GECCO 2016
Tutorial on
Evolutionary Multiobjective Optimization

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updated slides will be available at
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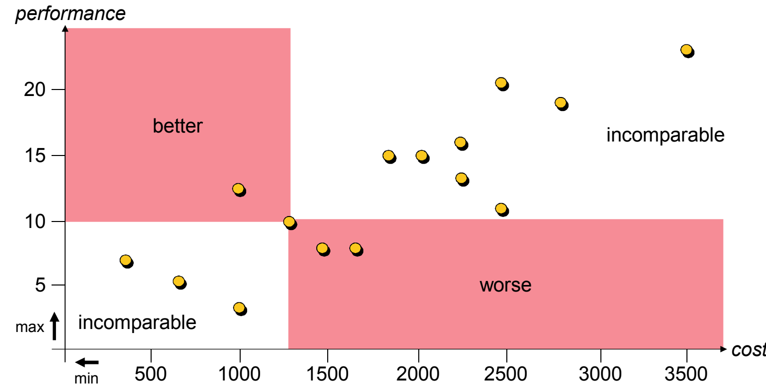
GECCO'16 Companion, July 20-24, 2016, Denver, CO, USA, ACM, ISBN 978-1-4503-4323-7/16/07, DOI:10.1145/2908961.2926974

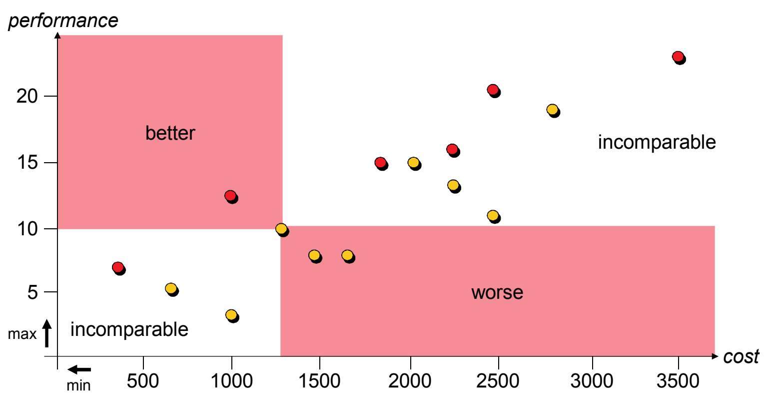
A Brief Introduction to Multiobjective Optimization

Multiobjective Optimization
Multiple objectives that have to be optimized simultaneously



A Brief Introduction to Multiobjective Optimization

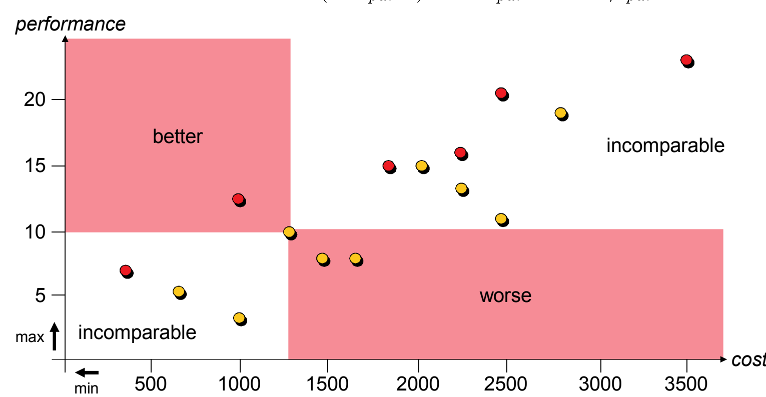
Observations: ❶ there is no single optimal solution, but
❷ some solutions (●) are better than others (●)



A Brief Introduction to Multiobjective Optimization

u *weakly Pareto dominates* v $(u \preccurlyeq_{par} v)$: $\forall 1 \le i \le k : f_i(u) \le f_i(v)$

u *Pareto dominates* v $(u \prec_{par} v)$: $u \preccurlyeq_{par} v \;\wedge\; v \not\preccurlyeq_{par} u$



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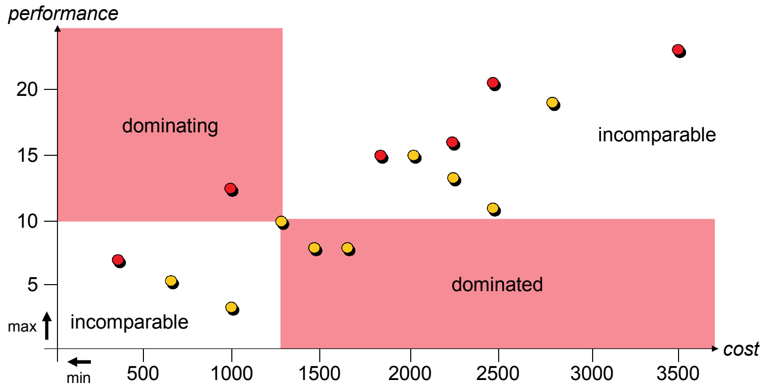
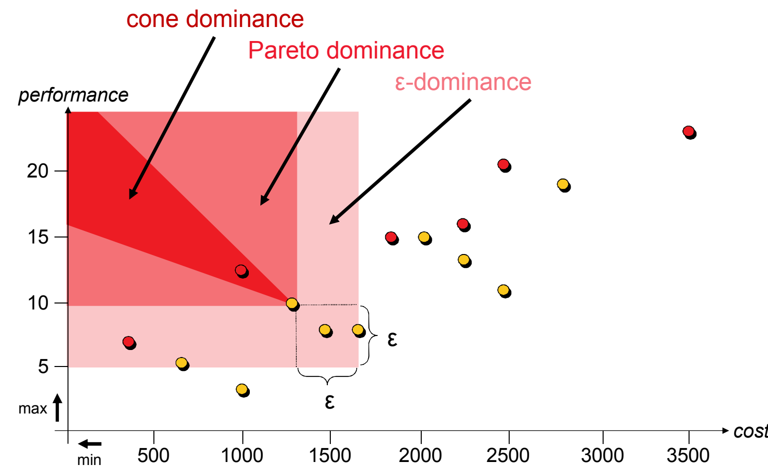
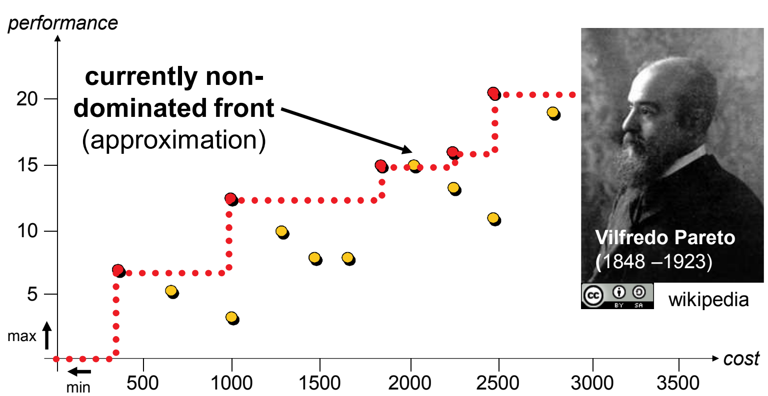


A Brief Introduction to Multiobjective Optimization



A Brief Introduction to Multiobjective Optimization

Pareto set: set of all non-dominated solutions (decision space)
Pareto front: its image in the objective space



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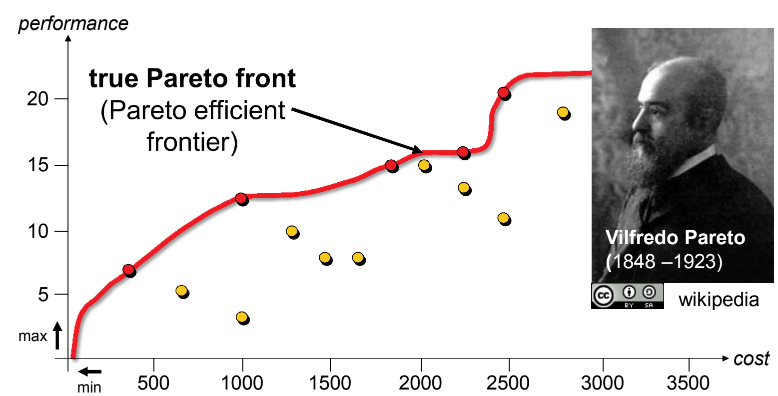

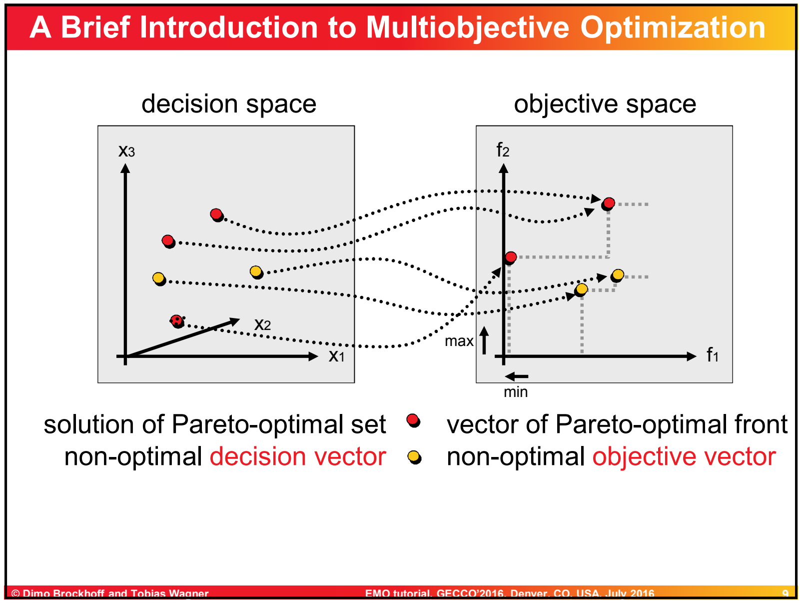
A Brief Introduction to Multiobjective Optimization
decision space
objective space
x3
x2
x1
f2
f1
max
min
solution of Pareto-optimal set
non-optimal decision vector
vector of Pareto-optimal front
non-optimal objective vector
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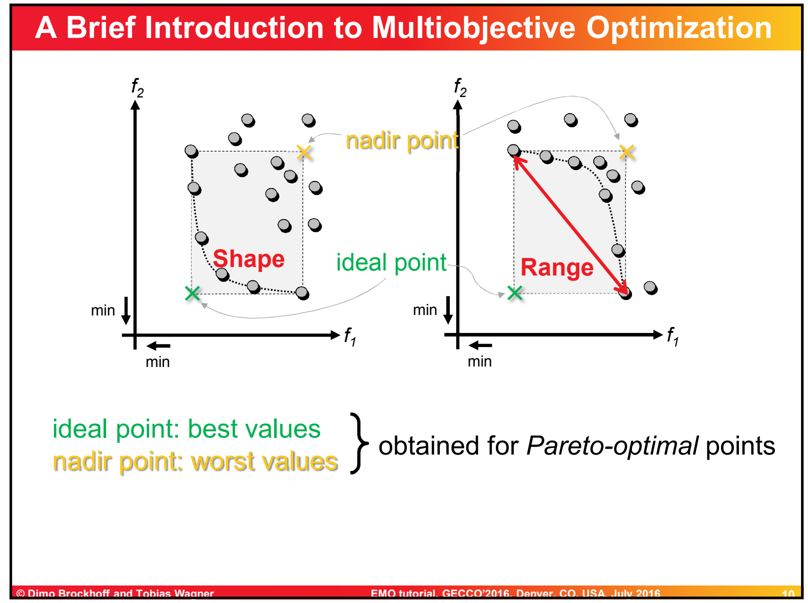
A Brief Introduction to Multiobjective Optimization
f2
f1
min
nadir point
ideal point
Shape
Range
ideal point: best values
nadir point: worst values
obtained for Pareto-optimal points
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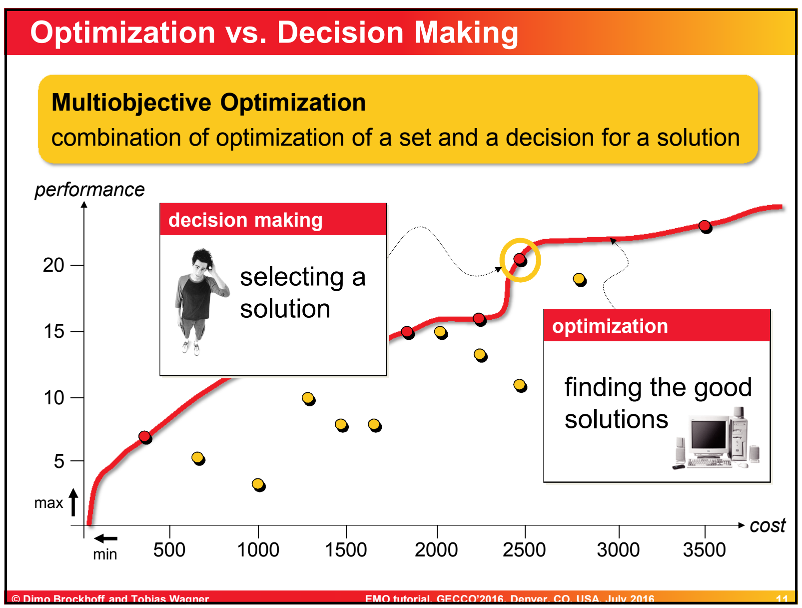
Optimization vs. Decision Making
Multiobjective Optimization
combination of optimization of a set and a decision for a solution
performance
decision making
selecting a solution
optimization
finding the good solutions
20
15
10
5
max
min
500
1000
1500
2000
2500
3000
3500
cost
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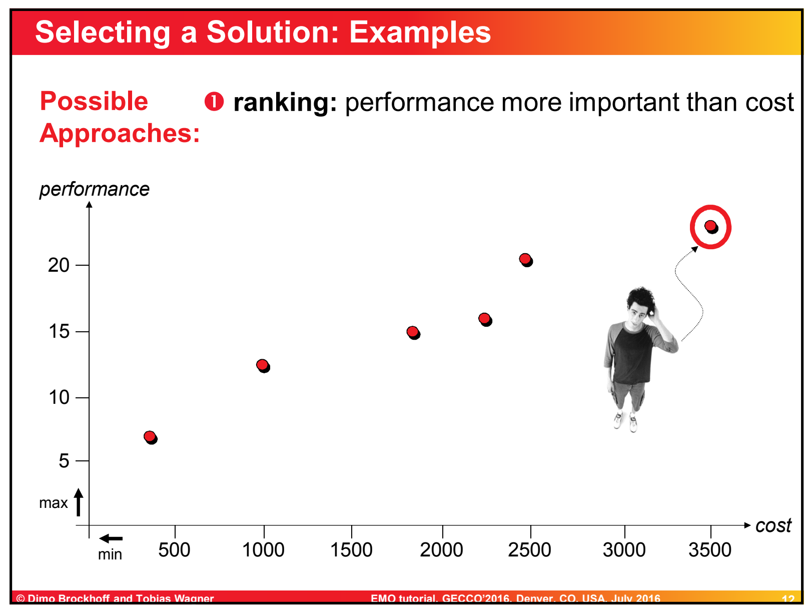
Selecting a Solution: Examples
Possible Approaches:
❶ ranking: performance more important than cost
performance
20
15
10
5
max
min
500
1000
1500
2000
2500
3000
3500
cost
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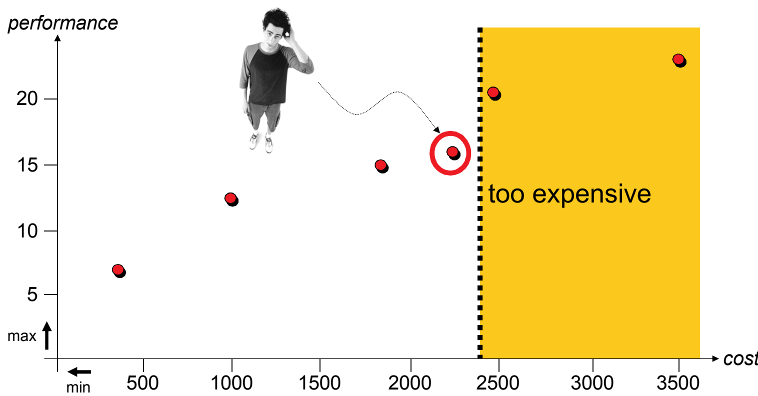
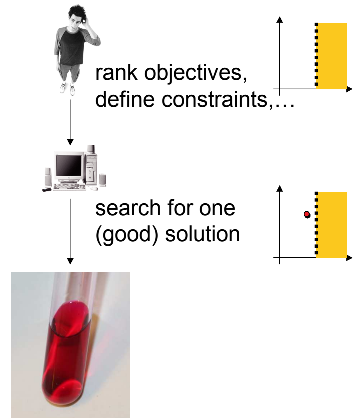
Selecting a Solution: Examples

Possible Approaches:
❶ ranking: performance more important than cost
❷ **constraints:** cost must not exceed 2400



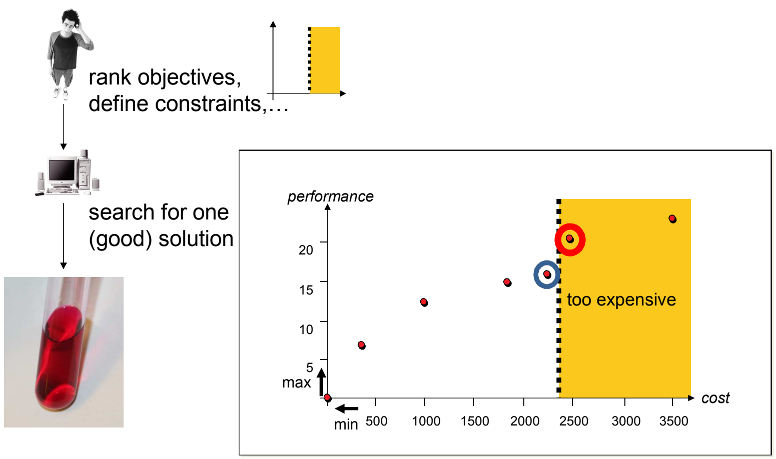
When to Make the Decision

Before Optimization:



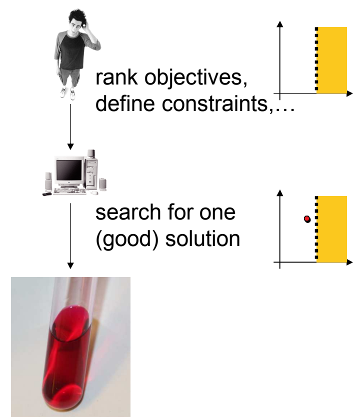
When to Make the Decision

Before Optimization:

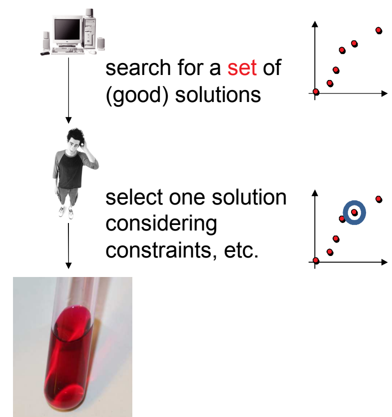


When to Make the Decision

Before Optimization:



After Optimization:



When to Make the Decision

Before Optimization:

rank objectives, define constraints,…

search for one (good) solution

After Optimization:

search for a set of (good) solutions

select one solution considering constraints, etc.

Focus: learning about a problem
- trade-off surface
- interactions among criteria
- structural information
- also: interactive optimization

Two Communities...

MCDM International Society on Multiple Criteria Decision Making

- established field (beginning in 1950s/1960s)
- bi-annual conferences since 1975
- background in economics, math, management and social sciences
- focus on optimization and decision making

EMO

- quite young field (first papers in mid 1980s)
- bi-annual conference since 2001
- background in computer science, applied math and engineering
- focus on optimization algorithms

...Slowly Merge Into One

MCDM International Society on Multiple Criteria Decision Making EMO

- MCDM track at EMO conference since 2009
- special sessions on EMO at the MCDM conference since 2008
- joint Dagstuhl seminars since 2004

One of the Main Differences

Blackbox optimization

$$x \in X \longrightarrow \boxed{f} \longrightarrow (f_1(x), \dots, f_k(x))$$

only mild assumptions

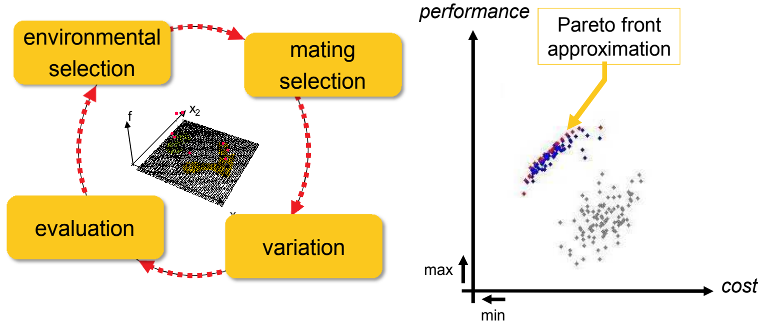
→ EMO therefore well-suited for real-world engineering problems

objectives: non-linear, noisy, uncertain, expensive (integrated simulations, real experiments), non-differentiable

problem: many objectives, huge search spaces, many constraints

The Other Main Difference

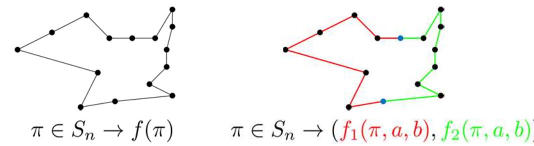
Evolutionary Multiobjective Optimization
- set-based algorithms
- therefore possible to approximate the Pareto front in one run



Multiobjectivization

Some problems are easier to solve in a multiobjective scenario

example: TSP
[Knowles et al. 2001]



Multiobjectivization

by addition of new "helper objectives" [Jensen 2004]

job-shop scheduling [Jensen 2004], frame structural design [Greiner et al. 2007], VRP [Watanabe and Sakakibara 2007], ...

by decomposition of the single objective

TSP [Knowles et al. 2001], minimum spanning trees [Neumann and Wegener 2006], protein structure prediction [Handl et al. 2008a], ...

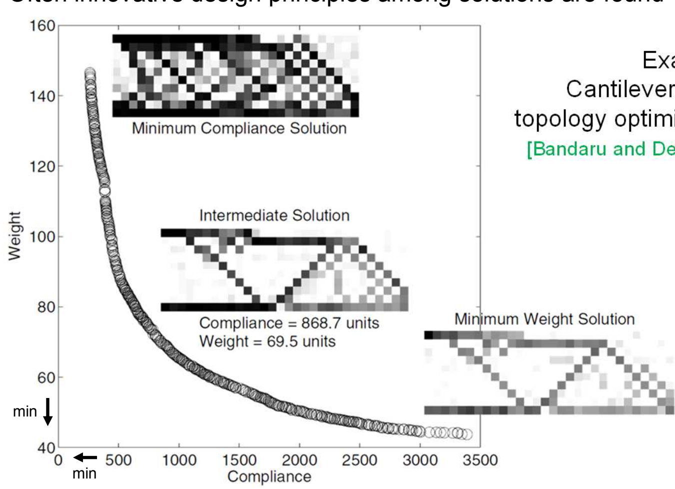
also backed up by theory e.g. [Brockhoff et al. 2009, Handl et al. 2008b]

related to *constrained* and *multimodal* single-objective optimization

see also this recent overview: [Segura et al. 2013]

Innovization

Often innovative design principles among solutions are found

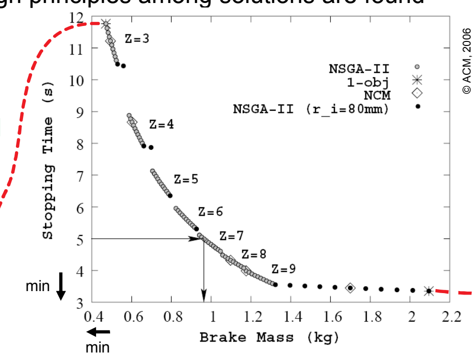


Example:
Cantilever beam topology optimization
[Bandaru and Deb 2015]

Innovization

Often innovative design principles among solutions are found

Example:
Clutch brake design
[Deb and Srinivasan 2006]



Solution	x_1	x_2	x_3	x_4	x_5	f_1	f_2
Min. f_1	70	90	1.5	1000	3	0.4704	11.7617
Min. f_2	80	110	1.5	1000	9	2.0948	3.3505

Innovization

Often innovative design principles among solutions are found

Innovization [Deb and Srinivasan 2006]

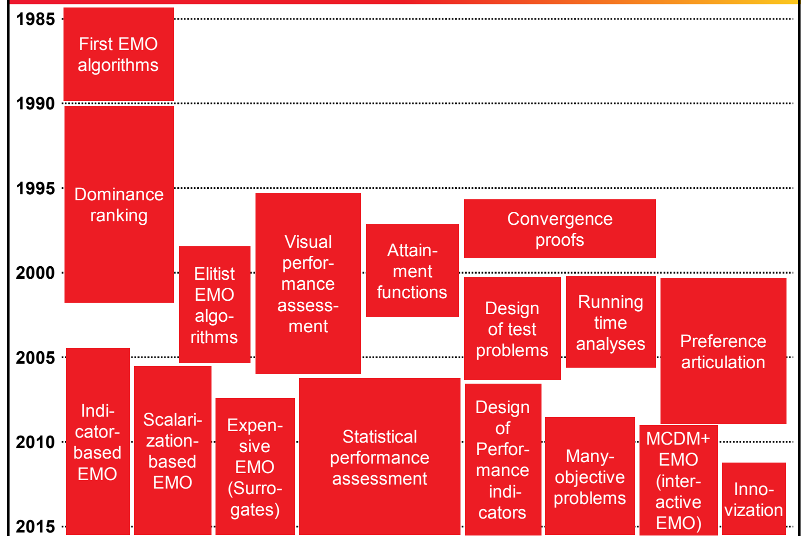
= using machine learning techniques to find new and innovative design principles among solution sets

= learning from/about a multi-objective optimization problem

Other examples:

- SOM for supersonic wing design [Obayashi and Sasaki 2003]
- Biclustering for processor design and knapsack [Ulrich et al. 2007]
- Successful case studies in engineering (noise barrier design, polymer extrusion, friction stir welding) [Deb et al. 2014]

The History of EMO At A Glance



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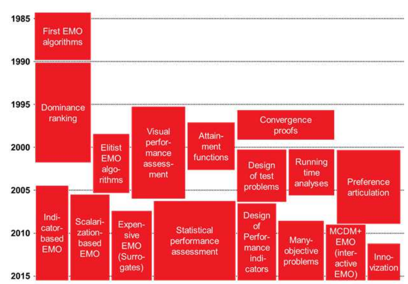
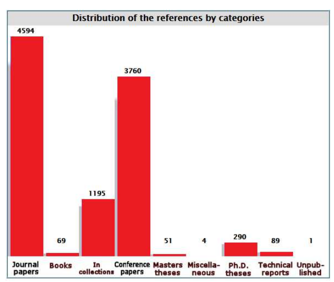
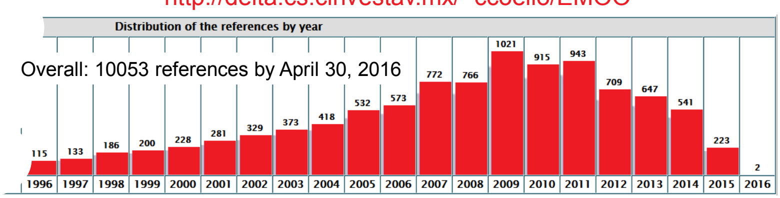
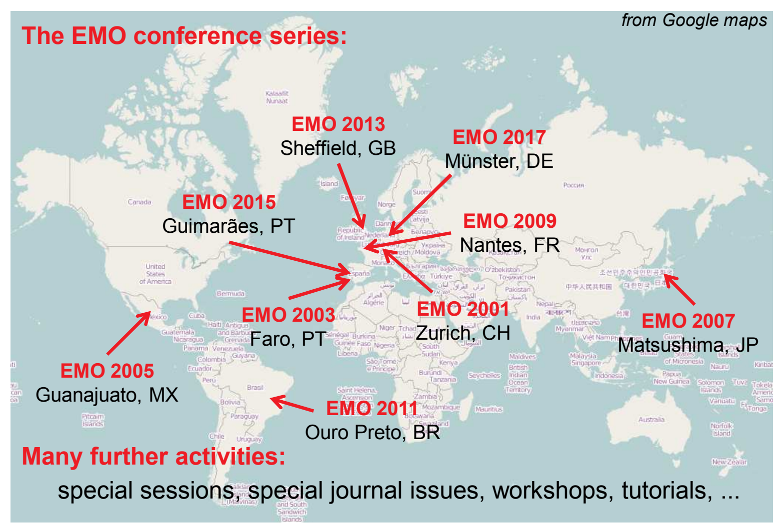




http://delta.cs.cinvestav.mx/~ccoello/EMOO



The EMO Community



Overview

The Big Picture

Basic Principles of Multiobjective Optimization

- algorithm design principles and concepts
- performance assessment

Selected Advanced Concepts

- preference articulation
- surrogate-based EMO

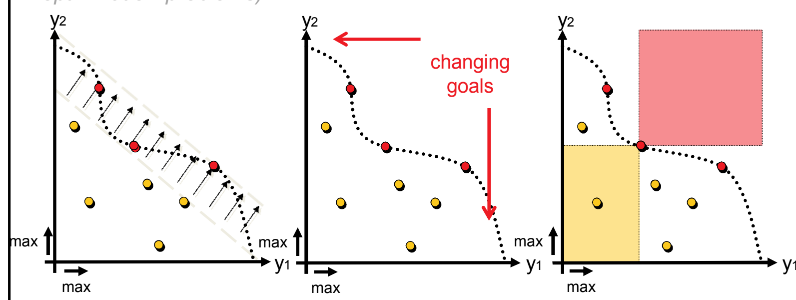
A Few Examples From Practice

Fitness Assignment: Principal Approaches

aggregation-based — *problem decomposition (multiple single-objective optimization problems)*

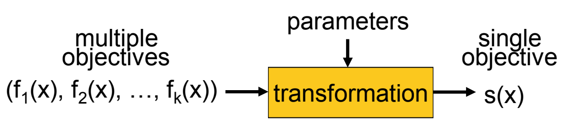
criterion-based — *VEGA*

dominance-based — *SPEA2, NSGA-II "modern" EMOA*



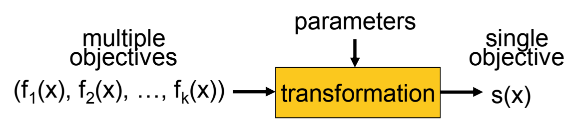
solution-oriented scaling-dependent ⟷ set-oriented scaling-independent

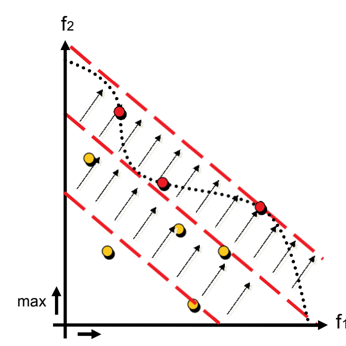
Solution-Oriented Problem Transformations



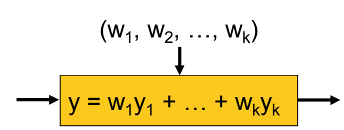
A scalarizing function s is a function $s: Z \to \mathbb{R}$ that maps each objective vector $u = (u_1, \ldots, u_n) \in Z$ to a real value $s(u) \in \mathbb{R}$

Solution-Oriented Problem Transformations



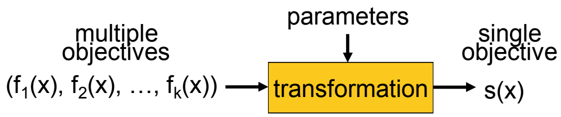


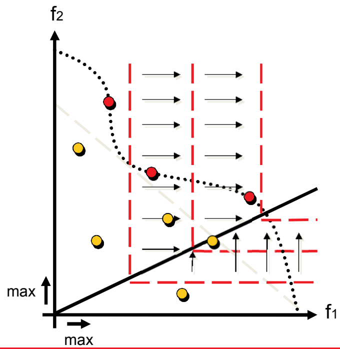
Example 1: weighted sum approach



Disadvantage: not all Pareto-optimal solutions can be found if the front is not convex

Solution-Oriented Problem Transformations





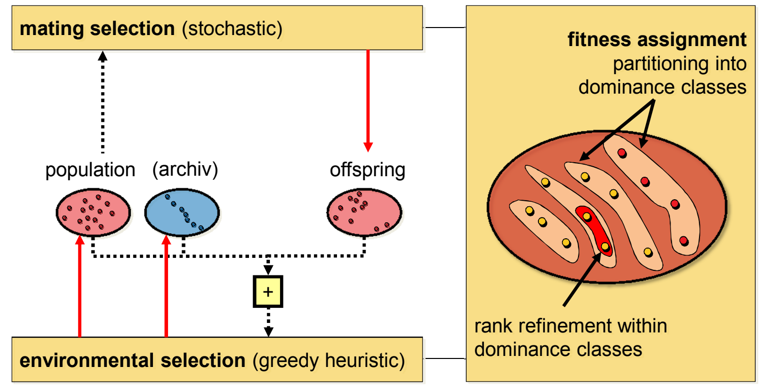
Example 2: weighted Tchebycheff

$(\lambda_1, \lambda_2, \ldots, \lambda_k)$

$$y = \max_i |\lambda_i(u_i - z_i)|$$

Several other scalarizing functions are known, see e.g. [Miettinen 1999]

General Scheme of Most Set-Oriented EMO



Ranking of the Population Using Dominance

... goes back to a proposal by David Goldberg in 1989.

... is based on pairwise comparisons of the individuals only.

- **dominance rank:** by how many individuals is an individual dominated?
  MOGA, NPGA
- **dominance count:** how many individuals does an individual dominate?
  SPEA, SPEA2
- **dominance depth:** at which front is an individual located?
  NSGA, NSGA-II, most of the recently proposed algorithms

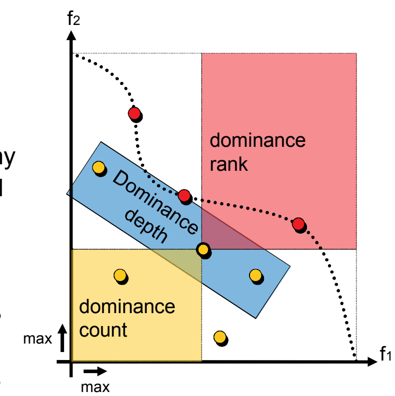
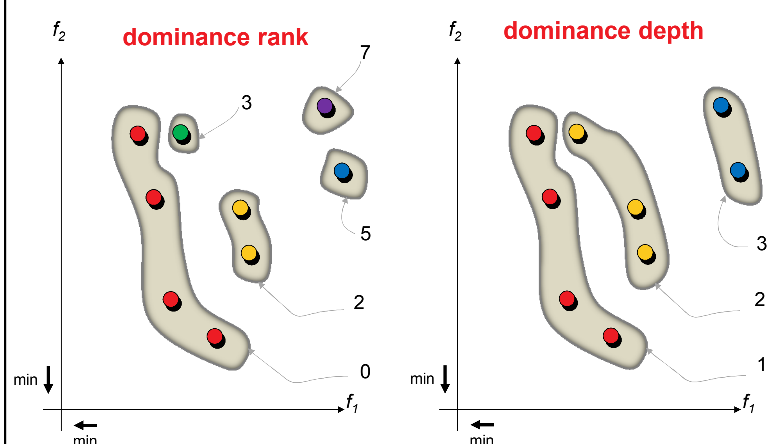


Illustration of Dominance-Based Partitioning

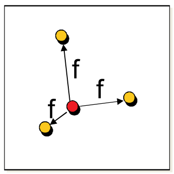


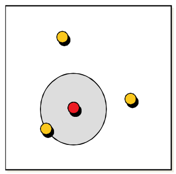
Refinement of Dominance Rankings

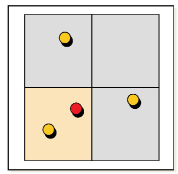
Goal: rank incomparable solutions within a dominance class

❶ Diversity information

Kernel method	k-th nearest neighbor	Histogram method
diversity = function of the distances	diversity = function of distance to k-th nearest neighbor	diversity = number of elements within box(es)





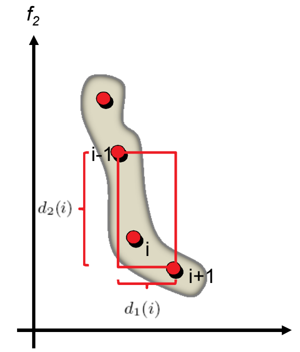


❷ (Contribution to a) quality indicator

Example: NSGA-II Diversity Preservation

Crowding Distance (CD)

- sort solutions with regard to each objective
- assign CD maximum value to extremal objective vectors
- compute CD based on the distance to the neighbors in each objective

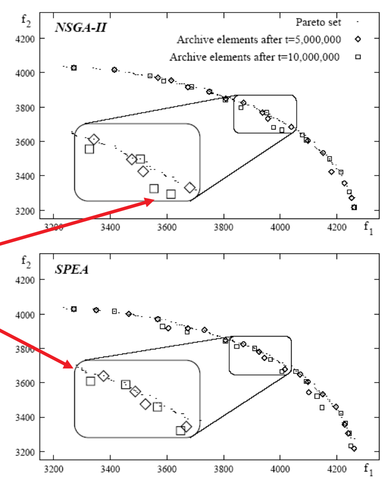


$$\mathrm{CD}(i) = \frac{d_1(i)}{f_{1,\max} - f_{1,\min}} + \cdots + \frac{d_m(i)}{f_{m,\max} - f_{m,\min}}$$

SPEA2 and NSGA-II: Deteriorative Cycles

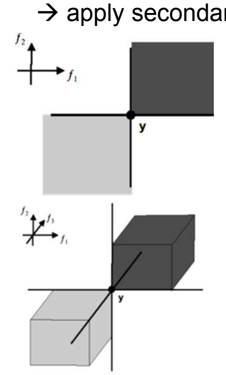
Selection in SPEA2 and NSGA-II can result in *deteriorative* cycles

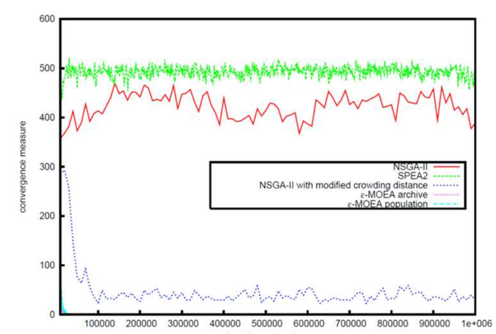
non-dominated solutions already found can be lost



Remark: Many-Objective Optimization

- high number of objectives
  → percentage of non-dominated solutions within a random sample quickly approaches 100 %
  → optimization is mainly guided by diversity criterion
  → apply secondary criterion compliant with dominance relation





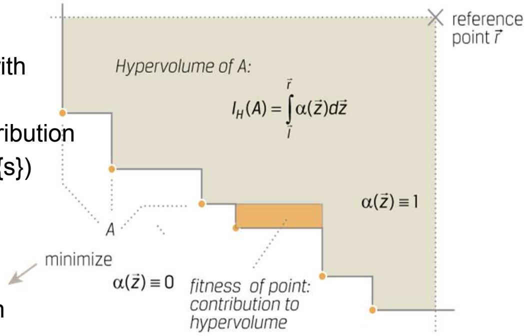
Hypervolume-Based Selection

Latest Approach (SMS-EMOA, MO-CMA-ES, HypE, …)

use hypervolume indicator to guide the search: refines dominance

Main idea

Delete solutions with the smallest hypervolume contribution $d(s) = I_H(P)-I_H(P / \{s\})$ iteratively



But: can also result in cycles if reference point is not constant [Judt et al. 2011]

and is expensive to compute exactly [Bringmann and Friedrich 2009]

Indicator-Based Selection

- Concept can be generalized to any quality indicator

A (unary) quality indicator I is a function $I : \Psi = 2^X \mapsto \mathbb{R}$ that assigns a Pareto set approximation a real value.

Multiobjective Problem $\xrightarrow{\text{Indicator}}$ Single-objective Problem

- for example: R2-indicator [Brockhoff et al. 2012], [Trautmann et al. 2013], [Díaz-Manríquez et al. 2013]
- Generalizable also to contribution to larger sets
  HypE [Bader and Zitzler 2011]: Hypervolume sampling + contribution if more than 1 (random) solution deleted

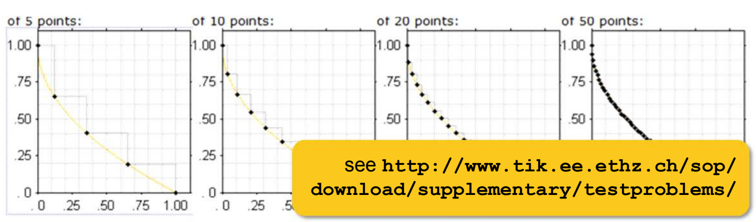
The Optimization Goal in Indicator-Based EMO

When the goal is to maximize a unary indicator…

- we have a single-objective problem on sets
- but what is the optimum?
- important: population size μ plays a role!

Optimal μ-Distribution:

A set of μ solutions that maximizes a certain unary indicator I among all sets of μ solutions is called optimal μ-distribution for I. [Auger et al. 2009a]



see `http://www.tik.ee.ethz.ch/sop/download/supplementary/testproblems/`

Optimal μ-Distributions for the Hypervolume

Hypervolume indicator refines dominance relation

$\Longrightarrow$ most results on optimal μ-distributions for hypervolume

Optimal μ-Distributions (example results)

[Auger et al. 2009a]:

- contain equally spaced points iff front is linear
- density of points $\propto \sqrt{-f'(x)}$ with f' the slope of the front

[Friedrich et al. 2011]:

optimal μ-distributions for the hypervolume correspond to ε-approximations of the front

OPT	$1+\frac{\log(\min\{A/a, B/b\})}{n}$
HYP	$1+\frac{\sqrt{A/a}+\sqrt{B/b}}{n-4}$
logHYP	$1+\frac{\sqrt{\log(A/a)\log(B/b)}}{n-2}$

! (probably) does not hold for > 2 objectives

Indicator-Based EMO

Open Questions:

- How do the optimal μ-distributions look like for >2 objectives?
- how to compute certain indicators quickly in practice?
  - several recent improvements for the hypervolume indicator [Yildiz and Suri 2012], [Bringmann 2012], [Bringmann 2013]
- how to do indicator-based subset selection quickly?
  - also here several recent improvements [Kuhn et al. 2014], [Bringmann et al. 2014], [Guerreiro et al. 2015]
- what is the best strategy for the subset selection?

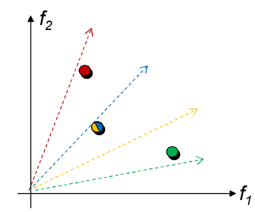
further open questions on indicator-based EMO available at
http://simco.gforge.inria.fr/doku.php?id=openproblems

Decomposition-Based Selection: MOEA/D

MOEA/D: Multiobjective Evolutionary Algorithm Based on Decomposition [Zhang and Li 2007]

Ideas:

- optimize N scalarizing functions in parallel
- use best solutions of neighbor subproblems for mating
- keep the best solution for each scalarizing function
- update neighbors
- use external archive for non-dominated solutions
- several variants and enhancements



Remark: Variation in EMO

- at first sight not different from single-objective optimization
- most research on selection mechanisms (until now)
- but: convergence to a set ≠ convergence to a point

Open Question:

- how to achieve fast convergence to a *set*?

Related work:

- set-based gradient of the HV [Emmerich et al. 2007]
- multiobjective CMA-ES [Igel et al. 2007] [Voß et al. 2010]
- RM-MEDA [Zhang et al. 2008]
- set-based variation [Bader et al. 2009]
- set-based fitness landscapes [Verel et al. 2011]
- offline and online configuration based on libraries of variation operators [Bezerra et al. 2015] [Hadka and Reed 2013]

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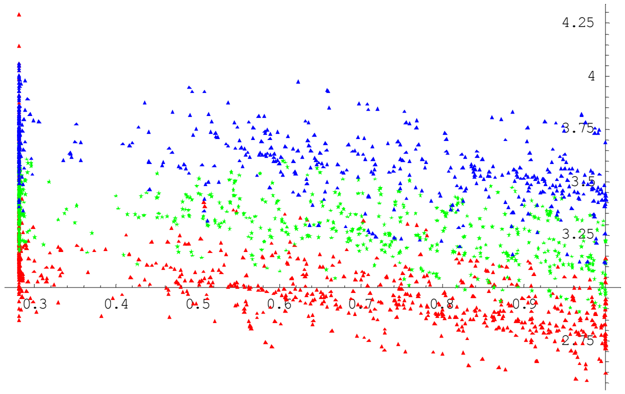
A Few Examples From Practice

Once Upon a Time...

... multiobjective EAs were mainly compared visually:

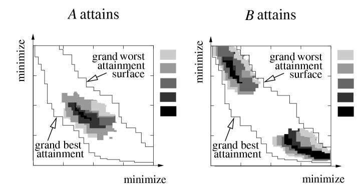


ZDT6 benchmark problem: IBEA, SPEA2, NSGA-II

Two Approaches for Empirical Studies

Attainment function approach

- applies statistical tests directly to the approximation set
- detailed information about how and where performance differences occur



Quality indicator approach

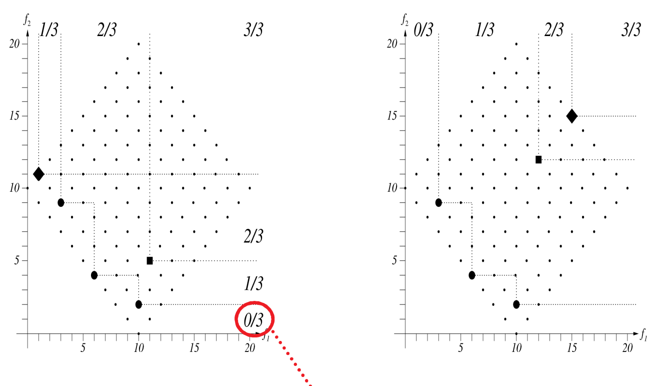
- reduces each approximation set to a single quality value
- applies statistical tests to the quality values

Indicator	A	B
Hypervolume indicator	6.3431	7.1924
ϵ-indicator	1.2090	0.12722
R_2 indicator	0.2434	0.1643
R_3 indicator	0.6454	0.3475

see e.g. [Zitzler et al. 2003]

Empirical Attainment Functions

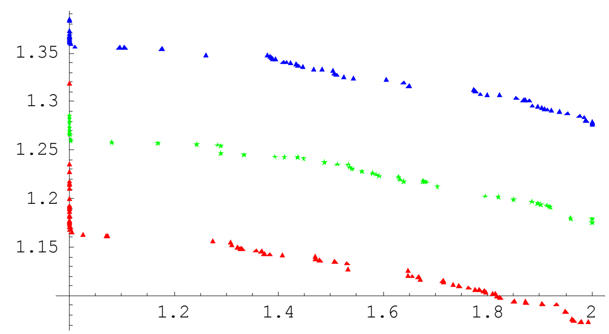
three runs of two multiobjective optimizers



frequency of attaining regions

Attainment Plots

50% attainment surface for IBEA, SPEA2, NSGA2 (ZDT6)



latest implementation online at
`http://eden.dei.uc.pt/~cmfonsec/software.html`
see [Fonseca et al. 2011]

Quality Indicator Approach

Idea:

- transfer multiobjective problem into a set problem
- define an objective function ("quality indicator") on sets
- use the resulting total (pre-)order (on the quality values)

Question:

Can any total (pre-)order be used or are there any requirements concerning the resulting preference relation?

$\Rightarrow$ Underlying dominance relation should be reflected!

$$A \preceq B :\Leftrightarrow \forall_{y \in B} \exists_{x \in A} \, x \preceq_{par} y$$

Refinements and Weak Refinements

❶ $\overset{\text{ref}}{\preccurlyeq}$ **refines** a preference relation $\preccurlyeq$ iff

$$A \preccurlyeq B \wedge B \not\preccurlyeq A \Rightarrow A \overset{\text{ref}}{\preccurlyeq} B \wedge B \overset{\text{ref}}{\not\preccurlyeq} A \qquad \textit{(better} \Rightarrow \textit{better)}$$

$\Rightarrow$ fulfills requirement

❷ $\overset{\text{ref}}{\preceq}$ **weakly refines** a preference relation $\preceq$ iff

$$A \preceq B \wedge B \not\preceq A \Rightarrow A \overset{\text{ref}}{\preceq} B \qquad \textit{(better} \Rightarrow \textit{weakly better)}$$

$\Rightarrow$ does not fulfill requirement, but $\overset{\text{ref}}{\preceq}$ does not contradict $\preccurlyeq$

! sought are total refinements… [Zitzler et al. 2010]

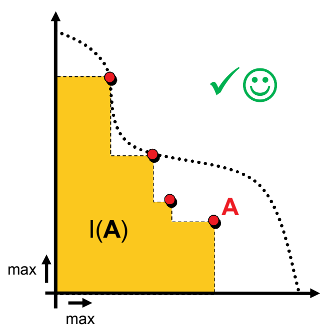
Example: Refinements Using Indicators

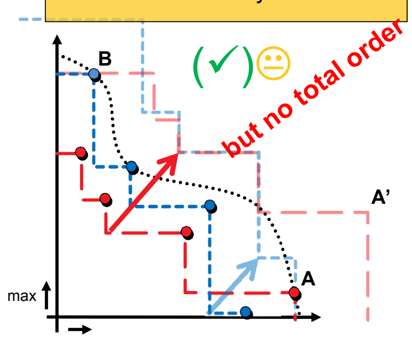
$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A) \geq I(B)$

$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A,B) \leq I(B,A)$

I(A) = volume of the weakly dominated area in objective space

I(A,B) = how much needs A to be moved to weakly dominate B





unary hypervolume indicator

binary epsilon indicator

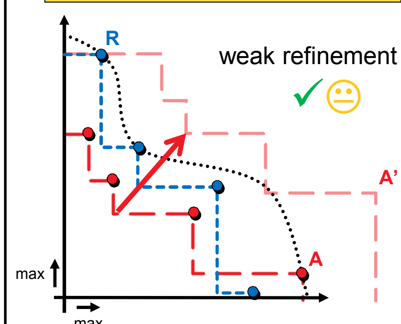
Example: Weak Refinement / No Refinement

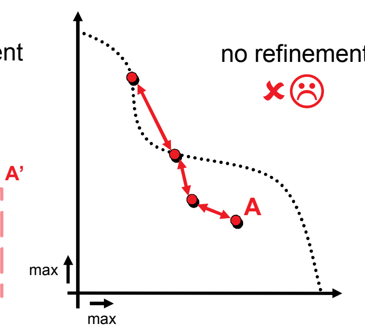
$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A,R) \leq I(B,R)$

$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A) \leq I(B)$

I(A,R) = how much needs A to be moved to weakly dominate R

I(A) = variance of pairwise distances



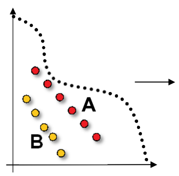


unary epsilon indicator

unary diversity indicator

Quality Indicator Approach

Goal: compare two Pareto set approximations A and B



	A	B
hypervolume	*432.34*	*420.13*
distance	*0.3308*	*0.4532*
diversity	*0.3637*	*0.3463*
spread	*0.3622*	*0.3601*
cardinality	*6*	*5*

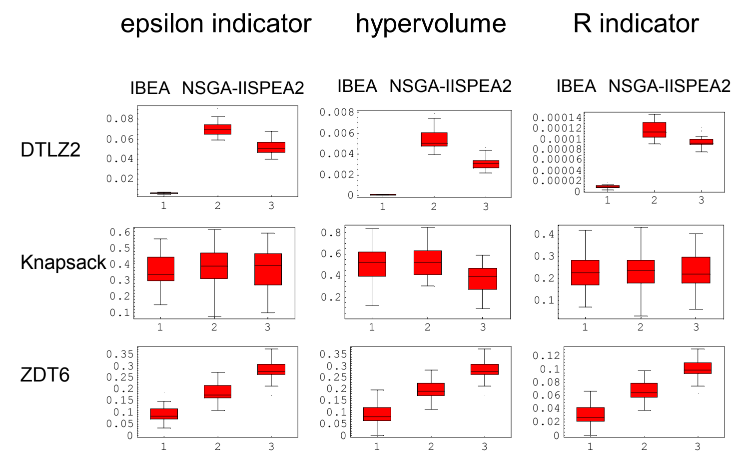
→ "**A** better"

Comparison method C = quality measure(s) + Boolean function

$$A, B \xrightarrow{\text{quality measure}} \mathbb{R}^n \xrightarrow{\text{Boolean function}} \text{statement}$$

reduction **interpretation**

Example: Box Plots



Statistical Assessment (Kruskal Test)

ZDT6
Epsilon

is better than

	IBEA	NSGA2	SPEA2
IBEA		~0 ☺	~0 ☺
NSGA2	1		~0 ☺
SPEA2	1	1	

Overall p-value = 6.22079e-17.
Null hypothesis rejected (alpha 0.05)

DTLZ2
R

is better than

	IBEA	NSGA2	SPEA2
IBEA		~0 ☺	~0 ☺
NSGA2	1		1
SPEA2	1	~0 ☺	

Overall p-value = 7.86834e-17.
Null hypothesis rejected (alpha 0.05)

Knapsack/Hypervolume: H_0 = No significance of any differences

Set Quality Indicators

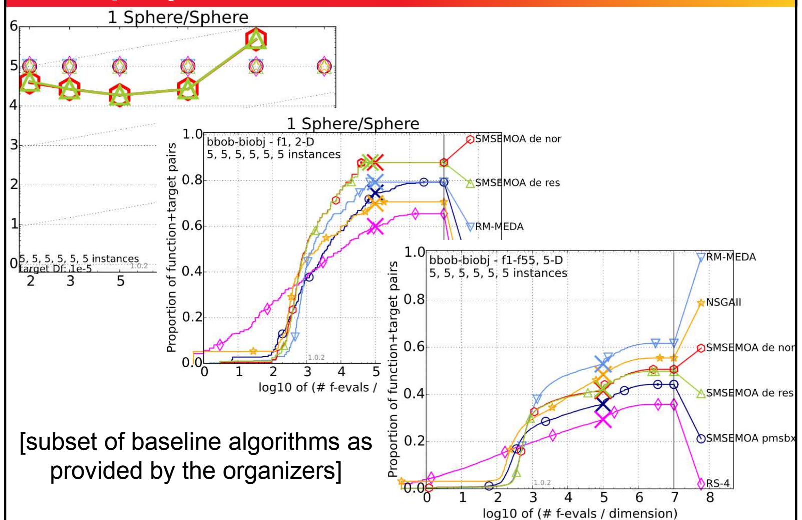
Open Questions:

- are there other unary indicators that are (weak) refinements?
- how to compute indicators efficiently (enough for practice)?
  - especially for >3 objective functions
- how to achieve good indicator values?

Automated Benchmarking

- State-of-the-art in single-objective optimization: Blackbox Optimization Benchmarking (BBOB) with COCO platform
  `https://github.com/numbbo/coco`
- This year: first release of a bi-objective test suite and corresponding BBOB-2016 workshop @ GECCO
- Focus on target-based runlengths
  - gives (nearly) anytime, interpretable results
  - defines problem=(test function instance, single-objective goal e.g. min. indicator difference to reference set, target precision)
  - reports Expected Runtimes (ERT) to reach target precision
- COCO provides data profiles, scaling plots, scatter plots, tables, statistical tests, etc. automatically

Exemplary BBOB-2016 Results



[subset of baseline algorithms as provided by the organizers]

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A Few Examples From Practice

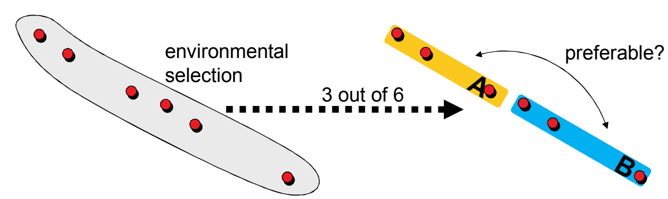
Articulating User Preferences During Search

What we thought: EMO is preference-less

> **Search before decision making:** Optimization is performed without any preference information given. The result of the search process is a set of (ideally Pareto-optimal) candidate solutions from which the final choice is made by the DM.

[Zitzler 1999]

What we learnt: EMO just uses weaker preference information



Incorporation of Preferences During Search

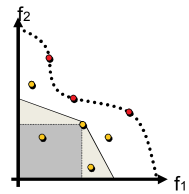
Nevertheless...

- the more (known) preferences incorporated the better
- in particular if search space is large

[Branke and Deb 2004] [Branke 2008] [Bechikh et al. 2015]

❶ **Refine/modify dominance relation, e.g.:**

- using goals, priorities, constraints
  [Fonseca and Fleming 1998a,b]
- using different types of dominance cones
  [Branke and Deb 2004]

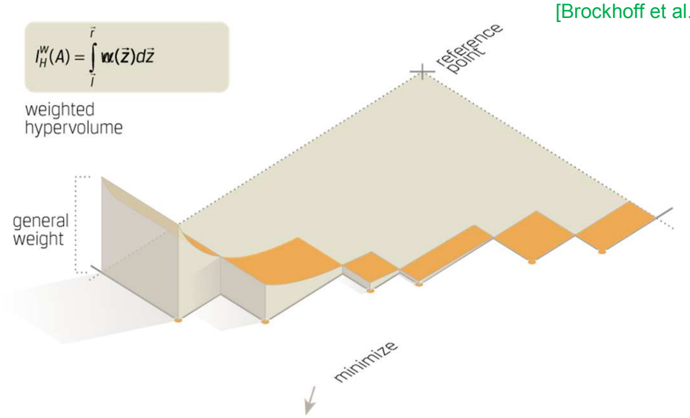


❷ **Use quality indicators, e.g.:**

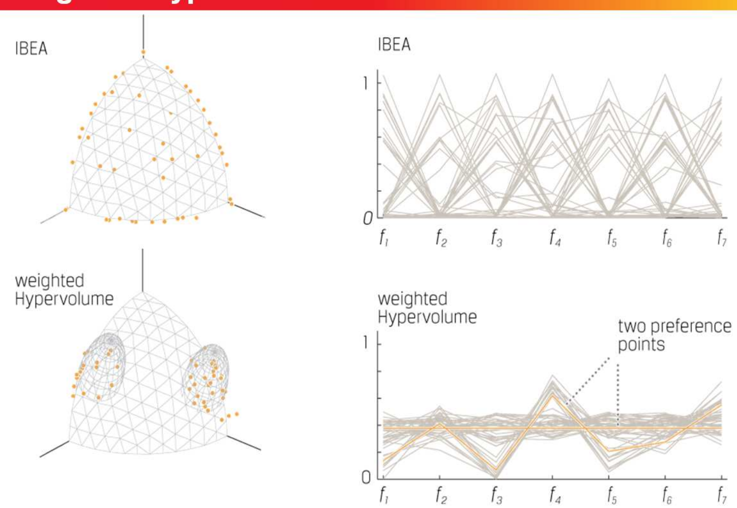
- based on reference points and directions [Deb and Sundar 2006, Deb and Kumar 2007]
- based on the hypervolume indicator
  [Brockhoff et al. 2013] [Wagner and Trautmann 2010]
- based on the R2 indicator [Trautmann et al. 2013]

Example: Weighted Hypervolume Indicator

[Brockhoff et al. 2013]



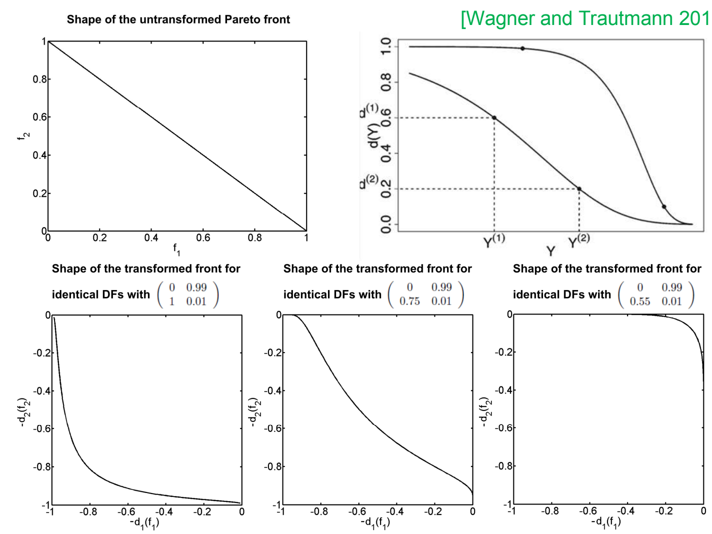
Weighted Hypervolume in Practice



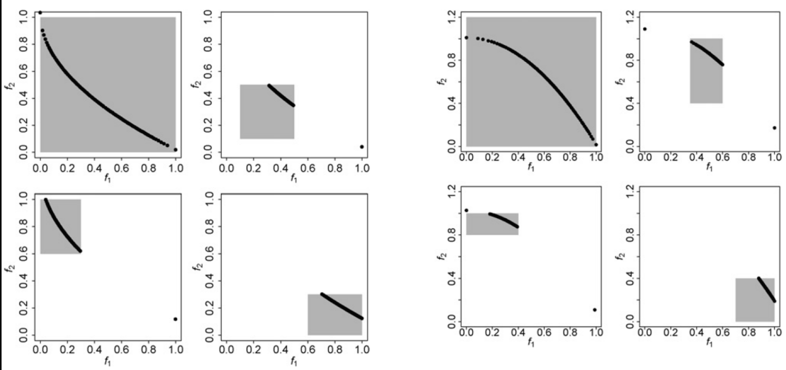
[Auger et al. 2009b]

Example: Desirability Function (DF)-SMS-EMOA

[Wagner and Trautmann 2010]



DF-SMS-EMOA in Practice

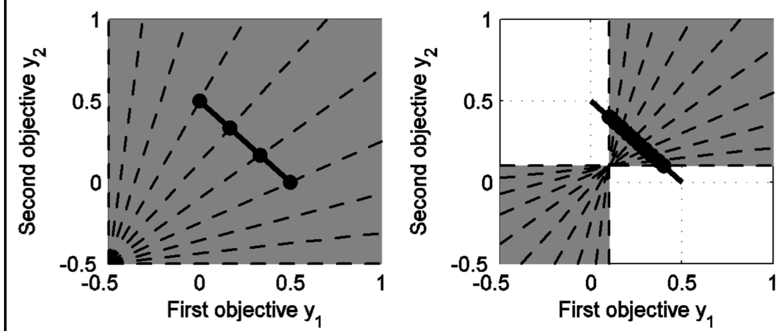


Example: R2-EMOA

Concept

Integration of preferences by varying the scalarizing functions

Position of ideal point

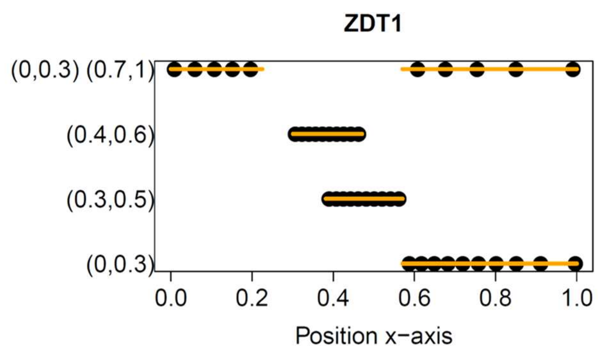


Example: R2-EMOA

Concept

Integration of preferences by varying the scalarizing functions

Restriction of the weight space



Interactive Approaches

Successive Preference Articulation = Interactive EMO

- recent interest of both EMO and MCDM community
- important in practice

Examples

- first interactive EMO: [Tanino et al. 1993]
- good overview: [Jaszkiewicz and Branke 2008]
- more recent work: [Brockhoff et al. 2014] [Branke et al. 2014]

Issues/Open Questions

- realistic scenarios/ value functions
- evaluation of interactive algorithms [López-Ibáñez and Knowles 2015]

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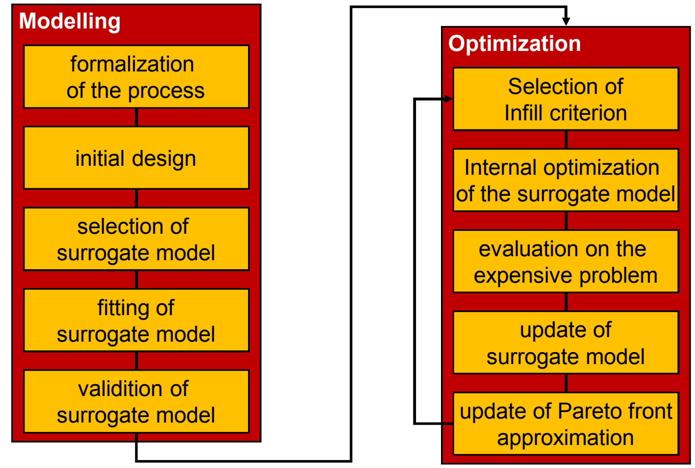
A Few Examples From Practice

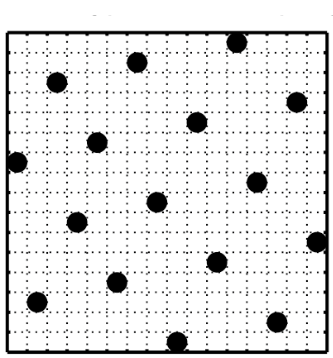
Surrogate-Based EMO

EMO + modeling and sequential experimental design



Initial design

Latin Hypercube Sampling (LHS)
- Space-filling coverage of the decision space
- Maximum resolution for each parameter

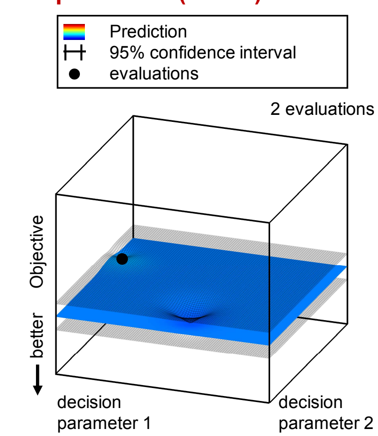


Selection of Surrogate Model

Design and Analysis of Computer Experiments (DACE)

[Sacks et al. 1989]

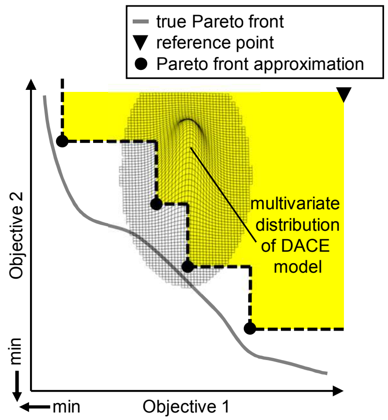
- based on kriging models
- local modeling of the available evaluations
- possibility of adaptive refinement



Selection of infill criterion

Aim

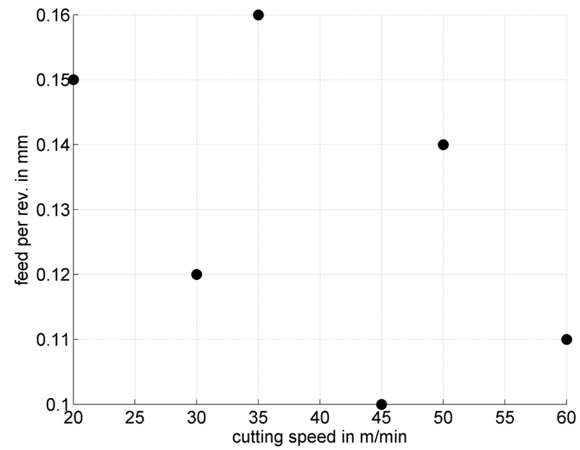
- refinement of the Pareto front approximation
- maximization of the dominated hypervolume
- improvements through experiments in the currently nondominated area
- see [Wagner et al. 2010] for a survey and first theoretical results



Practical application (drilling of Inconel 708)

Initial design

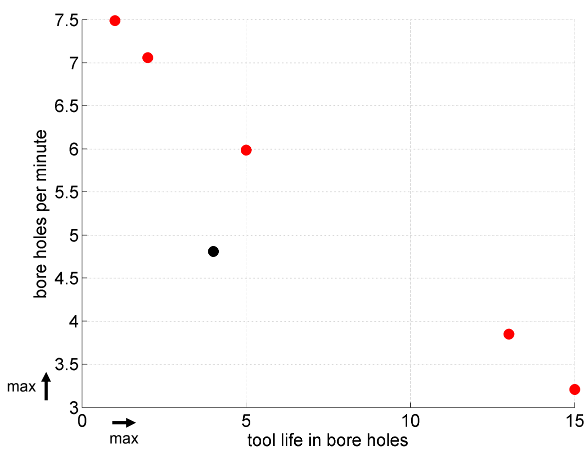
[Zhang et al. 2012]



Practical application (drilling of Inconel 708)

Initial Pareto front approximation

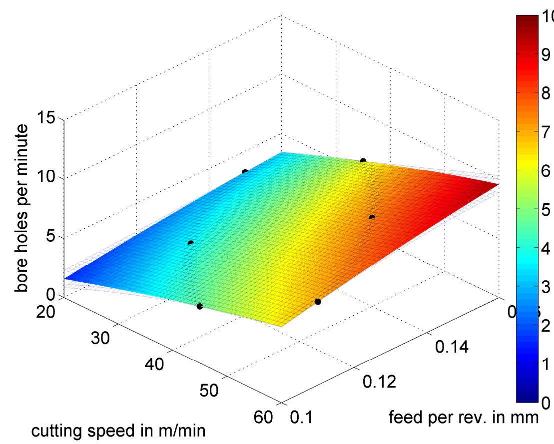
[Zhang et al. 2012]



Practical application (drilling of Inconel 708)

Initial model for objective 1 (productivity)

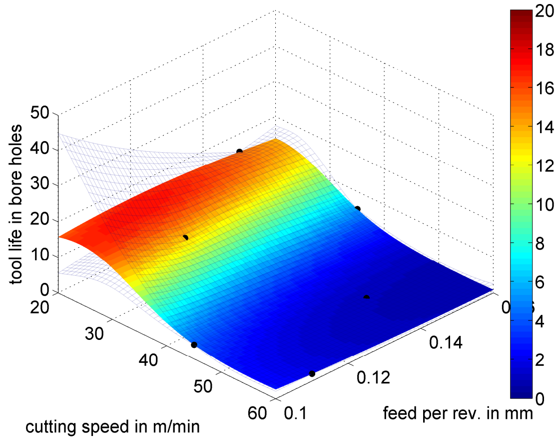
[Zhang et al. 2012]



Practical application (drilling of Inconel 708)

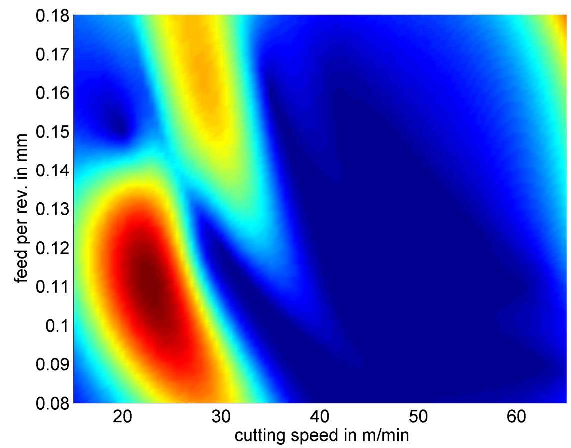
Initial model for objective 2 (tool life)

[Zhang et al. 2012]



Practical application (drilling of Inconel 708)

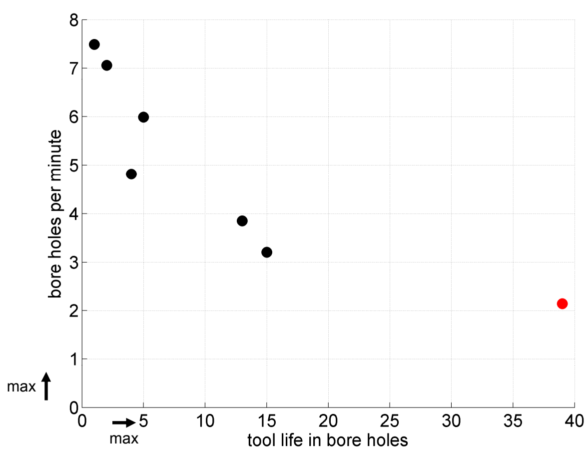
Optimization of Infill criterion



Practical application (drilling of Inconel 708)

Updated Pareto front approximation

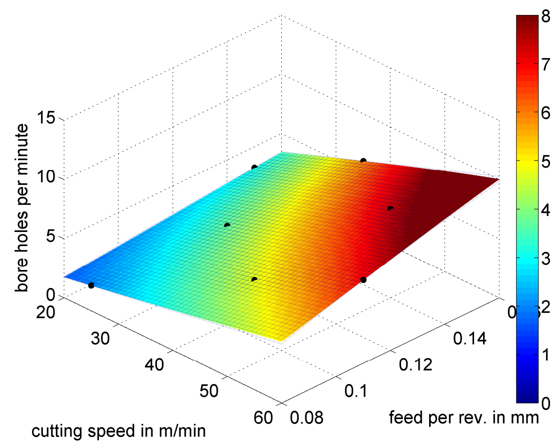
[Zhang et al. 2012]



Practical application (drilling of Inconel 708)

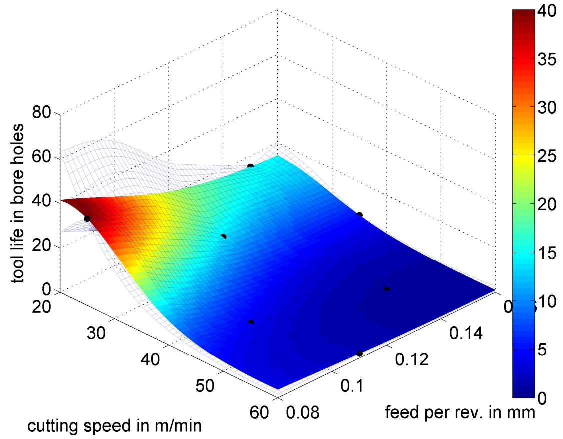
Updated model for objective 1 (productivity)

[Zhang et al. 2012]



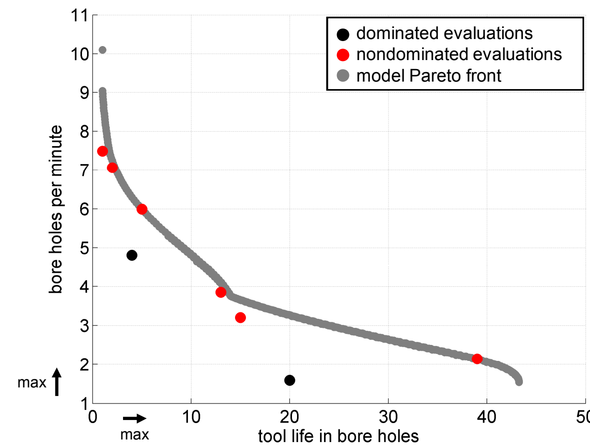
Practical application (drilling of Inconel 708)

Updated model for objective 2 (tool life) [Zhang et al. 2012]



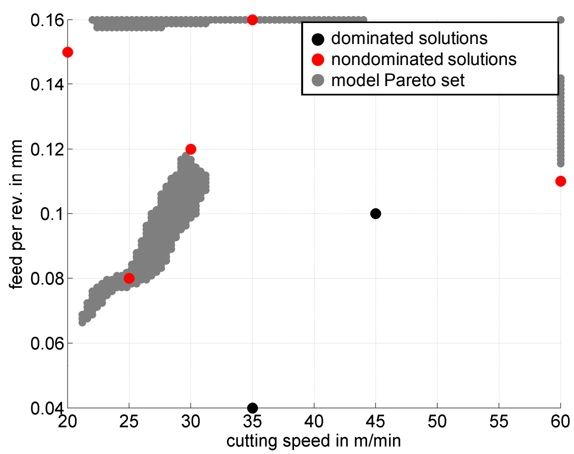
Practical application (drilling of Inconel 708)

Final Pareto front approximation [Zhang et al. 2012]



Practical application (drilling of Inconel 708)

Final Pareto set approximation (Innovization) [Zhang et al. 2012]



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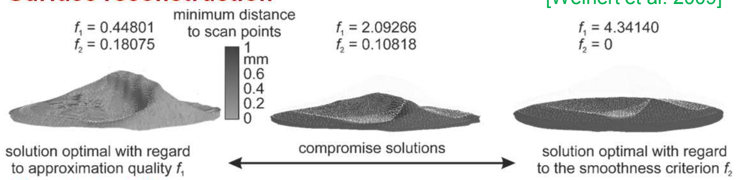
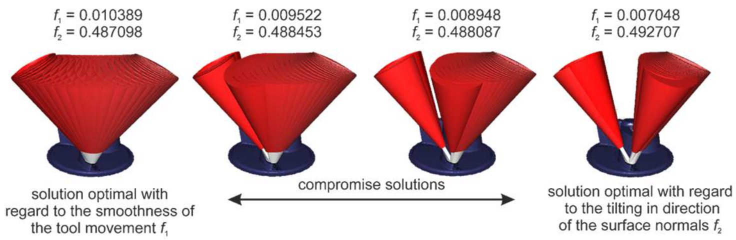
Applications of EMO

Surface reconstruction

[Weinert et al. 2009]



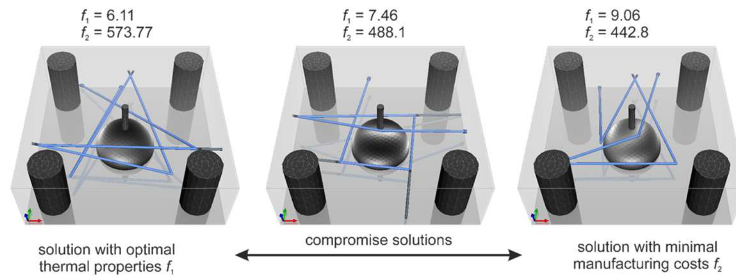
Five-Axis Milling



Applications of EMO

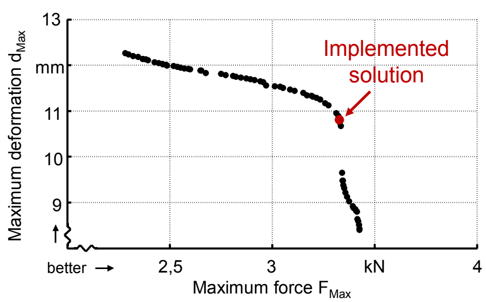
Mold Temperature Cooling Systems

[Weinert et al. 2009]



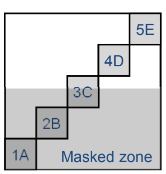
Applications of EMO

Hot Compaction of Thermoplastic Composites



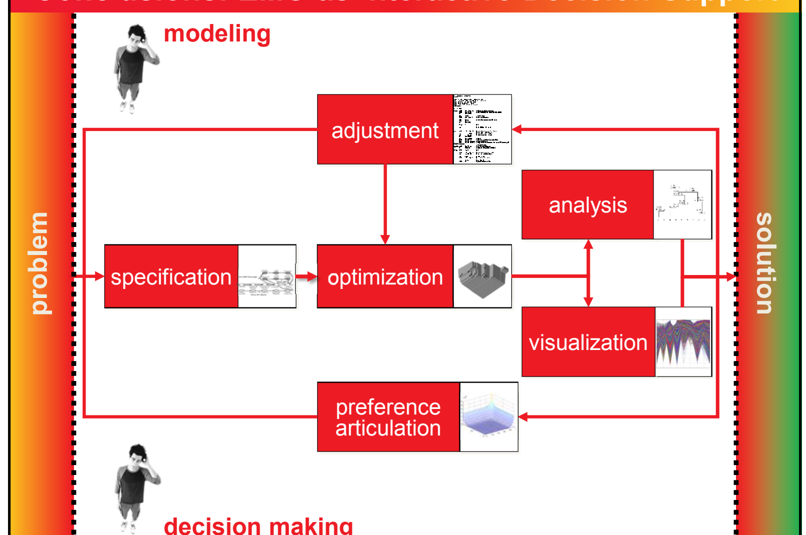
[Wagner et al. 2008]

T_{PH}^* = 146 °C,
t_{PH}^* = 174 s,
t_M^* = 170 s,
p_P^* = 1.1 MPa,
T_p^* = 169 °C,
t_P^* = 180 s



Position	T_M [°C]	F_{Max} [kN]	d_{Max} [mm]
1A	54	2.4	10.7
2B	84	2.8	9.8
3C	121	3.3	8.8
4D	147	3.3	8.3
5E	100	3.2	9.3

Conclusions: EMO as Interactive Decision Support



The EMO Community

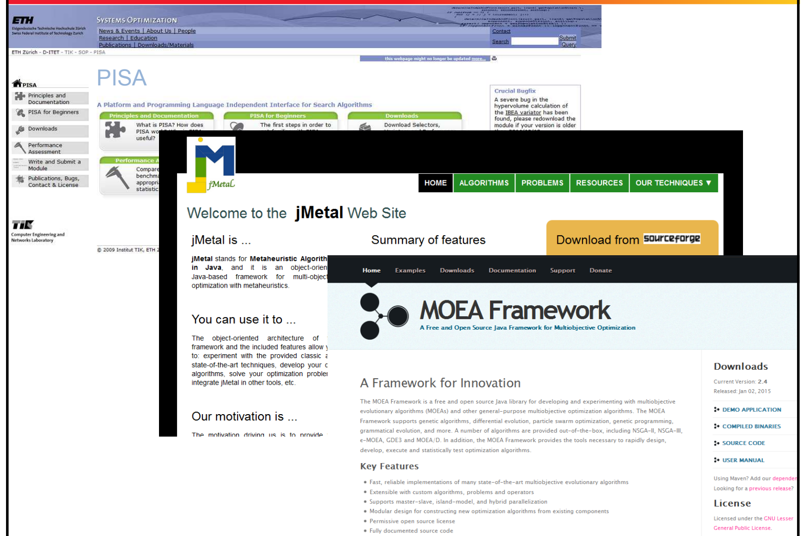
Links:

- EMO mailing list: *https://lists.dei.uc.pt/mailman/listinfo/emo-list*
- MCDM mailing list: *http://lists.jyu.fi/mailman/listinfo/mcdm-discussion*
- EMO bibliography: *http://www.lania.mx/~ccoello/EMOO/*
- EMO conference series: *http://www.dep.uminho.pt/EMO2015/*

Books:

- ***Multi-Objective Optimization using Evolutionary Algorithms*** Kalyanmoy Deb, Wiley, 2001
- ***Evolutionary Algorithms for Solving Multi Evolutionary Algorithms for Solving Multi-Objective Problems Objective Problems***, Carlos A. Coello Coello, David A. Van Veldhuizen & Gary B. Lamont, Kluwer, 2nd Ed. 2007
- **Multiobjective Optimization—Interactive and Evolutionary Approaches**, J. Branke, K. Deb, K. Miettinen, and R. Slowinski, editors, volume 5252 of *LNCS*. Springer, 2008 [(still) many open questions!]
- and more…

Software



Perspectives

Challenging Open (Research) Directions

- from algorithms to toolkits
  - libraries of modules for each task (selection, variation, etc.)
  - problem-specific algorithm configuration/ parameter tuning
- benchmarking
  - comparison with classical approaches
  - design/selection of practically relevant problems
  - Algorithm/toolkit recommendations for practice
- integration of EMO and MCDM into one field
- interactive preference articulation and learning
- interactive problem design
- integration of problem-specific knowledge

Questions?

Additional Slides

Instructor Biography: Dimo Brockhoff

Dimo Brockhoff

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