Evaluating the Population Size Adaptation Mechanism for CMA-ES on the BBOB Noisy Testbed

Kouhei Nishida
Graduate School of Science and Engineering,
Shinshu University
15tm232a@shinshu-u.ac.jp

Youhei Akimoto
Faculty of Engineering,
Shinshu University
y_akimoto@shinshu-u.ac.jp

ABSTRACT

The CMA-ES with a population size adaptation mechanism is benchmarked on the BBOB noisy testbed. The population size is adapted online based on the signal-to-noise ratio of the update of the distribution parameters such as the mean vector and the covariance matrix. Four variants of the population adaptation mechanism with a random restart strategy and the BIPOP-CMA-ES are compared.

Keywords

Benchmarking, Black-Box Optimization, Covariance Matrix Adaptation, Population Size Adaptation, Noise Handling

1. INTRODUCTION

The Covariance Matrix Adaptation Evolution Strategy (CMA-ES) is a comparison-based stochastic search algorithm for continuous optimization. It is recognized as a state-of-the-art search algorithm for black-box continuous optimization problems with several difficulties such as non-convexity, ill-conditioning, and non-separability. One of the strong points of the CMA-ES is its quasi-parameter-free feature. All of the parameters such as the population size and the learning rates are set depending only on the search space dimension D.

When the objective function is noisy, it is known that a larger population size helps to reach a better solution. However, finding a reasonable population size in advance is often a prohibitively expensive task. In the reference [4], a restart strategy with increasing population sizes is applied, where designing a reasonable stopping condition for noisy optimization is rather important and the standard termination conditions used for noiseless testbed usually fails. In the reference [9], a mechanism to adapt the population size is proposed for the rank-μ update CMA-ES. It is based on the accuracy of the update of the distribution parameters such as the mean vector and the covariance matrix of the multivariate normal distribution. It is shown that the population adaptation mechanism works both for well-structured multimodal functions and for noisy functions.

In this paper, we evaluate the rank-μ update CMA-ES with the population size adaptation mechanism [9] on the BBOB noisy testbed. We compare four variants of the population size adaptation mechanism with the BIPOP-CMA-ES [4].

GECCO'16 Companion, July 20 - 24, 2016, Denver, CO, USA

ISBN 978-1-4503-4323-7/16/07...$15.00

DOI: http://dx.doi.org/10.1145/2908961.2931701

2. ALGORITHM

The population size adaptation mechanism is proposed in [9]. It estimates the accuracy of the update of the distribution parameters, i.e., the mean vector $\boldsymbol{m}$ and the covariance matrix $\mathbf{C}$ of the multivariate normal distribution $\mathcal{N}(\boldsymbol{m}, \mathbf{C})$. The population size λ is increased or decreased if the estimated accuracy of the parameter update is less or greater than a given threshold. The threshold is determined by the expected accuracy of the parameter update on a random function. The population size adaptation mechanism is applied to the pure rank-μ update $(\mu/\mu_w, \lambda)$-CMA-ES. The pseudocode of the algorithm is described below.

Algorithm 1 Rank-μ Update CMA-ES with λ-Adaptation

Require: $\boldsymbol{m}$, $\mathbf{C}$, $c_{\boldsymbol{m}}$, α, and initialize $\lambda = \lambda_{\min} = 4$, $c_\mu = c_{\boldsymbol{m}}/\sqrt{(D+1)/2}$, $\beta = \min(c_{\boldsymbol{m}}, 0.9)$.

repeat
- $\mu \leftarrow \lfloor \lambda/2 \rfloor$, $w_i \leftarrow (\ln((\lambda+1)/2) - \ln(i))/\sum_{i=1}^{\mu}(\ln((\lambda+1)/2) - \ln(i))$ for $i = 1, \ldots, \lambda$
- Sample $x_i \sim \mathcal{N}(\boldsymbol{m}, \mathbf{C})$ for $i = 1, \ldots, \lambda$
- Evaluate $f(x_i)$ for $i = 1, \ldots, \lambda$
- $x_{i:\lambda} \leftarrow$ the ith best among $\{x_i\}_{i=1,\ldots,\lambda}$ w.r.t. $f(x_i)$
- $\boldsymbol{m}' \leftarrow \boldsymbol{m}$, $\mathbf{C}' \leftarrow \mathbf{C}$ ▷ keep old values
- $\mathbf{C} \leftarrow \mathbf{C} + c_\mu \sum_{i=1}^{\mu}[w_i(x_{i:\lambda} - \boldsymbol{m})(x_{i:\lambda} - \boldsymbol{m})^{\mathrm{T}} - \mathbf{C}]$
- $\boldsymbol{m} \leftarrow \boldsymbol{m} + c_{\boldsymbol{m}} \sum_{i=1}^{\mu} w_i(x_{i:\lambda} - \boldsymbol{m})$
- $\boldsymbol{p}_{\boldsymbol{m}} \leftarrow (1-\beta)\boldsymbol{p}_{\boldsymbol{m}} + \sqrt{\beta(2-\beta)}(\boldsymbol{m} - \boldsymbol{m}')$
- $\mathbf{P}_{\boldsymbol{C}} \leftarrow (1-\beta)\mathbf{P}_{\boldsymbol{C}} + \sqrt{\beta(2-\beta)}(\mathbf{C} - \mathbf{C}')$
- $\gamma \leftarrow (1-\beta)^2\gamma + \beta(2-\beta)(c_{\boldsymbol{m}}^2 D + c_\mu^2 D(D+1)/2)\sum_{i=1}^{\mu} w_i^2$
- $\Delta = \boldsymbol{p}_{\boldsymbol{m}}^{\mathrm{T}}(\mathbf{C}')^{-1}\boldsymbol{p}_{\boldsymbol{m}} + \mathrm{Tr}((\mathbf{P}_{\mathbf{C}}(\mathbf{C}')^{-1})^2)/2$
- **if** $\Delta/\gamma < \alpha$ **then**
  - $\lambda \leftarrow \lfloor \lambda \exp(\beta(\alpha - \Delta/\gamma)) \rfloor \vee \lambda + 1$
- **else**
  - $\lambda \leftarrow \lfloor \lambda \exp(\beta(\alpha - \Delta/\gamma)) \rfloor \vee \lambda_{\min}$

until a termination condition is met.

Note that the mean vector update and the rank-μ covariance matrix update take the stochastic natural gradient step [1,10]. As written in [10], the stochastic natural gradient converges as $\lambda \to \infty$ on noiseless problems. Moreover, it has been shown in [9] that it converges to a different limit as $\lambda \to \infty$ on noisy problems. It implies that the accuracy of the parameter update, i.e., Δ/γ, can be arbitrarily high as we increase the population size. Indeed, $\gamma \to 0$ as $\lambda \to \infty$. Therefore, there exists a stationary population size in both cases.

In this paper, a simple restart strategy is employed, where the mean vector is drawn from $\mathcal{U}[-4,4)^D$ and the initial covariance matrix is $2^2\mathbf{I}$ for each restart. The termination conditions for each (re-)start are as follows:

tolf `median(fiqr_hist)` $< 10^{-12}$`abs(median(fmin_hist))`, where `fiqr_hist` and `fmin_hist` are the histories of the interquartile ranges and the minimum values, respectively, of the function values for the last 20 iterations. If this is true, we consider that the objective function value differences are too small to sort them without being affected by numerical errors.

tolx `median(xiqr_hist`$_i$`)` $< 10^{-12}$`min(abs(xmed_hist`$_i$`))` for some $i = 1, \ldots, D$, where `xiqr_hist`$_i$ and `xmed_hist`$_i$ are the histories of the interquartile ranges and the median values, respectively, of the ith coordinate of the candidate solutions for the last 20 iterations. If this is true, we consider that the coordinate value differences are too small to update parameters without being affected by numerical errors.

maxcond `cond`$(\mathbf{C}) > 10^{-14}$, where `cond`$(\mathbf{C})$ is the condition number, i.e., the ratio of the maximum and minimum eigenvalues of $\mathbf{C}$. If this is true, we consider the matrix operations using $\mathbf{C}$ are not reliable due to numerical errors.

maxeval $\#f$-call $\geqslant 10^5 D$, while the maximum number of function calls per function instance is $10^5 D$.

This is the same setting as the noiseless benchmarking except `maxeval`. As is written in [4], standard termination conditions usually fail on noisy problems. In our case, the last two conditions, `maxcond` and `maxeval`, are relevant. The condition number may exceed its maximal value when the population size is small while the signal-to-noise is very low. In this case, the distribution parameters do a random walk on the parameter space and the condition number of the covariance matrix tends to diverse.

We test the following algorithm variants.

PSAaLmC $\alpha = \sqrt{2}$, $c_m = 0.1$

PSAaLmD $\alpha = \sqrt{2}$, $c_m = 1/D$

PSAaSmC $\alpha = 1.1$, $c_m = 0.1$

PSAaSmD $\alpha = 1.1$, $c_m = 1/D$

Here, c_m is the learning rate for the mean vector update, α is the threshold for the population size adaptation. The greater the α is, the higher the population size is kept. We compare these variants of the population size adaptation mechanism with the BIPOP-CMA-ES, for which the data is taken from [4]. For the CPU timing, refer to [3] for BIPOP-CMA-ES and refer to [8] for PSAaLmD and the other variants.

3. RESULTS

Results from experiments according to [5] on the benchmark functions given in [2,6] are presented in Figures 1, 2, and 3 and Tables 1, 2, and 3. The **average running time** (aRT), used in the figures and tables, depends on a given target function value, $f_t = f_{opt} + \Delta f$, and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach f_t, summed over all trials and divided by the number of trials that actually reached f_t [5,11]. **Statistical significance** is tested with the rank-sum test for a given target Δf_t using, for each trial, either the number of needed function evaluations to reach Δf_t (inverted and multiplied by -1), or, if the target was not reached, the best Δf-value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration if available.

From the results, we observe that the rank-μ update CMA-ES with the population size adaptation mechanism is not efficient on the Sphere functions. It is due to the lack of the step-size adaptation and is observed also on the noiseless testbed. The failure on the Rosenbrock functions is mainly because of the same reason.

On the noiseless testbed, we find that the variants of rank-μ update CMA-ES with the population size adaptation work effectively and are competitive with the best 2009 portfolio on the Step-Ellipsoid function and on the well-structured multimodal functions. On the noisy testbed, we still observe a competitive performance on the Step-Ellipsoid functions. On well-structured multimodal functions such as Schaffer functions, we observe a similar tendency for low dimensions, $D \leqslant 5$. For higher dimensional cases, the algorithms have been interrupted by the maximum number of function evaluations and were not able to reach the target function values.

4. SUMMARY AND FUTURE WORK

The population size adaptation mechanism for the pure rank-μ update CMA-ES has been benchmarked on the BBOB noisy testbed. Generally, the algorithm is slower than the BIPOP-CMA-ES on unimodal functions, not because that the population size increases, but because the step-size adaptation is not yet incorporated. On a function where the step-size adaptation is less important such as the Step-Ellipsoid function, the rank-μ update CMA-ES with the population size adaptation is competitive with the best 2009 portfolio. It is consistent with the result on the BBOB noiseless testbed. On well-structured multimodal functions such as Schaffer functions, we have observed a promising performance on low dimensional problems. To study the effect of the population size mechanism on noisy multimodal functions more carefully, we need to run the algorithm with a larger budget.

In this work we focused on the effect of the population size for noisy and multimodal functions. Another approach to noisy optimization is to increase the number of function evaluation per candidate solution and average them to estimate the expected objective value [7]. Comparing the effect of increasing population size and the effect of increasing the number of resampling of the function value will be an interesting future work.

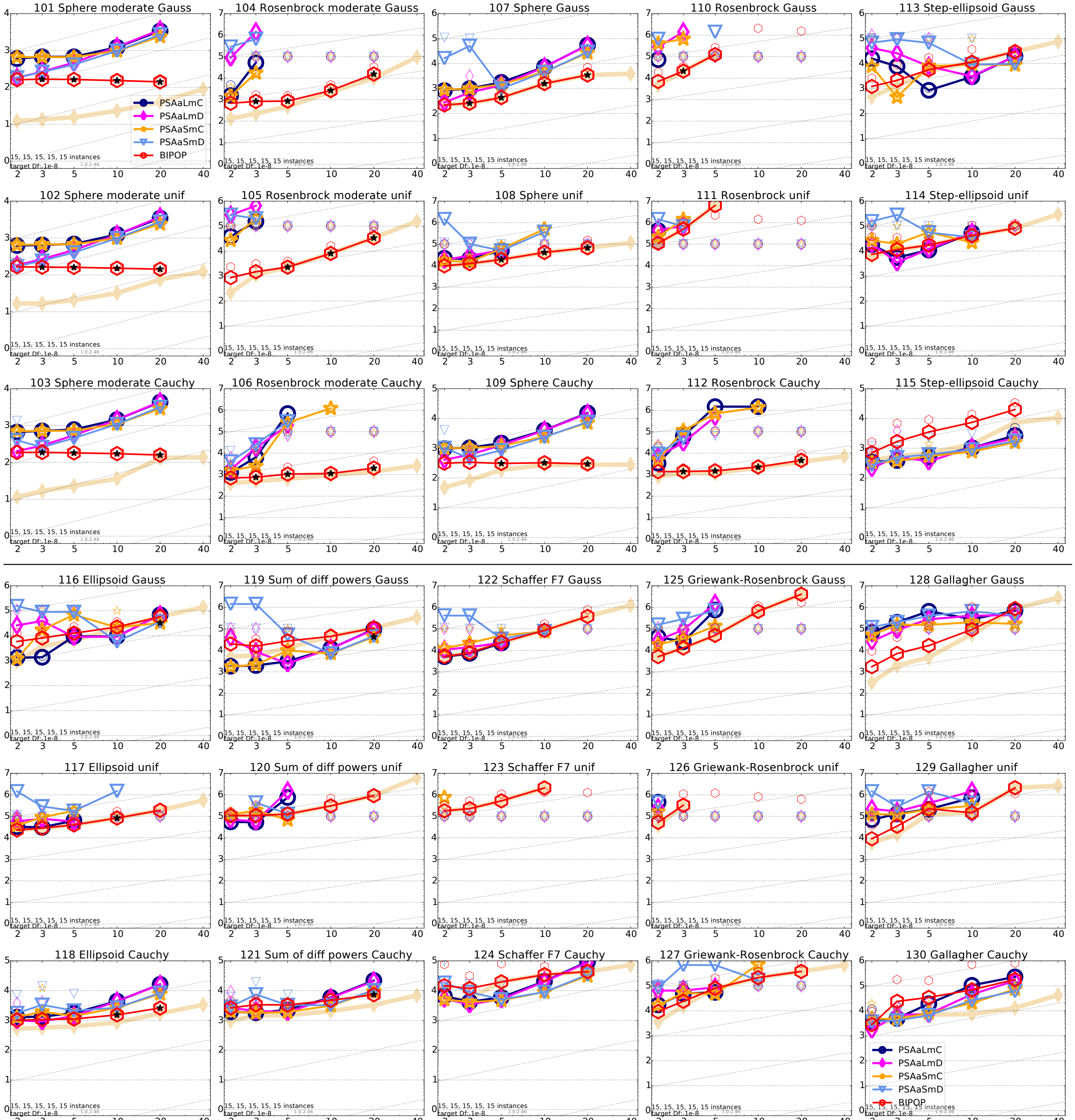

Figure 1: Average running time (aRT in number of f-evaluations as $\log_{10}$ value), divided by dimension for target function value 10^{-8} versus dimension. Slanted grid lines indicate quadratic scaling with the dimension. Different symbols correspond to different algorithms given in the legend of f_{101} and f_{130}. Light symbols give the maximum number of function evaluations from the longest trial divided by dimension. Black stars indicate a statistically better result compared to all other algorithms with $p < 0.01$ and Bonferroni correction number of dimensions (six). Legend: ○: PSAaLmC, ◇: PSAaLmD, ⋆: PSAaSmC, ▽: PSAaSmD, ⬡: BIPOP

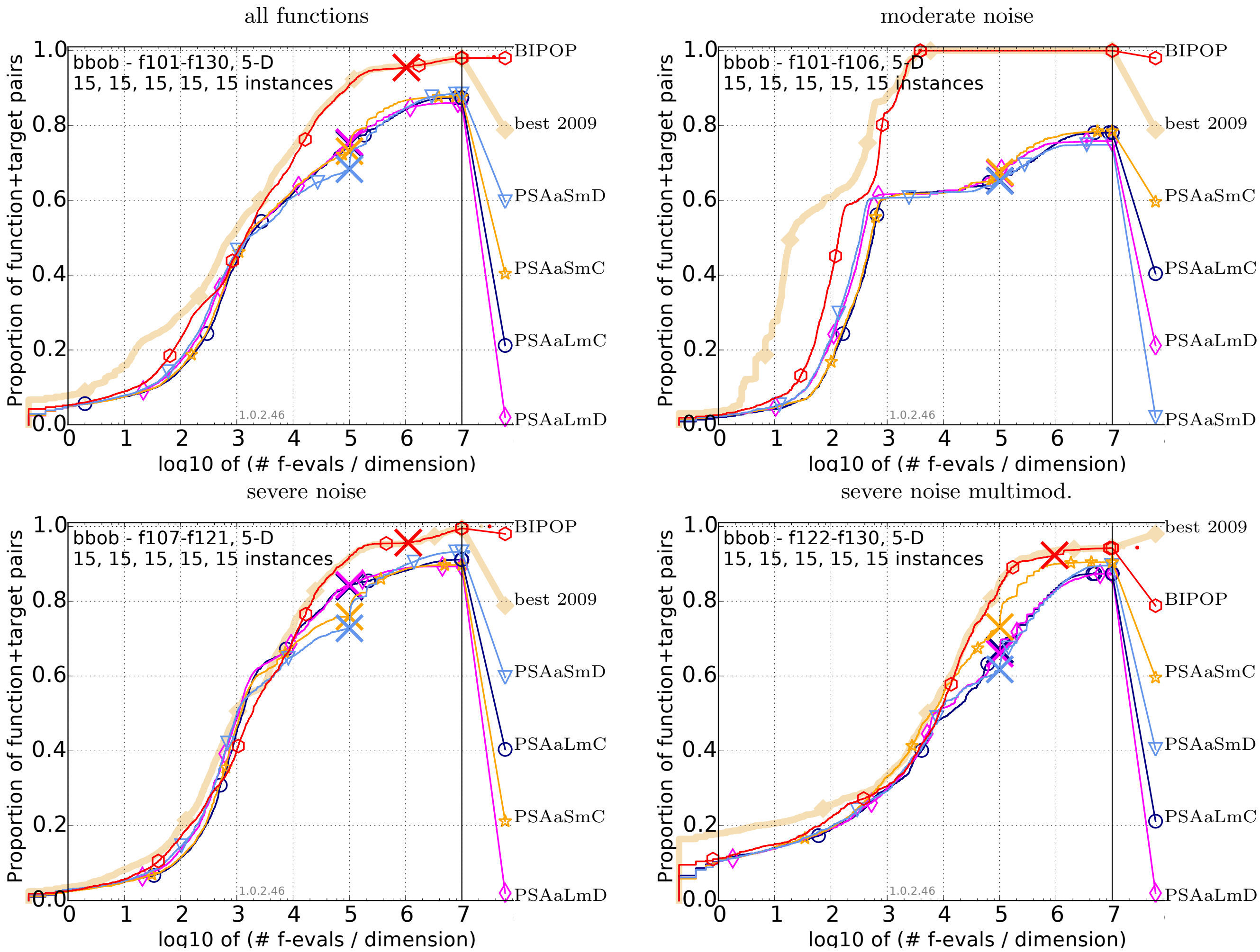

Figure 2: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 51 **targets with target precision in** $10^{[-8..2]}$ **for all functions and subgroups in 5-D. The "best 2009" line corresponds to the best** aRT **observed during BBOB 2009 for each selected target.**

Acknowledgements. This work is partially supported by JSPS KAKENHI Grant Number 15K16063 and the Kayamori Foundation of Informational Science Advancement.

5. REFERENCES

[1] Y. Akimoto, Y. Nagata, I. Ono, and S. Kobayashi. Bidirectional relation between CMA evolution strategies and natural evolution strategies. In *Parallel Problem Solving from Nature – PPSN XI*, pages 154–163, Springer-Verlag, 2010.

[2] S. Finck, N. Hansen, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2010: Presentation of the noisy functions. Technical Report 2009/21, Research Center PPE, 2010.

[3] N. Hansen. Benchmarking a BI-population CMA-ES on the BBOB-2009 function testbed. In *Workshop Proceedings of the GECCO Genetic and Evolutionary Computation Conference*, pages 2389–2395, New York, New York, USA, 2009. ACM Press.

[4] N. Hansen. Benchmarking a BI-population CMA-ES on the BBOB-2009 noisy testbed. In *GECCO '09: Proceedings of the 11th Annual Conference Companion on Genetic and Evolutionary Computation Conference: Late Breaking Papers*. ACM Request Permissions, July 2009.

[5] N. Hansen, A. Auger, S. Finck, and R. Ros. Real-parameter black-box optimization benchmarking 2012: Experimental setup. Technical report, INRIA, 2012.

[6] N. Hansen, S. Finck, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Noisy functions definitions. Technical Report RR-6869, INRIA, 2009. Updated February 2010.

[7] N. Hansen, A. S. P. Niederberger, L. Guzzella, and P. Koumoutsakos. A Method for Handling Uncertainty in Evolutionary Optimization With an Application to Feedback Control of Combustion. *IEEE Transactions on Evolutionary Computation*, 13(1):180–197, 2009.

[8] K. Nishida and Y. Akimoto. Evaluating the population size adaptation mechanism for CMA-ES on the BBOB

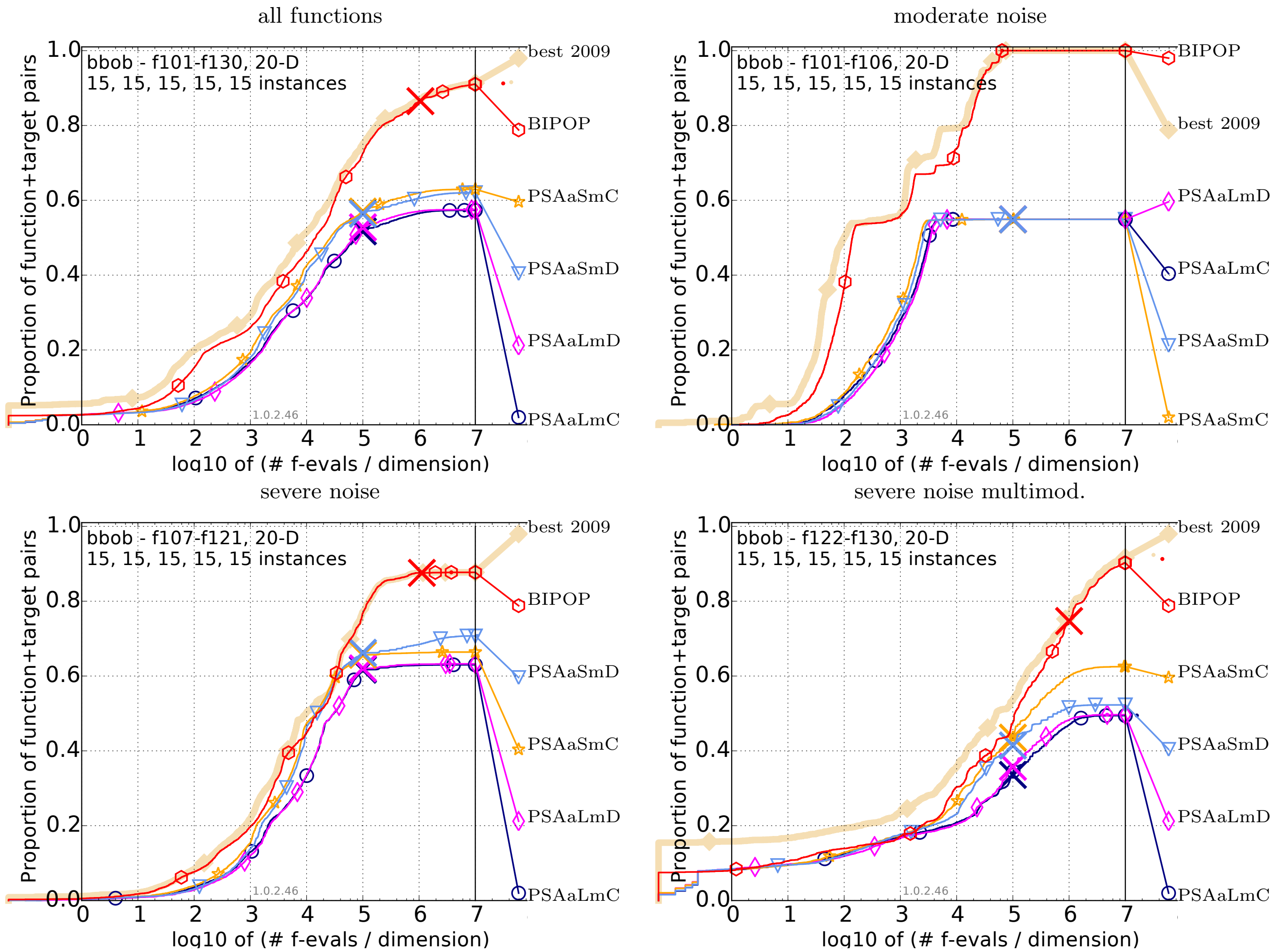

Figure 3: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 51 targets with target precision in $10^{[-8..2]}$ for all functions and subgroups in 20-D. The "best 2009" line corresponds to the best aRT observed during BBOB 2009 for each selected target.

noiseless testbed. In *Workshop Proceedings of Genetic and Evolutionary Computation Conference*, ACM, 2016. To appear.

[9] K. Nishida and Y. Akimoto. Population size adaptation for the cma-es based on the estimation accuracy of the natural gradient. In *Genetic and Evolutionary Computation Conference, GECCO 2016, Denver, Colorado, USA, July 20-24, 2016*, ACM, 2016. To appear.

[10] Y. Ollivier, L. Arnold, A. Auger, and N. Hansen. Information-geometric optimization algorithms: A unifying picture via invariance principles. *Preprint arXiv:1106.3708*.

[11] K. Price. Differential evolution vs. the functions of the second ICEO. In *Proceedings of the IEEE International Congress on Evolutionary Computation*, pages 153–157, 1997.

Table 1: Average running time (aRT in number of function evaluations) divided by the respective best aRT measured during BBOB-2009 in dimension 5 (left) and dimension 20 (right) on f_{101}–f_{110}. The aRT and in braces, as dispersion measure, the half difference between 10 and 90%-tile of bootstrapped run lengths appear for each algorithm and target, the corresponding best aRT in the first row. The different target Δf-values are shown in the top row. #succ is the number of trials that reached the (final) target $f_{\mathrm{opt}} + 10^{-8}$. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries, succeeded by a star, are statistically significantly better (according to the rank-sum test) when compared to all other algorithms of the table, with $p = 0.05$ or $p = 10^{-k}$ when the number k following the star is larger than 1, with Bonferroni correction by the number of instances. A ↓ indicates the same tested against the best algorithm of BBOB-2009.Best results are printed in bold.

Dimension 5:

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f101	11	37	44	49	62	69	75	15/15
aLmC	8.4(8)	8.5(4)	14(3)	18(5)	20(3)	31(4)	40(3)	15/15
aLmD	7.8(7)	7.1(2)	8.7(3)	13(3)	14(2)	21(2)	28(2)	15/15
aSmC	8.3(8)	9.0(5)	12(5)	17(3)	20(3)	30(3)	38(5)	15/15
aSmD	6.9(5)	6.2(3)	8.4(3)	12(0.9)	13(3)	19(2)	24(1)	15/15
BIPOP	**3.2**(1)	**3.1**(0.7)$^{\star 2}$	**4.6**(0.6)$^{\star 3}$	**6.0**(0.9)$^{\star 2}$	**6.1**(0.7)$^{\star 4}$	**8.0**(1)$^{\star 4}$	**10**(0.5)$^{\star 4}$	15/15
f102	11	35	50	66	72	86	99	15/15
aLmC	9.5(8)	9.2(7)	12(4)	14(2)	19(2)	26(3)	31(3)	15/15
aLmD	6.3(2)	5.9(3)	7.0(2)	9.3(3)	12(2)	17(2)	21(2)	15/15
aSmC	11(6)	9.4(3)	11(2)	13(2)	17(3)	25(3)	28(2)	15/15
aSmD	9.3(10)	6.5(1)	7.7(2)	8.7(2)	11(2)	15(2)	18(2)	15/15
BIPOP	**2.7**(1)	**3.0**(2)*	**4.0**(0.6)$^{\star 2}$	**4.3**(0.8)$^{\star 3}$	**5.1**(0.5)$^{\star 4}$	**6.3**(0.7)$^{\star 4}$	**7.2**(0.8)$^{\star 4}$	15/15
f103	11	28	30	30	31	35	115	15/15
aLmC	13(7)	12(4)	19(5)	30(7)	43(5)	67(5)	29(3)	15/15
aLmD	8.2(4)	8.4(3)	13(3)	21(4)	27(3)	44(2)	20(2)	15/15
aSmC	8.6(8)	12(3)	18(2)	28(4)	39(7)	59(7)	27(1)	15/15
aSmD	6.9(6)	8.4(5)	13(5)	18(3)	25(3)	37(5)	17(1)	15/15
BIPOP	**3.5**(3)	**4.7**(1)$^{\star 2}$	**7.4**(1)$^{\star 3}$	**10**(1)$^{\star 4}$	**13**(0.6)$^{\star 4}$	**17**(2)$^{\star 4}$	**6.9**(0.5)$^{\star 4}$	15/15
f104	173	773	1287	1574	1768	2040	2284	15/15
aLmC	4.1(1)	331(1144)	5502(5302)	4503(5216)	∞	∞	∞ *5e5*	0/15
aLmD	3.4(0.8)	572(651)	∞	∞	∞	∞	∞ *5e5*	0/15
aSmC	4.4(0.9)	577(655)	1578(1476)	4510(7243)	∞	∞	∞ *5e5*	0/15
aSmD	2.5(1.0)	979(1790)	∞	∞	∞	∞	∞ *5e5*	0/15
BIPOP	**1.4**(0.4)$^{\star 2}$	**1.9**(0.6)$^{\star 2}$	**2.0**(0.9)$^{\star 3}$	**2.0**(0.6)$^{\star 4}$	**2.0**(0.6)$^{\star 4}$	**1.9**(0.6)$^{\star 4}$	**1.8**(0.2)$^{\star 4}$	15/15
f105	167	1436	5174	9998	10388	10824	11202	15/15
aLmC	4.3(2)	239(180)	∞	∞	∞	∞	∞ *5e5*	0/15
aLmD	3.2(0.7)	258(438)	∞	∞	∞	∞	∞ *5e5*	0/15
aSmC	3.9(1)	405(529)	∞	∞	∞	∞	∞ *5e5*	0/15
aSmD	2.9(0.9)	1402(2014)	∞	∞	∞	∞	∞ *5e5*	0/15
BIPOP	**1.7**(0.3)*	**3.7**(3)	**1.7**(0.7)$^{\star 4}$	**1**(0.4)$^{\star 4}$	**1**(0.5)$^{\star 4}$	**1**(0.4)$^{\star 4}$	**1**(0.5)$^{\star 4}$	15/15
f106	92	529	1050	1770	2666	2887	3087	15/15
aLmC	8.3(2)	17(62)	384(553)	564(555)	1308(1400)	1213(1856)	1134(877)	2/15
aLmD	5.8(2)	10(3)	212(230)	263(167)	279(391)	344(209)	322(227)	3/15
aSmC	8.1(5)	31(26)	230(324)	214(179)	220(504)	456(305)	427(366)	5/15
aSmD	5.1(2)	43(128)	447(568)	549(704)	365(406)	513(618)	480(883)	2/15
BIPOP	**3.3**(1)	**4.3**(6)	**3.2**(3)$^{\star 2}$	**2.3**(0.2)$^{\star 2}$	**1.6**(1)$^{\star 2}$	**1.7**(1)$^{\star 2}$	**1.7**(0.1)$^{\star 2}$	15/15
f107	40	228	453	692	940	1376	1850	15/15
aLmC	5.1(5)	3.2(1)	2.8(1)	2.9(0.7)	3.3(0.6)	3.9(0.5)	4.2(0.2)	15/15
aLmD	6.6(8)	3.1(1)	2.8(0.7)	2.5(0.7)	2.8(0.6)	3.0(0.3)	3.2(0.5)	15/15
aSmC	6.2(6)	3.5(3)	2.9(2)	2.9(1)	2.9(1)	3.3(0.4)	3.4(0.3)	15/15
aSmD	2.6(4)	1.9(2)	1.9(0.7)	1.9(0.2)	2.1(0.4)	2.2(0.4)	2.4(0.2)	15/15
BIPOP	**1.7**(2)	**1**(0.6)	**1**(0.5)$^{\star 2}$	**1**(0.3)$^{\star 3}$	**1**(0.2)$^{\star 4}$	**1**(0.2)$^{\star 4}$	**1**(0.1)$^{\star 4}$	15/15
f108	87	5144	14469	24649	30935	58628	80667	15/15
aLmC	40(24)	1.7(1.0)	1.0(0.8)	1.2(0.4)	1.6(0.3)	1.9(0.3)	2.4(0.2)	15/15
aLmD	8.3(6)	**0.91**(0.6)	**0.82**(0.5)	1.1(0.3)	1.4(0.2)	1.9(0.2)	2.5(0.3)	15/15
aSmC	40(100)	11(9)	7.3(6)	5.5(5)	6.8(4)	4.1(6)	3.5(2)	11/15
aSmD	25(108)	16(10)	8.3(17)	5.8(11)	4.9(13)	3.1(0.1)	2.8(3)	12/15
BIPOP	**6.1**(6)	1.0(0.9)	1(1)	**1**(0.5)	**1**(0.6)	**1**(0.5)	**1**(0.3)	15/15
f109	11	57	216	375	572	873	946	15/15
aLmC	7.9(10)	5.6(2)	2.6(0.8)	2.8(0.5)	3.1(1)	4.4(0.8)	6.5(0.9)	15/15
aLmD	8.7(7)	4.3(2)	2.0(0.9)	2.3(1)	3.1(0.7)	4.0(0.7)	5.6(0.7)	15/15
aSmC	8.4(7)	5.1(2)	2.6(0.6)	2.8(0.7)	3.3(0.8)	4.1(0.5)	5.4(0.7)	15/15
aSmD	5.9(4)	3.9(1)	1.7(0.6)	2.0(0.8)	2.3(0.6)	2.9(0.7)	4.0(0.4)	15/15
BIPOP	**3.5**(2)	**2.2**(1)*	**1.1**(0.3)*	**1**(0.3)$^{\star 4}$	**1.1**(0.2)$^{\star 3}$	**1.1**(0.3)$^{\star 4}$	**1.5**(0.2)$^{\star 4}$	15/15
f110	949	33625	1.2e5	5.6e5	5.9e5	6.0e5	6.1e5	15/15
aLmC	1.3(0.9)	24(59)	59(70)	13(15)	∞	∞	∞ *5e5*	0/15
aLmD	0.86(0.4)	99(211)	∞	∞	∞	∞	∞ *5e5*	0/15
aSmC	1.1(0.3)	105(104)	∞	∞	∞	∞	∞ *5e5*	0/15
aSmD	**0.82**(0.2)	34(33)	59(91)	13(6)	12(7)	12(9)	12(12)	1/15
BIPOP	1(2)	**4.8**(1)	**3.7**(4)*	**1**(0.5)	**1**(0.4)	**1**(0.9)	**1**(0.7)	15/15

Dimension 20:

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f101	59	425	571	677	700	739	783	15/15
aLmC	23(6)	7.5(2)	11(1)	19(3)	31(4)	55(1)	75(2)	15/15
aLmD	32(8)	10(1)	14(2)	21(2)	34(3)	58(3)	81(2)	15/15
aSmC	21(3)	6.2(0.8)	8.0(0.7)	14(3)	23(3)	39(2)	53(1)	15/15
aSmD	30(8)	8.5(1)	11(2)	18(2)	27(2)	45(3)	61(2)	15/15
BIPOP	**6.1**(1)$^{\star 4}$	**1.5**(0.2)$^{\star 4}$	**1.6**(0.1)$^{\star 4}$	**1.8**(0.1)$^{\star 4}$	**2.1**(0.1)$^{\star 4}$	**2.7**(0.1)$^{\star 4}$	**3.3**(0.2)$^{\star 4}$	15/15
f102	231	399	579	755	921	1157	1407	15/15
aLmC	5.6(0.7)	7.5(1)	11(1)	17(2)	24(2)	35(1)	42(0.7)	15/15
aLmD	7.9(1)	10(1.0)	12(2)	19(3)	27(1)	38(2)	46(1)	15/15
aSmC	4.8(1)	6.2(0.6)	8.4(1)	13(0.9)	18(2)	25(0.7)	30(0.9)	15/15
aSmD	7.2(1)	8.7(0.8)	11(1)	15(2)	21(2)	29(2)	34(0.7)	15/15
BIPOP	**1.6**(0.3)$^{\star 4}$	**1.6**(0.1)$^{\star 4}$	**1.6**(0.2)$^{\star 4}$	**1.6**(0.1)$^{\star 4}$	**1.6**(0.1)$^{\star 4}$	**1.8**(0.1)$^{\star 4}$	**1.8**(0.1)$^{\star 4}$	15/15
f103	65	417	629	1043	1313	1893	2464	14/15
aLmC	21(4)	7.5(1)	10(1)	12(1)	16(1)	23(1)	29(0.6)	15/15
aLmD	29(6)	10(0.8)	12(0.9)	14(2)	18(1)	26(0.5)	31(0.5)	15/15
aSmC	19(4)	6.4(0.9)	7.7(0.8)	9.3(1)	12(1)	16(0.9)	20(0.7)	15/15
aSmD	26(6)	8.6(2)	10(0.7)	12(1)	15(0.9)	19(1)	22(1.0)	15/15
BIPOP	**5.5**(0.8)$^{\star 4}$	**1.6**(0.2)$^{\star 4}$	**1.5**(0.1)$^{\star 4}$	**1.2**(0.1)$^{\star 4}$	**1.2**(0.1)$^{\star 4}$	**1.2**(0.1)$^{\star 4}$	**1.2**(0.1)$^{\star 4}$	15/15
f104	23690	85656	1.7e5	1.8e5	1.8e5	1.9e5	2.0e5	15/15
aLmC	∞	∞	∞	∞	∞	∞	∞ *2e6*	0/15
aLmD	∞	∞	∞	∞	∞	∞	∞ *2e6*	0/15
aSmC	∞	∞	∞	∞	∞	∞	∞ *2e6*	0/15
aSmD	∞	∞	∞	∞	∞	∞	∞ *2e6*	0/15
BIPOP	**10**(9)$^{\star 4}$	**3.2**(2)$^{\star 4}$	**1.7**(1)$^{\star 4}$	**1.7**(1)$^{\star 4}$	**1.6**(0.5)$^{\star 4}$	**1.6**(0.9)$^{\star 4}$	**1.6**(1.0)$^{\star 4}$	15/15
f105	1.9e5	6.1e5	6.3e5	6.4e5	6.5e5	6.6e5	6.7e5	15/15
aLmC	∞	∞	∞	∞	∞	∞	∞ *2e6*	0/15
aLmD	∞	∞	∞	∞	∞	∞	∞ *2e6*	0/15
aSmC	∞	∞	∞	∞	∞	∞	∞ *2e6*	0/15
aSmD	∞	∞	∞	∞	∞	∞	∞ *2e6*	0/15
BIPOP	**2.7**(2)$^{\star 4}$	**1**(0.6)$^{\star 4}$	**1**(0.5)$^{\star 4}$	**1**(0.4)$^{\star 4}$	**1**(0.6)$^{\star 4}$	**1**(0.4)$^{\star 4}$	**1**(0.6)$^{\star 4}$	15/15
f106	11480	21668	23746	24788	25470	26492	27360	15/15
aLmC	∞	∞	∞	∞	∞	∞	∞ *2e6*	0/15
aLmD	∞	∞	∞	∞	∞	∞	∞ *2e6*	0/15
aSmC	∞	∞	∞	∞	∞	∞	∞ *1e6*	0/15
aSmD	∞	∞	∞	∞	∞	∞	∞ *2e6*	0/15
BIPOP	**1.0**(0.2)$^{\star 4}$	**1.3**(0.8)$^{\star 4}$	**1.4**(0.5)$^{\star 4}$	**1.4**(0.2)$^{\star 4}$	**1.5**(0.7)$^{\star 4}$	**1.5**(0.6)$^{\star 4}$	**1.5**(0.6)$^{\star 4}$	15/15
f107	8571	13582	16226	21100	27357	52486	65052	15/15
aLmC	1.6(0.7)	2.6(0.6)	4.9(0.3)	7.2(1)	11(1)	12(0.5)	15(0.8)	15/15
aLmD	2.0(0.5)	3.2(0.6)	5.3(1.0)	7.4(1)	11(0.9)	12(0.4)	14(0.4)	15/15
aSmC	1.1(0.3)	1.6(0.3)	2.5(0.2)	4.2(0.6)	5.8(0.6)	5.9(0.4)	7.1(0.3)	15/15
aSmD	1.5(0.6)	2.1(0.4)	3.2(0.3)	4.8(0.6)	6.4(0.7)	6.5(0.3)	7.7(0.3)	15/15
BIPOP	**1**(0.2)	**1**(0.8)*	**1**(0.7)$^{\star 3}$	**1**(0.4)$^{\star 4}$	**1**(0.4)$^{\star 4}$	**1**(0.8)$^{\star 4}$	**1**(0.8)$^{\star 4}$	15/15
f108	58063	97228	2.0e5	4.0e5	4.5e5	6.3e5	9.0e5	15/15
aLmC	2.6(0.9)	5.6(1)	6.4(1)	75(130)	∞	∞	∞ *2e6*	0/15
aLmD	2.4(0.9)	6.0(1)	6.4(0.9)	74(59)	∞	∞	∞ *2e6*	0/15
aSmC	1.4(0.5)	2.3(0.4)	2.4(0.5)	2.7(0.3)	∞	∞	∞ *2e6*	0/15
aSmD	1.5(0.6)	2.5(0.6)	2.5(0.4)	2.8(0.5)	68(109)	∞	∞ *2e6*	0/15
BIPOP	**1**(0.4)	**1**(0.6)$^{\star 3}$	**1**(0.5)$^{\star 3}$	**1**(0.5)$^{\star 4}$	**1**(0.5)$^{\star 4}$	**1**(0.5)$^{\star 4}$	**1**(1)$^{\star 4}$	15/15
f109	333	632	1138	1679	2287	3583	4952	15/15
aLmC	4.2(0.4)	5.1(0.7)	6.2(1)	11(2)	22(3)	42(3)	51(2)	15/15
aLmD	6.0(2)	6.9(1)	7.1(1)	12(2)	23(3)	42(2)	49(2)	15/15
aSmC	3.6(0.8)	4.4(1)	4.7(0.9)	8.7(0.9)	15(1)	21(0.9)	24(1)	15/15
aSmD	5.2(1)	6.0(0.7)	6.2(0.5)	10(1)	17(1)	23(1)	26(2)	15/15
BIPOP	**1.2**(0.2)$^{\star 4}$	**1.2**(0.1)$^{\star 4}$	**1.1**(0.2)$^{\star 4}$	**1.1**(0.1)$^{\star 4}$	**1.1**(0.2)$^{\star 4}$	**1.1**(0.1)$^{\star 4}$	**1.0**(0.2)$^{\star 4}$	15/15
f110	∞	∞	∞	∞	∞	∞	∞	0
aLmC	.	.	.	.	.	.	.	0/15
aLmD	.	.	.	.	.	.	.	0/15
aSmC	.	.	.	.	.	.	.	0/15
aSmD	.	.	.	.	.	.	.	0/15
BIPOP	.	.	.	.	.	.	.	0/15

Table 2: Average running time (aRT in number of function evaluations) divided by the respective best aRT measured during BBOB-2009 in dimension 5 (left) and dimension 20 (right) on f_{111}–f_{120}. The aRT and in braces, as dispersion measure, the half difference between 10 and 90%-tile of bootstrapped run lengths appear for each algorithm and target, the corresponding best aRT in the first row. The different target Δf-values are shown in the top row. #succ is the number of trials that reached the (final) target $f_{\mathrm{opt}} + 10^{-8}$. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries, succeeded by a star, are statistically significantly better (according to the rank-sum test) when compared to all other algorithms of the table, with $p = 0.05$ or $p = 10^{-k}$ when the number k following the star is larger than 1, with Bonferroni correction by the number of instances. A ↓ indicates the same tested against the best algorithm of BBOB-2009.Best results are printed in bold.

Dimension 5:

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f111	6856	6.1e5	8.8e6	2.3e7	2.3e7	3.1e7	3.1e7	3/15
aLmC	0.70(0.2)	12(9)	∞	∞	∞	∞	∞ *5e5*	0/15
aLmD	**0.44**(0.2)	∞	∞	∞	∞	∞	∞ *5e5*	0/15
aSmC	0.63(0.6)	12(8)	∞	∞	∞	∞	∞ *5e5*	0/15
aSmD	5.7(19)	12(7)	**0.80**(1)	**0.32**(0.4)	**0.33**(0.4)	∞	∞ *5e5*	0/15
BIPOP	1(0.2)	**2.5**(6)	1(0.6)	1(1)	1(0.9)	**1**(0.8)	**1**(2)	3/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f112	107	1684	3421	4162	4502	5132	5596	15/15
aLmC	7.7(2)	31(49)	101(86)	841(1010)	1623(1946)	1424(1183)	1306(1272)	1/15
aLmD	4.9(1)	5.6(8)	172(365)	167(62)	250(146)	490(338)	450(511)	2/15
aSmC	6.8(3)	47(95)	359(226)	825(791)	763(1056)	669(1038)	614(395)	2/15
aSmD	4.2(2)	36(19)	101(77)	261(242)	∞	∞	∞ *2e5*	0/15
BIPOP	**4.0**(6)	**1**(0.8)^{*2}	**1.2**(0.5)^{*2}	**1.2**(0.2)^{*2}	**1.3**(0.3)^{*2}	**1.3**(0.2)^{*2}	**1.3**(0.3)^{*3}	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f113	133	1883	8081	24021	24128	24128	24402	15/15
aLmC	2.2(4)	0.54(0.2)	**0.31**(0.1)	**0.15**(0.0)	**0.16**(0.0)	**0.16**(0.0)	**0.17**(0.0)	15/15
aLmD	2.1(2)	**0.53**(0.3)	4.7(31)	1.6(5)	1.6(5)	1.6(5)	1.6(10)	14/15
aSmC	3.3(0.8)	0.71(0.6)	4.7(0.1)	1.6(10)	1.6(5)	1.6(5)	1.6(5)	14/15
aSmD	2.4(2)	42(0.4)	23(47)	14(26)	14(21)	14(36)	14(26)	9/15
BIPOP	**1.5**(0.9)	1.3(1)	1.7(2)	1.1(1)	1.1(1)	1.1(1)	1.1(1)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f114	767	14720	56311	78890	83272	83272	84949	15/15
aLmC	**1.4**(2)	**0.56**(0.4)	**0.40**(0.2)	**0.59**(0.1)	**0.63**(0.1)	**0.63**(0.1)	**0.64**(0.1)	15/15
aLmD	5.6(10)	0.91(2)	0.49(0.4)	0.62(0.4)	0.68(0.2)	0.68(0.2)	0.67(0.3)	15/15
aSmC	3.4(7)	3.2(17)	3.6(5)	3.6(7)	3.4(3)	3.4(3)	3.4(5)	10/15
aSmD	4.5(12)	6.1(0.2)	3.5(9)	3.5(6)	3.3(6)	3.3(8)	3.3(3)	10/15
BIPOP	2.2(2)	1(0.4)	1(0.5)	1(0.6)	1(0.6)	1(0.7)	1(0.7)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f115	64	485	1829	2274	2550	2550	2970	15/15
aLmC	3.8(3)	1.5(0.5)	0.73(0.2)	0.91(0.1)	0.91(0.2)	0.91(0.3)	0.81(0.1)	15/15
aLmD	2.5(2)	**0.88**(0.2)	**0.43**(0.1)	**0.62**(0.1)	**0.62**(0.1)	**0.62**(0.1)	**0.56**(0.1)	15/15
aSmC	3.2(2)	1.3(0.3)	0.69(0.5)	0.99(1)	0.92(0.2)	0.92(0.2)	0.87(0.5)	15/15
aSmD	3.2(2)	1.3(3)	0.74(0.7)	0.97(0.5)	1.2(1)	1.2(0.5)	1.0(1)	15/15
BIPOP	**1.5**(0.8)	2.6(6)	6.5(5)	6.6(4)	5.9(3)	5.9(8)	5.7(6)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f116	5730	14472	22311	26243	26868	30329	31661	15/15
aLmC	0.37(0.3)	2.7(9)	1.8(6)	1.6(0.1)	1.6(5)	1.5(8)	1.5(8)	14/15
aLmD	**0.29**(0.1)	**0.27**(0.4)	**0.27**(0.5)	**1.5**(5)	**1.5**(0.0)	**1.4**(0.1)	**1.4**(4)	14/15
aSmC	14(44)	13(44)	12(23)	13(19)	13(19)	11(21)	11(8)	9/15
aSmD	17(10)	18(54)	20(28)	17(24)	17(28)	15(46)	14(16)	8/15
BIPOP	1.2(1)	2.0(2)	1.9(2)	2.1(1)	2.1(0.8)	2.0(1)	2.0(2)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f117	26686	76052	1.1e5	1.3e5	1.4e5	1.7e5	1.9e5	15/15
aLmC	1.8(5)	0.86(6)	0.78(2)	0.78(2)	0.87(0.1)	1.0(0.8)	1.4(0.9)	14/15
aLmD	**0.84**(2)	**0.51**(1)	**0.45**(0.4)	**0.51**(0.3)	**0.62**(0.5)	**0.85**(0.4)	1.2(0.2)	15/15
aSmC	7.4(14)	4.4(7)	5.8(8)	5.3(6)	5.9(9)	4.9(6)	4.6(7)	6/15
aSmD	13(28)	7.7(3)	7.0(7)	6.1(12)	5.8(10)	4.7(8)	4.5(3)	6/15
BIPOP	1(0.6)	1(1.0)	1(0.8)	1(0.5)	1(0.3)	1(0.5)	**1**(0.5)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f118	429	1217	1555	1774	1998	2430	2913	15/15
aLmC	2.8(0.6)	1.3(0.2)	1.2(0.3)	1.3(0.2)	1.5(0.2)	2.2(0.2)	2.7(0.2)	15/15
aLmD	1.8(0.5)	**0.88**(0.2)	**0.85**(0.1)	**0.95**(0.1)	**1.1**(0.2)	**1.8**(0.2)	2.0(0.2)	15/15
aSmC	2.3(0.6)	1.1(0.5)	1.0(0.1)	1.2(0.2)	1.5(0.2)	1.8(0.1)	2.1(0.1)	15/15
aSmD	**1.7**(2)	2.4(1)	3.3(11)	3.3(0.8)	3.9(6)	3.7(6)	3.5(5)	15/15
BIPOP	3.2(1)	2.0(0.7)	1.9(0.7)	2.1(0.5)	2.1(0.3)	2.0(0.3)	**1.8**(0.3)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f119	12	657	1136	2363	10372	35296	49747	15/15
aLmC	7.6(8)	1.3(1)	1.6(0.8)	1.3(0.4)	0.47(0.1)	0.26(0.0)	0.26(0.0)	15/15
aLmD	2.9(3)	1.1(0.5)	1.4(2)	1.2(0.8)	**0.41**(0.2)	**0.21**(0.1)	**0.21**(0.0)	15/15
aSmC	7.0(6)	2.1(5)	2.0(3)	1.4(0.3)	0.48(0.1)	0.26(0.0)	0.93(3)	14/15
aSmD	5.5(7)	**0.97**(1)	1.2(0.8)	1.0(0.2)	0.51(1)	2.4(7)	4.1(8)	10/15
BIPOP	**1.9**(3)	1(0.9)	**1**(0.6)	**1**(0.9)	1(0.4)	1.5(0.9)	2.3(1)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f120	16	2900	18698	34491	72438	3.3e5	5.5e5	15/15
aLmC	7.9(12)	1.3(2)	**0.71**(0.3)	1.1(0.4)	1.1(0.4)	0.71(0.1)	0.83(0.1)	2/15
aLmD	19(4)	7.1(8)	1.7(2)	1.7(2)	1.4(0.6)	0.73(0.1)	0.91(0.1)	1/15
aSmC	**4.7**(7)	2.4(0.9)	1.6(4)	1.9(0.7)	1.3(2)	**0.47**(0.0)	**0.52**(0.5)	13/15
aSmD	21(57)	18(24)	3.7(5)	2.6(5)	2.3(3)	0.90(0.8)	1.1(0.7)	9/15
BIPOP	17(14)	**1.1**(0.8)	1(0.7)	**1**(0.4)	**1**(0.5)	1(0.5)	1(0.2)	15/15

Dimension 20:

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f111	∞	∞	∞	∞	∞	∞	∞	0
aLmC	.	.	.	.	.	.	.	0/15
aLmD	.	.	.	.	.	.	.	0/15
aSmC	.	.	.	.	.	.	.	0/15
aSmD	.	.	.	.	.	.	.	0/15
BIPOP	.	.	.	.	.	.	.	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f112	25552	64124	69621	72175	73557	76137	78238	15/15
aLmC	∞	∞	∞	∞	∞	∞	∞ *2e6*	0/15
aLmD	∞	∞	∞	∞	∞	∞	∞ *2e6*	0/15
aSmC	∞	∞	∞	∞	∞	∞	∞ *2e6*	0/15
aSmD	∞	∞	∞	∞	∞	∞	∞ *2e6*	0/15
BIPOP	**1**(0.1)^{*4}	**1.1**(0.8)^{*4}	**1.1**(0.8)^{*4}	**1.2**(0.4)^{*4}	**1.2**(0.4)^{*4}	**1.2**(0.4)^{*4}	**1.2**(0.4)^{*4}	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f113	50123	3.6e5	5.6e5	5.9e5	5.9e5	5.9e5	5.9e5	15/15
aLmC	0.85(0.5)	0.36(0.1)	0.39(0.1)	0.64(0.1)	0.64(0.1)	0.64(0.1)	0.66(0.1)	15/15
aLmD	1.0(0.3)	0.38(0.1)	0.39(0.1)	0.62(0.1)	0.62(0.1)	0.62(0.0)	0.65(0.0)	15/15
aSmC	**0.52**(0.2)	**0.20**(0.1)	**0.20**(0.0)	**0.28**(0.0)^{*2}	**0.28**(0.0)^{*2}	**0.28**(0.0)^{*2}	**0.29**(0.0)*	15/15
aSmD	0.63(0.2)	0.22(0.0)	0.22(0.1)	0.32(0.0)	0.32(0.0)	0.32(0.0)	0.32(0.0)	15/15
BIPOP	1(0.8)	1(0.6)	1(0.4)	1(0.4)	1(0.4)	1(0.3)	1(0.4)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f114	2.1e5	1.1e6	1.4e6	1.6e6	1.6e6	1.6e6	1.6e6	15/15
aLmC	2.7(1)	2.6(2)	∞	∞	∞	∞	∞ *2e6*	0/15
aLmD	2.5(1)	2.6(1)	∞	∞	∞	∞	∞ *2e6*	0/15
aSmC	1.4(0.8)	0.82(0.4)	1.1(0.2)	∞	∞	∞	∞ *2e6*	0/15
aSmD	1.1(0.5)	**0.68**(0.3)	**0.91**(0.3)	19(34)	19(23)	19(35)	19(22)	0/15
BIPOP	**1**(0.4)	1(0.5)	1(0.5)	**1**(0.5)	**1**(0.3)	**1**(0.5)	**1**(0.5)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f115	2405	30268	91749	1.3e5	1.3e5	1.3e5	1.3e5	15/15
aLmC	1.9(0.3)	0.37(0.1)	0.20(0.0)	0.27(0.1)	0.28(0.0)	0.28(0.0)	0.31(0.1)	15/15
aLmD	2.1(0.5)	0.40(0.1)	0.20(0.0)	0.28(0.1)	0.29(0.1)	0.29(0.1)	0.30(0.1)	15/15
aSmC	1.5(0.3)	**0.27**(0.1)^{*3}	**0.14**(0.0)^{*2}	**0.18**(0.0)^{*2}	**0.18**(9e-3)^{*2}	**0.18**(0.0)^{*2}	**0.19**(0.0)*	15/15
aSmD	1.9(0.3)	0.36(0.0)	0.18(0.0)	0.22(0.0)	0.22(0.0)	0.22(0.0)	0.24(0.0)	15/15
BIPOP	**1**(0.6)	6.5(4)	3.9(2)	3.0(2)	3.0(1)	3.0(1)	3.0(1)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f116	5.0e5	6.9e5	8.9e5	1.0e6	1.0e6	1.1e6	1.1e6	15/15
aLmC	0.33(0.0)$_{\downarrow}$	0.32(0.1)	0.33(0.0)	0.42(0.0)	0.56(0.0)	0.83(0.0)	1.1(0.0)	15/15
aLmD	0.31(0.1)$_{\downarrow3}$	0.31(0.0)	0.33(0.0)	0.43(0.0)	0.57(0.0)	0.84(0.0)	1.1(0.0)	15/15
aSmC	**0.18**(0.1)$_{\downarrow4}$	**0.17**(0.0)$_{\downarrow4}$	**0.17**(0.0)$^{\star}_{\downarrow4}$	**0.20**(0.0)^{*3}	**0.28**(1e-2)^{*3}	**0.40**(0.0)^{*3}	**0.51**(0.0)^{*3}	15/15
aSmD	0.22(0.1)$_{\downarrow4}$	0.20(0.0)$_{\downarrow4}$	0.20(0.0)$_{\downarrow4}$	0.24(0.0)	0.31(0.0)	0.44(0.0)	0.57(0.0)	15/15
BIPOP	1.4(1)	1.2(0.6)	1.1(0.5)	1(0.4)	1(0.4)	1(0.5)	1(0.4)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f117	1.8e6	2.5e6	2.6e6	2.8e6	2.9e6	3.2e6	3.6e6	15/15
aLmC	1.2(0.7)	∞	∞	∞	∞	∞	∞ *2e6*	0/15
aLmD	1.0(0.7)	∞	∞	∞	∞	∞	∞ *2e6*	0/15
aSmC	0.42(0.1)$_{\downarrow3}$	0.45(0.1)	0.59(0.0)	∞	∞	∞	∞ *2e6*	0/15
aSmD	**0.39**(0.0)$_{\downarrow3}$	**0.41**(0.1)	**0.55**(0.0)	∞	∞	∞	∞ *2e6*	0/15
BIPOP	1(0.6)	1(0.2)	1(0.2)	**1**(0.2)	**1**(0.2)	**1**(0.2)	**1**(0.2)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f118	6908	11786	17514	22206	26342	30062	32659	15/15
aLmC	2.0(0.2)	1.5(0.1)	1.3(0.1)	1.8(0.2)	2.8(0.3)	5.7(0.2)	8.3(0.1)	15/15
aLmD	2.4(0.2)	1.8(0.2)	1.5(0.1)	1.9(0.1)	2.8(0.4)	5.7(0.4)	8.3(0.3)	15/15
aSmC	**1.5**(0.2)	**1.2**(0.1)^{*3}	**0.96**(0.1)^{*4}	**1.2**(0.1)*	1.7(0.2)	3.1(0.2)	4.1(0.2)	15/15
aSmD	2.0(0.2)	1.5(0.1)	1.2(0.1)	1.4(0.2)	2.0(0.2)	3.2(0.3)	4.4(0.2)	15/15
BIPOP	1.9(0.3)	1.8(0.3)	1.6(0.2)	1.6(0.2)	**1.5**(0.2)*	**1.6**(0.1)^{*4}	**1.6**(0.1)^{*4}	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f119	2771	29365	35930	63288	4.1e5	1.4e6	1.9e6	15/15
aLmC	1.4(1)	1.6(0.6)	3.0(0.7)	3.5(1)	1.2(0.1)	0.75(0.0)	0.85(0.0)	15/15
aLmD	2.0(0.8)	1.7(0.4)	3.2(0.3)	3.3(0.5)	1.2(0.1)	0.73(0.0)	0.83(0.0)	15/15
aSmC	**1.4**(0.8)	**0.87**(0.1)	1.4(0.2)	1.9(0.3)	**0.56**(0.1)	**0.33**(0.0)*	**0.38**(0.0)^{*3}	15/15
aSmD	1.8(1)	1.1(0.2)	1.8(0.4)	2.2(0.5)	0.64(0.0)	0.37(0.0)	0.42(0.0)	15/15
BIPOP	1.6(1)	1(0.8)	**1**(0.3)	**1**(0.5)^{*3}	1(0.4)	1.3(0.3)	1.1(0.3)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f120	36040	1.8e5	2.8e5	8.5e5	1.6e6	6.7e6	1.4e7	13/15
aLmC	0.71(0.3)	4.6(1)	35(27)	∞	∞	∞	∞ *2e6*	0/15
aLmD	1.1(1)	4.2(1)	20(17)	∞	∞	∞	∞ *2e6*	0/15
aSmC	**0.26**(0.2)$^{\star}_{\downarrow}$	1.7(0.9)	2.9(0.8)	2.4(1)	∞	∞	∞ *2e6*	0/15
aSmD	0.70(0.5)	1.8(0.5)	3.0(0.4)	2.0(0.3)	∞	∞	∞ *2e6*	0/15
BIPOP	1(0.6)	**1**(0.6)	**1**(0.4)^{*3}	**1**(0.1)^{*3}	**1**(0.4)^{*3}	**1**(0.4)	**1**(0.3)	13/15

Table 3: Average running time (aRT in number of function evaluations) divided by the respective best aRT measured during BBOB-2009 in dimension 5 (left) and dimension 20 (right) on f_{121}–f_{130}. The aRT and in braces, as dispersion measure, the half difference between 10 and 90%-tile of bootstrapped run lengths appear for each algorithm and target, the corresponding best aRT in the first row. The different target Δf-values are shown in the top row. #succ is the number of trials that reached the (final) target $f_{\mathrm{opt}} + 10^{-8}$. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries, succeeded by a star, are statistically significantly better (according to the rank-sum test) when compared to all other algorithms of the table, with $p = 0.05$ or $p = 10^{-k}$ when the number k following the star is larger than 1, with Bonferroni correction by the number of instances. A ↓ indicates the same tested against the best algorithm of BBOB-2009.Best results are printed in bold.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f121	8.6	111	273	533	1583	3870	6195	15/15
aLmC	3.7(5)	3.2(2)	2.9(1)	2.9(0.9)	1.8(0.6)	1.6(0.2)	1.5(0.2)	15/15
aLmD	3.4(0.7)	2.3(0.7)	1.9(0.9)	2.1(0.8)	1.6(0.3)	1.4(0.3)	1.3(0.2)	15/15
aSmC	5.2(5)	2.7(2)	2.6(0.7)	2.6(0.9)	1.6(0.4)	1.3(0.2)	**1.2**(0.2)	15/15
aSmD	**2.6**(3)	1.7(0.4)	1.6(0.5)	1.7(0.7)	1.2(0.4)	**1.2**(0.2)*	1.9(2)	15/15
BIPOP	2.7(3)	**1.1**(0.4)	**1**(0.2)$^{\star 2}$	**1**(0.2)$^{\star 3}$	**1.1**(0.6)	2.0(0.3)	2.2(0.2)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f122	10	1727	9190	21579	30087	53743	1.1e5	15/15
aLmC	8.7(12)	**0.99**(0.6)	0.67(0.2)	0.64(0.1)	0.73(0.1)	**0.73**(0.1)	**0.79**(0.1)	15/15
aLmD	10(10)	1.0(0.9)	**0.64**(0.3)	**0.63**(0.1)	**0.70**(0.1)	0.74(0.1)	0.86(0.2)	15/15
aSmC	11(11)	22(147)	14(41)	6.3(6)	4.7(13)	3.9(7)	2.1(3)	11/15
aSmD	5.6(13)	2.1(4)	8.9(27)	4.0(12)	4.6(8)	2.7(9)	1.5(3)	12/15
BIPOP	**2.2**(0.9)	1(1)	1(0.7)	1(0.9)	1(0.5)	1(0.5)	1(0.2)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f123	11	16066	81505	2.3e5	3.4e5	6.7e5	2.2e6	15/15
aLmC	**5.0**(4)	1.2(1)	1.6(0.6)	1.9(0.4)	∞	∞	∞ *5e5*	0/15
aLmD	9.4(9)	1.1(1)	1.5(0.7)	2.0(0.8)	∞	∞	∞ *5e5*	0/15
aSmC	6.2(5)	**0.76**(0.7)	**0.72**(0.3)	**0.66**(0.1)	**0.82**(0.1)	∞	∞ *5e5*	0/15
aSmD	10(13)	8.3(39)	3.0(3)	1.5(1)	1.6(1)	∞	∞ *5e5*	0/15
BIPOP	8.1(1)	1(0.8)	1(0.5)	1(0.9)	1(0.7)	**1**(0.5)	**1**(0.5)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f124	10	202	1040	8974	20478	45337	95200	15/15
aLmC	5.7(12)	3.4(2)	1.9(0.7)	0.61(0.2)	0.44(0.1)	0.41(0.0)	0.29(0.0)	15/15
aLmD	2.5(4)	2.5(1)	1.5(0.3)	**0.48**(0.1)	0.39(0.1)	0.37(0.1)	0.25(0.0)	15/15
aSmC	5.4(11)	3.4(1.0)	1.6(0.5)	0.50(0.1)	**0.33**(0.1)	**0.28**(0.0)	**0.23**(0.1)	15/15
aSmD	5.7(7)	2.9(3)	3.6(9)	0.61(0.0)	0.34(0.9)	0.42(0.4)	0.25(0.4)	15/15
BIPOP	**1.5**(1)	**1.1**(0.4)$^{\star 2}$	**1**(0.3)*	1.2(1)	1.1(0.5)	1.2(1)	1(1)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f125	1	1	1	1.3e5	2.4e5	2.4e5	2.5e5	15/15
aLmC	1.9(2)	28(29)	3562(2794)	2.1(0.7)	16(14)	15(14)	15(16)	2/15
aLmD	1.7(0.5)	23(19)	2721(4301)	2.2(1)	31(49)	31(13)	30(35)	1/15
aSmC	1.3(0.8)	28(34)	2145(1672)	1.7(0.9)	1.3(1)	2.3(2)	2.2(4)	8/15
aSmD	2.0(2)	27(14)	**2054**(1911)	1.4(2)	2.4(3)	4.0(4)	15(28)	2/15
BIPOP	**1.1**(0.2)	**17**(18)	3443(2682)	**1**(0.9)	**1**(0.6)	**1**(0.9)	**1**(0.6)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f126	1	1	1	8.8e5	∞	∞	∞	0
aLmC	1.7(1)	46(59)	8647(8136)	8.2(14)	.	.	.	0/15
aLmD	2.1(1)	46(14)	1.2e4(1e4)	2.7(2)	.	.	.	0/15
aSmC	1.5(0.8)	**42**(44)	**6393**(7302)	3.9(3)	.	.	.	0/15
aSmD	1.7(1)	**42**(82)	4.1e4(1e5)	**1.5**(2)	.	.	.	0/15
BIPOP	**1**(0)	160(480)	1.3e4(8264)	2.1(2)	.	.	.	0/15
Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f127	1	1	1	1.3e5	3.4e5	3.9e5	4.0e5	15/15
aLmC	1.7(1)	44(62)	1764(702)	1.1(0.5)	0.78(0.1)	0.70(0.1)	0.69(0.2)	15/15
aLmD	1.9(1)	30(16)	1975(2018)	1.4(0.5)	1.1(0.9)	0.98(0.8)	1.1(1.0)	12/15
aSmC	2.2(2)	35(43)	**1511**(1438)	**0.64**(0.1)	**0.41**(1)	**0.47**(0.4)	**0.60**(1)	12/15
aSmD	2.1(2)	22(16)	1839(1260)	1.4(1)	1.3(1)	2.4(2)	8.5(11)	2/15
BIPOP	**1**(0)	**19**(18)	2136(1484)	1.2(1)	1(0.9)	1(0.6)	1(1.0)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f128	111	4248	7808	10500	12447	17217	21162	15/15
aLmC	2.0(2)	474(621)	419(450)	311(299)	263(394)	190(197)	155(166)	2/15
aLmD	**2.0**(2)	178(207)	178(258)	132(215)	111(141)	81(66)	66(83)	4/15
aSmC	3.2(5)	136(267)	97(226)	72(96)	61(41)	44(73)	36(42)	6/15
aSmD	2.1(1)	236(413)	257(289)	191(239)	161(81)	117(95)	95(53)	3/15
BIPOP	2.2(9)	**6.9**(18)	**10**(42)	**7.8**(22)	**6.6**(11)	**4.8**(5)	**3.9**(9)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f129	64	10710	59443	2.3e5	2.8e5	5.1e5	5.8e5	15/15
aLmC	**3.3**(4)	34(47)	17(13)	4.5(6)	3.7(6)	2.1(2)	1.8(3)	5/15
aLmD	7.0(6)	75(128)	23(46)	8.7(12)	7.1(6)	4.0(5)	3.5(3)	3/15
aSmC	3.8(3)	54(117)	13(32)	**4.4**(5)	**3.6**(8)	**2.0**(2)	**1.8**(5)	5/15
aSmD	10(32)	189(363)	119(122)	30(29)	25(18)	14(8)	12(10)	1/15
BIPOP	12(20)	**7.1**(5)	**9.2**(1)	4.8(8)	3.9(0.2)	2.2(0.1)	1.9(0.1)	13/15
Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f130	55	812	3034	8198	32823	33889	34528	10/15
aLmC	3.2(3)	71(81)	31(75)	11(15)	2.9(2)	2.8(4)	2.8(5)	9/15
aLmD	**1.9**(2)	28(27)	12(22)	4.3(3)	1.1(2)	1.1(0.5)	1.1(0.4)	14/15
aSmC	2.6(2)	34(54)	13(9)	4.8(7)	1.2(1.0)	1.2(1)	1.2(2)	12/15
aSmD	10(64)	**22**(28)	**10**(13)	**3.9**(2)	**0.97**(0.7)	**0.95**(0.9)	**0.95**(0.7)	14/15
BIPOP	1.9(1)	57(99)	55(48)	20(47)	5.1(7)	5.0(10)	5.0(3)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f121	249	769	1426	3433	9304	34434	57404	15/15
aLmC	5.3(2)	6.3(0.8)	7.2(1)	10(3)	10(1)	6.7(0.4)	6.4(0.3)	15/15
aLmD	6.4(1)	7.9(2)	8.1(2)	10(2)	10(2)	6.4(0.1)	6.1(0.2)	15/15
aSmC	3.9(0.7)	4.8(0.4)	5.2(1)	6.5(1)	5.2(0.6)	3.1(0.2)	2.8(0.1)	15/15
aSmD	5.6(2)	6.7(1.0)	6.6(0.8)	6.9(1.0)	5.7(0.6)	3.3(0.1)	3.0(0.2)	15/15
BIPOP	**1.2**(0.4)$^{\star 3}$	**1.0**(0.1)$^{\star 4}$	**1.2**(0.3)$^{\star 4}$	**1.1**(0.1)$^{\star 4}$	**1.1**(0.2)$^{\star 4}$	**1.3**(0.1)$^{\star 4}$	**1.9**(0.1)$^{\star 4}$	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f122	692	52008	1.4e5	3.8e5	7.9e5	2.0e6	5.8e6	15/15
aLmC	2.4(2)	2.9(2)	3.7(0.7)	∞	∞	∞	∞ *2e6*	0/15
aLmD	2.3(1)	3.0(0.8)	3.8(1)	∞	∞	∞	∞ *2e6*	0/15
aSmC	**1.5**(1)	1.4(0.3)	2.1(0.4)	3.2(0.2)	2.6(0.1)	∞	∞ *2e6*	0/15
aSmD	2.4(2)	1.6(0.3)	2.0(0.6)	3.2(0.3)	2.3(0.1)	∞	∞ *2e6*	0/15
BIPOP	1.8(4)	**1**(0.3)	**1**(0.8)$^{\star 2}$	**1**(0.6)$^{\star 3}$	**1**(1.0)$^{\star 2}$	**1**(0.4)$^{\star 2}$	**1**(0.6)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f123	1063	5.3e5	1.5e6	3.3e6	5.3e6	2.7e7	1.6e8	0
aLmC	8.6(10)	56(43)	∞	∞	∞	∞	∞ *2e6*	0/15
aLmD	7.1(3)	∞	∞	∞	∞	∞	∞ *2e6*	0/15
aSmC	**3.2**(3)	2.4(0.5)	∞	∞	∞	∞	∞ *2e6*	0/15
aSmD	5.9(5)	2.1(0.9)	∞	∞	∞	∞	∞ *2e6*	0/15
BIPOP	5.7(4)	**1**(0.8)*	**1**(1.0)$^{\star 2}$	**1**(0.6)	**1**(0.5)	**1**(0.7)	**1**(1)	0/15
Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f124	192	1959	40840	64491	1.3e5	3.9e5	8.0e5	15/15
aLmC	3.4(2)	3.5(0.7)	0.99(0.5)	4.2(0.3)	3.7(0.2)	2.7(0.1)	2.1(0.1)	15/15
aLmD	4.4(2)	4.6(0.7)	0.93(0.1)	4.0(0.4)	3.6(0.3)	2.6(0.1)	2.0(0.1)	15/15
aSmC	2.7(1)	2.9(0.4)	**0.71**(0.1)	1.7(0.1)	1.4(0.1)	**0.95**(0.0)	**0.71**(0.0)	15/15
aSmD	4.1(1)	3.4(0.4)	0.81(0.2)	1.9(0.2)	1.6(0.1)	1.0(0.0)	0.78(0.0)	15/15
BIPOP	**1.1**(0.5)$^{\star 2}$	**1.0**(0.5)$^{\star 4}$	1(1)	**1**(0.5)*	**1**(0.9)*	1(0.8)	1(0.5)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f125	1	1	1	1.2e7	2.5e7	8.0e7	8.1e7	4/15
aLmC	4.3(2)	832(227)	∞	∞	∞	∞	∞ *2e6*	0/15
aLmD	3.7(2)	944(338)	∞	∞	∞	∞	∞ *2e6*	0/15
aSmC	2.7(1)	675(295)	∞	∞	∞	∞	∞ *2e6*	0/15
aSmD	3.7(2)	857(621)	∞	∞	∞	∞	∞ *2e6*	0/15
BIPOP	**1**(0)	**383**(383)	**9.8e6**(5e6)	**1**(0.4)	**1**(1)	**1**(1)	**1**(0.7)	4/15
Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f126	1	1	1	∞	∞	∞	∞	0
aLmC	3.4(2)	2736(1796)	∞	.	.	.	.	0/15
aLmD	3.0(2)	2927(900)	∞	.	.	.	.	0/15
aSmC	3.3(2)	**1845**(1498)	∞	.	.	.	.	0/15
aSmD	4.3(2)	2609(785)	∞	.	.	.	.	0/15
BIPOP	**1**(0)*	5781(4159)	∞	.	.	.	.	0/15
Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f127	1	1	1	1.6e6	4.4e6	7.3e6	7.4e6	15/15
aLmC	3.2(2)	560(198)	∞	∞	∞	∞	∞ *2e6*	0/15
aLmD	3.3(2)	732(244)	∞	∞	∞	∞	∞ *2e6*	0/15
aSmC	3.9(0.5)	475(141)	∞	∞	∞	∞	∞ *2e6*	0/15
aSmD	3.5(2)	651(241)	∞	∞	∞	∞	∞ *2e6*	0/15
BIPOP	**1**(0)	**176**(60)$^{\star 3}$	**9.0e5**(1e6)*	**1**(2)*	**1**(0.7)	**1**(0.8)	**1**(0.5)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f128	1.4e5	1.3e7	1.7e7	1.7e7	1.7e7	1.7e7	1.7e7	9/15
aLmC	19(8)	0.66(0.8)	0.80(2)	0.80(0.7)	0.80(0.9)	0.80(1)	0.80(0.5)	2/15
aLmD	11(7)	0.46(0.9)	0.50(0.3)	0.50(0.7)	0.50(0.4)	0.51(0.7)	0.51(0.4)	3/15
aSmC	2.7(8)	**0.19**(0.2)	**0.19**(0.2)	**0.19**(0.2)	**0.19**(0.2)	**0.19**(0.2)	**0.19**(0.2)	6/15
aSmD	11(11)	0.62(0.8)	0.48(0.4)	0.48(1)	0.49(0.6)	0.49(0.4)	0.49(0.4)	3/15
BIPOP	**1**(2)	1(0.9)	1(0.9)	1(1)	1(0.9)	1(3)	1(2)	9/15
Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f129	7.8e6	4.1e7	4.2e7	4.2e7	4.2e7	4.2e7	4.2e7	5/15
aLmC	∞	∞	∞	∞	∞	∞	∞ *2e6*	0/15
aLmD	3.8(5)	∞	∞	∞	∞	∞	∞ *2e6*	0/15
aSmC	1.8(0.9)	**0.35**(0.4)	**0.35**(0.4)	**0.35**(0.2)	**0.35**(0.4)	**0.35**(0.3)	**0.35**(0.3)	0/15
aSmD	∞	∞	∞	∞	∞	∞	∞ *2e6*	0/15
BIPOP	**1**(1)	1(2)	1(0.7)	1(0.9)	1(1)	1(0.7)	1(2)	5/15
Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f130	4904	93149	2.5e5	2.5e5	2.5e5	2.6e5	2.6e5	7/15
aLmC	43(24)	34(12)	18(36)	18(15)	18(18)	18(16)	18(12)	5/15
aLmD	20(55)	24(21)	12(14)	12(9)	12(5)	12(17)	12(5)	7/15
aSmC	25(60)	14(34)	5.6(10)	5.6(4)	5.6(5)	5.6(10)	5.7(9)	10/15
aSmD	12(22)	**11**(25)	**4.7**(6)	**4.7**(5)	**4.7**(8)	**4.7**(2)	**4.7**(3)	11/15
BIPOP	**1.9**(5)	33(49)	14(16)	14(26)	14(29)	14(18)	14(16)	15/15