suboptimal arms

15

Multi-armed bandits: type of algorithms

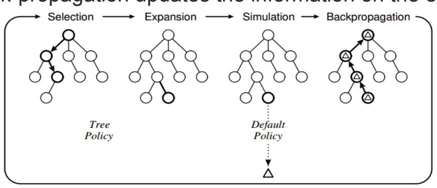
[Bubeck & Cesa-Bianchi, 2015]

- Stochastic multi-armed bandits
  - Online selection of the arm with the maximum expected mean (i.e., the arm with higher expected reward)
    - The best arm can change over time
  - Best arm identification algorithms
    - Fixed confidence vs fixed budget
    - Multiple best arm identification
- Adversarial multi-armed bandits
  - a game is played between a forecaster and an environment assuming that the adversarial process controls the rewards
- Contextual multi-armed bandits
  - uses the context to adapt the multi-armed bandit long term behaviour, or regret
- Bayesian multi-armed bandits: Thompson sampling
- Continuous multi-armed bandits: X-armed bandits
- Monte Carlo tree search: Upper confidence tree search

16

Monte Carlo Tree Search (MTCS)

- [Browne et al, 2012]
- A heuristic used to solve intractable problems, i.e. huge search spaces, like playing computer Go
- MCTS → builds a search tree using a search policy selecting the most probable nodes to expand
- A top down approach, i.e. root to leaves, with the following steps
  - Selection of the most promising children
  - Expansion creates new nodes using a tree policy
  - Simulation plays at random from the current node to the end of the game
  - Back-propagation updates the information on the explored path



17

Exploration / exploitation trade-off

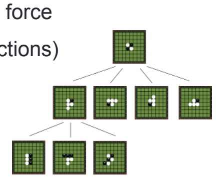
- Model free RL
  - exploration means try different actions to see their results
  - exploitation means to exploit the knowledge about good actions
- Monte Carlo Tree Search
  - exploration means generate new branches in the tree
  - exploitation means to focus the search on the tree branches that returned good rewards
- Multi-armed bandits
  - exploration of suboptimal arms
  - exploitation means to pull often optimal or close to optimal arms
- Evolutionary multi-objective optimisation
  - exploration of unknown regions of the search space
  - exploitation of good parts of the solution space

18

Real-world applications for reinforcement learning

- Playing online games:
  - Atari games (Pacman)
    - generative RL using Bayesian rules
    - QV - learning
    - Monte Carlo Tree Search

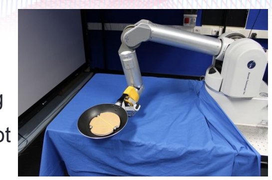
  - Go games --> Monte Carlo Tree Search using Multi-armed bandits
  - (Chess can be efficiently played with brute force
  - MinMax methods and good evaluation functions)



  - Othello, Pacman
  - Mazes --> Q-learning

Real-world applications for RL

- Robotics
  - Vision with model-based RL
  - Control of a robot arm with Q-learning
  - POMDPs for navigating a mobile robot



- Traffic light control
  - Model based RL and game theory
  - Multi-agent systems in
    - traffic distribution in a network

- *RL is considered slow and resource consuming compared with other heuristics but can learn online under changing environmental conditions*

20

Multi-criteria decision making (MCDM)

- Assumes the involvement of a decision maker (DM) → the main difference with multi-objective optimisation
- Classification of MCDM methods
  - Pareto-based ranking → DM is indifferent
    - Dominance relation → a partial order relation where two solutions can be better in one objective and worse in another objective
  - Scalarized relation → DM prefers a region of the search space
    - A function that transforms the value vector of a solution in a scalar value
  - Preference ranking → DM expresses the preference for some objectives
    - A utility function or a lexicographic order to rank the objectives
  - Interactive methods → DM permanently interacts with the environment
    - Games

Pareto dominance relation

- *A reward vector can be better than another reward vector in one objective and worse in another objective*
- The natural order relationship for multi-objective search spaces
- Examples of relationships between reward vectors

relationship	notation	relationships
μ_1 dominates μ_2	$\mu_2 \prec \mu_1$	$\exists j, \mu_2^j < \mu_1^j$ and $\forall o, j \neq o, \mu_2^o \leq \mu_1^o$
μ_1 weakly domin μ_2	$\mu_2 \preceq \mu_1$	$\forall j, \mu_2^j \leq \mu_1^j$
μ_1 is incomp with μ_2	$\mu_2 \| \mu_1$	$\mu_2 \nprec \mu_1$ and $\mu_1 \nprec \mu_2$
μ_1 is non-domin by μ_2	$\mu_2 \nprec \mu_1$	$\mu_2 \prec \mu_1$ or $\mu_2 \| \mu_1$

- *The Pareto front is* the set of expected reward vectors that are *non-dominated* by the other expected reward vectors
- All the solutions in the Pareto front are considered equally important

22

Lp scalarization function

- *Goal*: Lp transforms the multi-objective search space into a single objective space using a scalarization function
- Weighted power p sums of reward values, where a set of predefined weights is considered

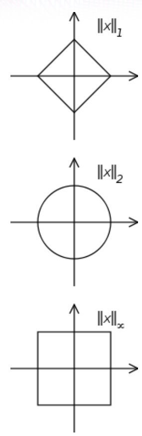
$$f_p(\mu_i) = \sqrt[p]{\sum_{j=1}^{D} \omega^j \cdot (\mu_i^j - z^j)^p}$$

L_p function can find all solutions of any shape, i.e. non-convex

The reference point $\mathbf{z} = (z^1, \ldots, z^D)$ is an extra parameter

L_1 function is *a linear scalarization* function

L_∞ function is *a Chebyshev scalarization* function



23

Evolutionary Computation into RL

- Evolutionary Multi-objective Optimisation
- Multi-criteria decision making (MCDM)
- Learning in games
- Evolutionary Algorithms for Reinforcement Learning
- Multi-objective dynamic programming (MODP)
- Multi-objective reinforcement learning (MORL)
- Online learning in multi-objective games
- Multi-objective multi-armed bandits

24

Evolutionary Algorithms for RL [Moriarty et al, 1999]

- EARL evolves policies with EAs
  - Generate new policies using EA operators
  - Associate each policy a fitness value
  - Select the best policies to produce the next generation
- Policy representation
  - Rule-based policy representation
    - each gene is a condition-action rule that maps a set of states to an action
    - distributed rule-based representation of a policy over several EAs evolved separately for learning classifier systems (LCS)
  - Parameter representation for evolving neural networks
    - each gene is a weight in a neural network
    - distributed network based policies constructs different parts of a neural network that optimise different tasks using EAs

25

Evolutionary Algorithms for RL

- Fitness and credit assignment
  - the agent interacts with an environment
  - the fitness values are averaged over time
  - what is the effect of current action vs past actions on the current reward for sparse pay-offs like reaching the goal with a robot
  - subpolicy credit assignment for distributed policies
- Selection
  - roulette selection proportional with fitness values
- Genetic operators are specific for each policy representation
  - triggered operators for learning classifier systems (LCS)
  - real coded operators for strings of weights for neural networks

26

Evolutionary Algorithms for RL

- Strengths of EARL
  - scaling up to large state spaces
    - policy generalisation is grouping together states for which the same action is required
      - may vary considerable with the rules they encode
      - level of abstraction is higher than for a normal policy
    - policy selectivity means the knowledge about bad decisions is not represented
      - the search is reduced by focusing on promising actions
  - non-stationary environments
    - tracking in non-stationary environments
    - a statistical model of agent uncertainty
  - incomplete state information
    - reward policies that avoid the ambiguous states
    - model hidden states

27

Evolutionary Algorithms for RL

Many variants of EARL algorithms improve state of the art RLs

- CMA-ES strategies are used to select RL policies [Heidrich-Meisner & Igel, 2009] or to evolve neural networks
- Neuroevolution is used to evolve neural networks [Whiteson & Stone, 2006]
- Representation of phenotype - genotype space in EAs is used to describe populations of single weights for evolving neural network controllers [Gomez et al, 2009]

28

Multi-objective reinforcement learning (MORL)

- Early research (80s): multi-objective dynamic programming and multi-objective MDPs
- Main differences with MDPs
  - Immediate reward values are replaced with reward vectors
  - A set of optimal policies of incomparable quality in all objectives
  - Computationally intractable even for simple small environments
- Techniques from MOO and MCDC are incorporated in RL with reward vectors
- Goal: to identify one or a set of Pareto optimal policies
- When compared with RL, MORL has:
  - extra computational challenges
  - different exploration / exploitation trade-offs
  - more complicated experimental setups

29

Evolutionary multi-objective optimisation (EMO) in RL

Multi-objective Markov Decision Processes (MOMDPs)

- compute all Pareto optimal policies
- tuples of rewards instead of a single reward
- stationary and non-stationary deterministic environments

Multi-objective Reinforcement Learning (MORL)

- important differences with single objective reinforcement learning
  - several actions can be considered to be the best according to their reward tuples.
  - techniques from EMO should be used in the multi-objective RL framework to improve the exploration/exploitation trade-off
  - complex and large multi-objective environments.

Multi-objective multi-armed bandits (MOMABs)

- single state reinforcement learning algorithm
- various variants of multi-armed bandits extended to reward vectors

30

Multi-objective Markov decision processes (MOMDPs)

[White, 1982] [Wiering and De Jong, 2007][Lizotte et al, 2012][Roijers et al, 2013][Wiering et al, 2014][Parisi et al, 2014]

- Multi-objective dynamic programming (MODP)
  - A reward vector $r = (r_1, r_2, \ldots, r_m)$ with m dimensions
  - A reward function $R(s, a, s') = (R_1(s, a, s'), \ldots, R_m(s, a, s'))$
  - A set of Pareto optimal policies
  - An agent will select one or more Pareto optimal policies
  - Value or policy iteration multi-objective dynamic programming
- Pareto dominance relation or scalarization functions are used to compute and track the Pareto optimal policies
- For both non-stationary and deterministic environments
- The corresponding algorithms converge to the unique Pareto optimal set of policies

31

Value iteration multi-objective dynamic programming

- [Wiering and de Jong, 2007][Wiering et al 2014]
- The value function is a vector $V^i(s) = (V_1^i(s), V_2^i(s), \ldots, V_m^i(s))$
- The set of non-dominated value functions is denoted with

$$V^O(s) = \{V^i(s) \mid V^i(s) \text{ isn't dominated by a policy in } s\}$$

- The set of non-dominated Q-values

$$Q^O(s,a) = \{Q^i(s,a) \mid Q^i(s,a) \text{ isn't dominated in } s,\ a\}$$

- The non-dominated operator tells whether a policy i is non-dominated in the state s by any value function in $V^D(s)$
- $$ND(V^i(s), V^D(s)) \leftrightarrow \nexists V^j(s) \in V^D(s);\ \ V^j(s) \succ V^i(s)$$
- The Pareto optimal operator

$$PO(Q^D(s,a)) = \{Q^i(s,a) \mid Q^i(s,a) \in Q^D(s,a) \wedge ND(Q^i(s,a), Q^D(s,a))\}$$

- The dynamic programming operator for deterministic environments

$$DP(Q^D(s,a)) = \{R(s,a,s') \oplus \gamma V^O(s') \mid P(s,a,s') = 1.0\}$$

32

Value iteration multi-objective dynamic programming

- The optimal set of Q – values vectors Q^{O^*}

$$Q^{O^*} = (PO(DP(Q_0)))^*$$

- where * means the operator is repeated an infinite number of times
- This dynamic programming algorithm *converges* to the maximal Pareto optimal set of policies because it eliminates all dominated policies
- *The set of Pareto optimal policies is independent of the usage of the Pareto optimal operator inside the dynamic programming operator*

$$PO(DP'(Q_0))^* = (PO(DP(Q_0)))^*$$

- where DP' does not make use of the PO operator at all
- $DP'(Q^D(s,a)) = \{R(s,a,s') \oplus \gamma V^D(s') \mid P(s,a,s') = 1.0\}$

33

Multi-objective RL (MORL) algorithms

- Pareto Q-learning algorithm applies a Pareto operator for sets to keep track of Pareto optimal policies [van Moffaert et al, 2014]
- Scalarization based RL [Vamplew et al, 2010][Brys et al, 2014] [van Moffaert et al, 2013a]
  - Weighted linear scalarization functions (most algorithms)
  - Goal: identifying all Pareto optimal policies on a convex front
- Hyper-volume unary indicator is used to measure the quality of solutions [Wang & Sebag, 2013] [van Moffaert et al, 2013b]
  - Another function that transforms the multi-objective search space
- Preference based dominance relations are used with multi-criteria RL [Gabor et al, 1998]
  - Only a region on the Pareto front is required
  - Interaction with the user to select the preferred solutions

34

Multi-objective Q-learning

- In MORL, *Q*- values are extended to ***Q***-vectors, one value for each objective

$$\mathbf{Q}(s,a) = (Q^1(s,a), \ldots, Q^M(s,a))$$

- A set of linear scalarization functions to identify convex fronts
- L_∞ norm identifies solutions on any Pareto front
- Q-values are updated separately in each objective → convergence properties inherited from Q-learning
- The selection of next actions depends on the used scalarization function
- Adaptive scalarization functions adapt the weights of the linear scalarization functions
- The main differences with single objective RL
  - ***Pareto front*** includes several reward values that are equally good (incomparable) and a set of ***Q***-vectors with incomparable best values.
  - ***Updating rules*** for the ***Q***-vectors and the selection of the best next action when compared with single objective MDPs.

35

Scalarized multi-objective Q-learning

- Multi-objective MDPs → immediate reward vectors

$$R(s,a) = (R^1(s,a), R^2(s,a), \ldots, R^m(s,a))$$

- Q values

$$Q(s,a) = (Q^1(s,a), Q^2(s,a), \ldots, Q^m(s,a))$$

- Scalarized Q-values

$$SQ(s,a) = \sum_{o=1}^{m} w^o Q^o(s,a)$$

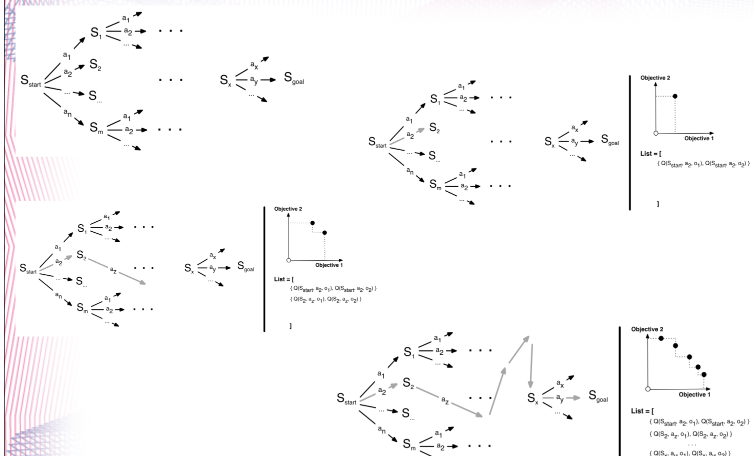
- ε -greedy selects the action a with the largest SQ value with a high probability
- Try action a
- For each objective o

$$Q(s,a,o) \leftarrow Q(s,a,o) + \alpha \cdot [r(s,a,o) + \gamma Q(s', max_{a'}, o) - Q(s,a,o)]$$

- s ← s'

36

A scalarized Q-learning algorithm

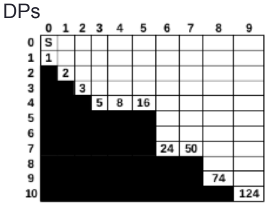


37

Model based multi-objective reinforcement learning

[Wiering et al, 2014]

- Most MORL are model free algorithms that use scalarization functions to transform the reward vectors into reward values
- Model based MORL
  - estimate the model of the environment
    - using frequencies stored in a table
  - solve this model
    - based on value iteration multi-objective DPs
- Exploration strategies
  - least visited exploration
    - counts the times each action was taken
  - random exploration
    - actions are selected randomly



38

Hypervolume based MORL

- [Wang & Sebag, 2013]
- Multi-objective Monte Carlo tree Search (MOMCTS)
- Hypervolume unary indicator is used to select a node in MOMCTS
- MOMCTS has inherited computational problems for hypervolume unary indicator
- MOMCTS is also combined with Pareto dominance to improve the diversity of solutions
- MOMCTS performs better than scalarized MORL on the bi-objective Deep Sea environment

- [van Moffaert et al, 2013b]
- Multi-objective Q-learning uses hypervolume indicator to evaluate the quality of the Pareto front
- Action selection mechanism that maximises the hypervolume indicator
- Compared with scalarization based MORL, hypervolume based MORL has
  - better performance
  - narrower Pareto front

39

Real-world applications for MORL

Many real-world problems are inherently multi-objective and were tackled with single objective techniques and predefined linear scalarization problems

- Traffic light control --> linear scalarized multi-objective Q-learning
- Control problems (like wet clutch) with two or more objectives --> adaptive linear scalarized MORL
- A benchmark or RL problems that were transformed into multi-objective environments
  - Mounting car problems
  - Maze like problems, i.e. Deep Sea World
  - Simple multi-objective MDPs --> these problems were approached with both scalarized and Pareto MORL
- Preference based MORL that requires user interactions --> scalarized MORL

40

Multi-objective multi-armed bandits (MOMABs)

- [Drugan & Nowe, 2013]
- Multi-armed bandits use reward vectors
- Evolutionary Computation (EC) techniques are used to design computationally efficient MOMABs
- The exploration / exploitation trade-off is common for both multi-armed bandits (MABs) and EC for multi-objective optimisation
  - In EC, exploration means evaluation of new solutions in a very large search space where states cannot be enumerated
  - In MAB, exploration means to pull arms that have suboptimal mean reward values
  - In EC, exploitation means to focus the search in promising regions where the global optimum could be located
  - In MAB, exploitation means to pull the currently identified best arm(s)
- MOMABs with a finite set of arms and reward vectors generated from stochastic distributions

41

Multi-objective multi-armed bandits (MOMABs)

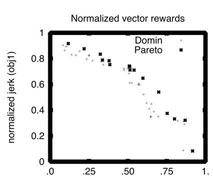
- The goal of MOMABs is to maximise the returned reward; or to minimise the regret of pulling suboptimal arms
- We assume that all Pareto optimal arms are equally important and need to be identified
- Performance measures
  - Pareto regret → sum of the distances between each suboptimal arm and the Pareto front
  - Variance regret → variance in using the Pareto optimal arms
  - KL divergence measure
- Theoretical analysis
  - Upper and lower bounds on expected cumulative regret
- Challenges
  - Large and complex *stochastic* multi-objective search spaces
  - Non-convex Pareto fronts

42

The bi-objective transmission problem of wet clutch

- *Wet clutch:* an application from control theory
- *Goal:* optimise the functionality of the clutch:
  - the optimal *current profile* of the electro-hydraulic valve that controls the pressure of the oil to the clutch
  - the *engagement time*.

- *Stochastic output data* —> some external factors, such as the surrounding temperature, cannot be exactly controlled.
- *Goal: optimise the parameters* —> that minimise the clutch's profile and the engagement time in varying environmental conditions.



43

Stochastic discrete MOMAB problems

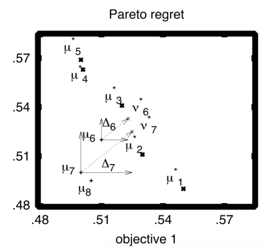
- *K*-armed bandit, $K \geq 2$, with independent arms
- The reward vectors have *D* –objectives, where *D* fixed
- An arm *i* is played at time steps $t_{1,i}, t_{2,i}, \ldots$
- The corresponding *reward vectors* $X_{i,t_1}, X_{i,t_2}, \ldots$ are independently and identically distributed according to an unknown law with unknown expectation vectors
- *The goal of MOMAB:*
  - Identify the set of best arms by simultaneously maximising rewards in all objectives
  - The arms in the Pareto front are considered equally important and should be pulled the same number of times.
  - Minimise the regret (or the loss) of not selecting the arms in the Pareto front

44

Performance metric: Pareto regret

- We denote with $\Delta_i = \|\nu_i^* - \mu_i\|_2$ the empirical distance between an arm and the Pareto front
- Let ν_i^* be the *virtual reward vector* of the arm i such that μ_i has the minimum distance to ν_i^*,
  - $\nu_i^* = \|\mu_i - \epsilon_i\|_2$ is incomparable with all reward vectors in the Pareto front
- The *expected Pareto regret* for a learning algorithm after n arm pulls is

$$\mathbb{E}[R_n] = \sum_{i=1}^{K} \Delta_i \cdot \mathbb{E}[T_i(n)]$$



Pareto MAB algorithms

- ***Definition:*** a multi-objective MAB algorithm that uses the *Pareto partial order relationship*
- The Pareto regret metric is used to upper bound the performance of the designed Pareto MAB algorithms
- *Challenges* in designing Pareto MAB algorithms:
  1. Pareto front identification
     1. Identification of a representative Pareto set of arms
  - The exploitation/exploration trade-off:
    - Exploration: pull suboptimal arms that might be unlucky
    - Exploitation: pull as much as possible the optimal arms
  - Optimising the performance of Pareto MABs in terms of upper and lower bounds on expected and/or immediate regret
  - Ameliorate the performance of Pareto MABs for large sets of arms

Pareto Upper Confidence Bound (PUCB1)

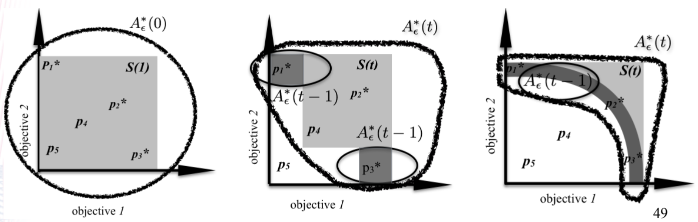
- Straightforward generalisation of UCB1
  - ✤operator selection [Fialho et al, 2009]
  - ✤learning the utility of swap operations in combinatorial optimisation [Puglierin et al, 2013]
- Maximises the reward index $\widehat{\boldsymbol{\mu}}_i + \sqrt{\frac{2\ln(n\sqrt[4]{D|\mathcal{A}^*|})}{n_i}}$
- The algorithm
  - Each iteration, a Pareto front is calculated using $\widehat{\boldsymbol{\mu}}_h + \sqrt{\frac{2\ln(n\sqrt[4]{D|\mathcal{A}^*|})}{n_h}} \succ \widehat{\boldsymbol{\mu}}_i + \sqrt{\frac{2\ln(n\sqrt[4]{D|\mathcal{A}^*|})}{n_i}}$
  - One of the arms from the Pareto front is selected
- The upper bound is $\sum_{i \notin \mathcal{A}^*} \frac{8 \cdot \log(n\sqrt[4]{D|\mathcal{A}^*|})}{\Delta_i} + (1 + \frac{\pi^2}{3}) \cdot \sum_{i \notin \mathcal{A}^*} \Delta_i$
- The worst-case performance of this algorithm is when the number of arms *K* equals the number of optimal arms
- The algorithm reduces to the standard UCB1 for *D = 1*.
- Pareto UCB1 performs *similarly* with the standard UCB1 for a small number of objectives and small Pareto optimal sets

Pareto front identification

- This policy is an extension of the *best arm identification algorithm* [Audibert et al.,2010] for a set of arms of equal quality.
- The *m*-best arm identification algorithm assumes that the *m*-best arms can be totally ordered.
- ***The algorithm***
- Let $A_1 = \{1, \ldots, K\}\,,\ \overline{\log}(K) = \frac{1}{2} + \sum_{i=2}^{K} \frac{1}{i}\,,\ n_0 = 0 \text{ and for } k \in \{1, \ldots, K-1\}$
- $n_k = \left\lceil \frac{\log(D|\mathcal{A}^*|)}{\overline{\log}(K) + \log(D|\mathcal{A}^*|)} \cdot \frac{n-K}{K+1-k} \right\rceil$
- For all rounds $k = 1, 2, \ldots, K-1$
- (1) For each arm $i \in A_k$, select it for $n_k - n_{k-1}$ rounds
- (2) Let $A_{k+1} = A_k \setminus \text{argmin}_{i \in A_k} \widehat{\mathbf{x}}_{i,n_k}$ the arm to dismiss in this round
- Let the remaining set of arms be the Pareto optimal set of arms $\mathcal{A}^*$

Annealing Pareto Knowledge gradient [Yahyaa et al, 2014]

- Knowledge gradient policy is a reinforcement learning algorithm where the reward vectors are updated using Bayesian rules
- Annealing like functions that decrease uncertainty around the arms
- The algorithm
    - At initialisation, all arms are considered
    - Iteratively, extreme arms are identified as either Pareto optimal or deleted as suboptimal arms
    - The iteration stops when there are no more arms to classify



49

Challenges in designing scalarized MOMABs

- [Drugan & Nowe, 2014] [Drugan, 2015a][Drugan, 2015b]
- Identify the entire Pareto front
    - Large Pareto fronts
    - Non-convex Pareto fronts
    - Non-uniform distributions of arms on the Pareto front
- Optimising the performance of scalarized MOMABs in terms of upper and lower regret bounds
    - The *scalarized* / Pareto regret metric
    - The Kullback-Leibler divergence regret metric
- Exploitation/exploration trade-off:
    - Exploration: *sample scalarization* functions, and pull arms that might be unluckily identified as suboptimal
    - Exploitation: pull as much as possible the Pareto optimal arms of *relevant scalarization* functions

50

Reinforcement learning in EC

- Adaptive operator selection for Evolutionary Computation
    - Online parameter selection as opposite to off-line parameter selection
    - Uses reinforcement learning to select operators
        - Adaptive pursuit →pursuit more often the operator that improves the most the results
        - Multi-armed bandits like UCB1 to adaptively select the best operator
        - SARSA
    - Applied in tuning the parameters of
        - Evolutionary Computation (Genetic algorithms, Evolution Strategies)
        - Iterated (Pareto) local search
        - Evolutionary Multi-objective optimisation
- Schemata bandits
    - Monte Carlo decision trees for schemata theory
- Monte Carlo Tree Search for learning in continuous search spaces

51

Adaptive operator selection

- Motivation:
    - the performance of EAs depends on the used parameters
    - the performance of a genetic operator depends on the landscape
    - an operator can have different performance in different regions of the landscape
- Tuning genetic operators
    - Selection of parameters
    - Mutation rates / Recombination exchange rates
    - Population size
    - Variable neighbourhood size (local search)
- Online learning strategy
    - The algorithms should learn relatively fast the best operator
    - There are several operators that perform similarly

52

Adaptive pursuit strategy (AP) [Thierens, 2005]

- Each operator i has associated a probability value $P_i^{(t)}$ of selection and an estimated reward value $Q_i^{(t)}$
- Online operator selection algorithm with fixed target probabilities is a step like distribution $D = \{p_M, p_m, \ldots, p_m\}$
  - p_M has a large probability value to select often the best operator
  - p_m has a small non zero probability to select any suboptimal operator
- The iterative algorithm
  - Pursuit with probability $P_v^{(t)}$ the operator *v* with the maximal estimated reward $Q_v^{(t)}$
  - Get reward vector $R_v^{(t)}$ for the operator *v*
  - Update reward value $Q_v^{(t)}$ using the immediate reward $R_v^{(t)}$
  - High rank the estimated reward distribution $Q_v^{(t)}$ and set the values in vector r
  - For each operator i, update the selection probabilities

$$P_i^{(t+1)} \leftarrow P_i^{(t)} + \beta \cdot (D_{r[i]} - P_i^{(t)})$$

53

UCB1 for online operator selection [Fialho at al, 2010]

- Each operator is considered an arm with unknown probability of getting a reward
- The reward function for operator i contains
  - the estimated value for the operator $\widehat{\mu}_i$
  - the exploitation coefficient $C\sqrt{\frac{2\log\sum_j n_j}{n_i}}$

where n_i is the number of times the operator *i* was selected and *C* the exploration constant

- Remarks
  - Originally, UCB1 has positive sub-unitary values
  - Setting up *C* is important for any fitness landscape
  - UCB1 detects changes in the environment but will react quite slow to them
  - UCB1 is combined with other optimisation techniques to improve the performance of the online operator selection algorithm

54

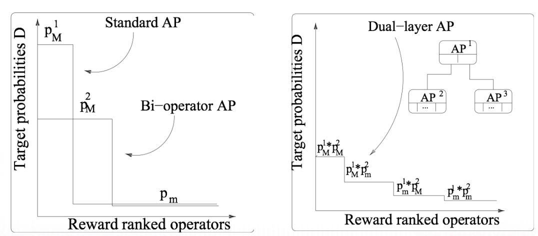
UCB1 for online operator selection

- Performance of operator selection depends on
  - the improvement measure considered like difference in fitness value and / or diversity
- Techniques to improve the performance of UCB1
  - Detect a change in the distribution with Page-Hinkley statistical tests
  - Weigh the operators using their frequency in applying it
  - Area under curve is also used as a measure of improvement in UCB1
  - Extreme values operator selection focuses on extremes to encourage exploration
- Hyper-parameter tuning, or tuning the tuner
- Off-line parameter tuning with F-race
- UCB1 is used to select solutions that adapt the CMA-ES matrix in continuous MO-CMA-ES [Loshchilov et al, 2011]

55

Generalised adaptive pursuit [Drugan & Thierens, 2011]

- An umbrella technique that could describe the main online adaptive selection algorithms
- Any target operator probabilities D can be considered, i.e. static or time dependent
- We assume that exploiting a set of related operators is beneficial for the performance of the algorithm
- We can consider two high value probabilities instead of one high value probability in D
- We consider multiple layers of adaptive pursuit algorithms



56

Online multi-operator selection [Drugan & Talbi, 2014]

- Optimise the usage of two or more operators simultaneously
- Motivated by the quadratic assignment problem:
  - Exploring large variable neighbourhoods is expensive
  - Iterated local search is efficient for QAPs
- ***Probability distribution*** of the mutation and the neighbourhood operators

$$\mathbf{P}_{\mathcal{N}}^{(t)} = \{\mathbf{P}_{11}^{(t)}, \ldots, \mathbf{P}_{1K}^{(t)}\}, \quad \mathbf{P}_{\mathcal{M}}^{(t)} = \{\mathbf{P}_{21}^{(t)}, \ldots, \mathbf{P}_{2P}^{(t)}\}$$

$$\mathbf{Q}_{\mathcal{N}}^{(t)} = \{\mathbf{Q}_{11}^{(t)}, \ldots, \mathbf{Q}_{1K}^{(t)}\}, \quad \mathbf{Q}_{\mathcal{M}}^{(t)} = \{\mathbf{Q}_{21}^{(t)}, \ldots, \mathbf{Q}_{2P}^{(t)}\}$$

- ***Quality distribution*** of the mutation and the neighbourhood operators

$$\mathcal{Q}_{\mathcal{N}}^{(t)} = \frac{\#\ improv\ of\ v_{\mathcal{N}}}{\#\ trials\ of\ v_{\mathcal{N}}}, \quad \mathcal{Q}_{\mathcal{M}}^{(t)} = \frac{\#\ improv\ of\ v_{\mathcal{M}}}{\#\ trials\ of\ v_{\mathcal{M}}}$$

- ***Update reward vectors***: an improvement in the cost of the candidate solution when compared with the current solution

$$\mathcal{P}_{\mathcal{N}i}^{(t+1)} \leftarrow \mathcal{P}_{\mathcal{N}i}^{(t)} + \beta \cdot (\mathcal{D}_{r_{\mathcal{N}i}} - \mathcal{P}_{\mathcal{N}i}^{(t)}), \quad \forall 1 \leq i \leq P$$

$$\mathcal{P}_{\mathcal{M}i}^{(t+1)} \leftarrow \mathcal{P}_{\mathcal{M}j}^{(t)} + \beta \cdot (\mathcal{D}_{r_{\mathcal{M}j}} - \mathcal{P}_{\mathcal{M}j}^{(t)}), \quad \forall 1 \leq j \leq K$$

- ***Update probabilities***: the probability distributions are independently updated

57

Generic Parameter Control with RL [Karafotias et al, 2014]

- On fly parameter tuning of parameters of EA algorithms based on
  - Diversity (phenotypic and genotypic diversity)
  - Improvement (fitness variation and improvement and stagnation counter)
- SARSA is used by the parameter control algorithm
  - States are represented in a binary decision tree
  - Actions set each parameter to a certain value
  - Each pair state-action has associated a value to determine how much impact this pair has on the estimated reward
- Evolution Strategy applied on
  - population size
  - the generation gap (the ratio of offsprings)
  - mutation step size
  - tournament size for survivor selection
- CMA-ES

58

Schemata bandits [Drugan et al, 2014]

- A hierarchical bandit where each arm is a schemata
- A synergy between Schemata Theory and Monte Carlo tree Search
- Genetic algorithms that do not use the genetic operators to generate new individuals
- Current version is computationally challenging
- The schemata with the maximal estimated mean fitness is selected the most often using an UCB1 algorithm
- A schemata is a L – dimensional hypercube, 2L binary strings

0011111001

0111011001 *0001111010*

0101101010 **0**1**10**** *0101011001*

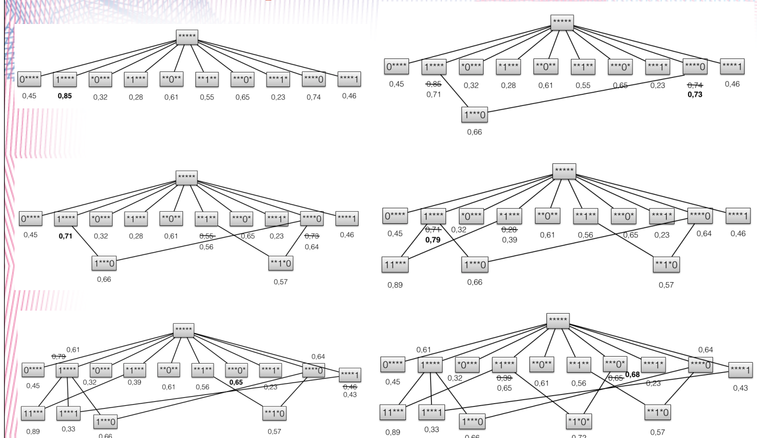
0111111011 *0101001000*

0011101001 *0001001010*

0001001001

59

An example of schemata net



60

Schemata bandits

- Schemata are generated from good solutions
- Random solutions are generated from a schemata node
- The mean of solutions represents the value of the schemata

- Schemata net structure
    - 3L schemata of the form $H \in \{0, 1, *\}$
    - o(H) – the order of schemata is the number of 0s or 1s
    - d(H) – the dimension of the schema H (number of * symbol)
  - *root* - the most general schema **…*,
  - *the leaves* – each * is replaced with 0 or 1
  - Each *node has*:
    - a value that is the mean fitness of the individuals belonging to the schema
    - 2 * d(H) children replace a * symbol with 0 or 1
    - o(H) parents replace of 0 or 1 with * symbol

61

A baseline schemata algorithm

- Initialise n random individuals
- Repeat
  - Select the root schemata
  - Select the most promising child of the current schemata using UCB1

$$argmax_i \overline{f}(H_i) + C \cdot \sqrt{\frac{2\log(t)}{t_i}}$$

  - Update counters
  - Back-propagate the information to update the value of schemata nodes

  - Parameter free optimisation algorithm
  - Schemata net is densely connected
    - computationally infeasible for large L
    - expand only a part of the schemata net
  - hybrid between the two approaches

62

Bandit trees for real coded optimisation

Monte Carlo tree search variants are used in optimisation of real-coded multi-dimensional functions

- The search space is partitioned in subdomains
- Each node in MCTS contains a multi-dimensional domain
- The search focuses on the most promising partitions, i.e. that contain the best solutions
- The other regions are explored with small probability

- Simultaneous optimistic optimisation (SOO) [Preux et al, 2014] is successfully applied on many dimensional test problems from the CEC'2014 competition on single objective real-parameter numerical optimisation.
- Hierarchical CMA-ES solver [Drugan, 2015c] uses CMA-ES solvers in each node of MCTS

63

Concluding remarks on MORL algorithms

- Multi-objective reinforcement learning
  - Follows closely the latest developments in RL and MOO, but also MCDM
- Multi-objective multi-armed bandits
  - New theoretical tools needed to study the performance of MORL algorithms
- Open research questions
  - Computationally efficient exploitation / exploration trade-off
  - Adequate performance measures for MORL and MOMABs
  - Advanced MOO and MCDM techniques to improve the performance of MORL and MOMAB algorithms
  - Challenging real world problems to motivate MORL and MOMABs paradigms

64

Concluding remarks on EC using RL

- Most EC algorithms use model free RL or multi-armed bandits techniques for parameter tuning
- Schemata theorem as initially used in association with multi-armed bandits by [Holland, 1975]
- Learning Classifier Systems [Holland, 1975] is used with RL to optimise and learn in complex non-stationary environments
- Variants of Monte Carlo Tree Search are used to optimise multi-dimensional real world problems
- Hyper-heuristics use reinforcement learning to select the best heuristic for a given task [Ozcan et al, 2010]
- Pareto Local search is used in combination with RL for optimising in multi-objective search spaces [Inja et al, 2014]

65

Conclusions

- Hybrid algorithms between reinforcement learning and evolutionary computation
  - perform better than many standard settings of both algorithms
  - can represent realistic models of problems in, for example, engineering and management
    - incomplete observations
    - large stochastic and changing environments
  - new methodological and theoretical challenges
    - in reinforcement learning, the convergence proofs need to take in account the multiple dimensions and the possible interactions between them
    - in evolutionary computation, some performance metrics to measure the adaptability of the algorithm needs to be considered
  - potential to develop new algorithms for automatic parameter tuning

66

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