

Expressive Genetic Programming: Concepts and Applications

A Tutorial at the Genetic and Evolutionary Computation Conference
(GECCO-2016), in Denver, Colorado, USA

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Instructors (1)



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Instructors (2)



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Scope and Content (1)

The language in which evolving programs are expressed can have significant impacts on the dynamics and problem-solving capabilities of a genetic programming system. In GP these impacts are driven by far more than the absolute computational power of the languages used; just because a computation is theoretically possible in a language, it doesn't mean it's readily discoverable or leveraged by evolution. Highly expressive languages can facilitate the evolution of programs for any computable function using, as appropriate, multiple data types, evolved subroutines, evolved control structures, evolved data structures, and evolved modular program and data architectures. In some cases expressive languages can even support the evolution of programs that express methods for their own reproduction and variation (and hence for the evolution of their offspring).

Scope and Content (2)

This tutorial will present a range of approaches that have been taken for evolving programs in expressive programming languages. We will then provide a detailed introduction to the Push programming language, which was designed specifically for expressiveness in genetic programming systems. Push programs are syntactically unconstrained but can nonetheless make use of multiple data types and express arbitrary control structures, potentially supporting the evolution of complex, modular programs in a particularly simple and flexible way.

Interleaved with our description of the Push language will be demonstrations of the use of analytical tools such as graph databases and program diff/merge tools to explore ways in which evolved Push programs are actually taking advantage of the language's expressive features. We will illustrate, for example, the effective use of multiple types and type-appropriate functions, the evolution and modification of code blocks and looping/recursive constructs, and the ability of Push programs to handle multiple, potentially related tasks.

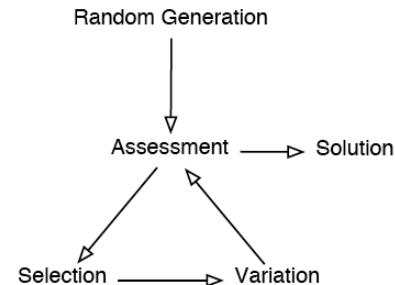
Scope and Content (3)

We will conclude with a brief review of over a decade of Push-based research, including the production of human-competitive results, along with recent enhancements to Push that are intended to support the evolution of complex and robust software systems.

Course Agenda

- Genetic programming
- Expressiveness and evolvability
- The Push programming language and PushGP
- Demo
- Results
- Ongoing Research

Evolutionary Computation



Genetic Programming

- Evolutionary computing to produce executable computer programs
- Programs are assessed by executing them
- Automatic programming; producing software

Program Representations

- Lisp-style symbolic expressions (Koza, ...)
- Purely functional/lambda expressions (Walsh, Yu, ...)
- Linear sequences of machine/byte code (Nordin et al., ...)
- Artificial assembly-like languages (Ray, Adami, ...)
- Stack-based languages (Perkis, Spector, Stoffel, Tchernev, ...)
- Graph-structured programs (Teller, Globus, ...)
- Object hierarchies (Bruce, Abbott, Schmutter, Lucas, ...)
- Fuzzy rule systems (Tunstel, Jamshidi, ...)
- Logic programs (Osborn, Charif, Lamas, Dubossarsky, ...)
- Strings, grammar-mapped to arbitrary languages (O'Neill, Ryan, McKay, ...)

Mutating Lisp (1)

```
(+ (* X Y)
  (+ 4 (- Z 23)))
```

Mutating Lisp (2)

```
(+ (* X Y)
  (+ 4 (- Z 23)))
```

Mutating Lisp (3)

```
(+ (- (+ 2 2) Z)
  (+ 4 (- Z 23)))
```

Recombining Lisp

```
(+ (* X Y)
  (+ 4 (- Z 23)))
(- (* 17 (+ 2 X))
  (* (- (* 2 Z) 1)
    (+ 14 (/ Y X))))
```

(+ (- (* 2 Z) 1) ←
(+ 4 (- Z 23)))

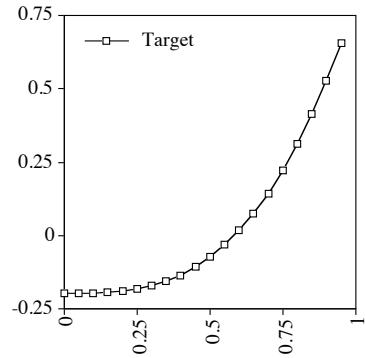
Symbolic Regression

- A simple example
- Given a set of data points, evolve a program that produces y from x.
- Primordial ooze: +, -, *, %, x, 0.1
- Fitness = error (smaller is better)

Genetic Programming Parameters

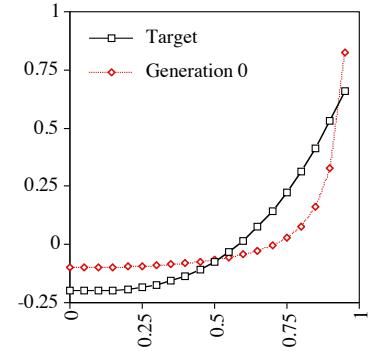
- Maximum number of Generations: 51
- Size of Population: 1000
- Maximum depth of new individuals: 6
- Maximum depth of new subtrees for mutants: 4
- Maximum depth of individuals after crossover: 17
- Fitness-proportionate reproduction fraction: 0.1
- Crossover at any point fraction: 0.3
- Crossover at function points fraction: 0.5
- Selection method: FITNESS-PROPORTIONATE
- Generation method: RAMPED-HALF-AND-HALF
- Randomizer seed: 1.2

Evolving $y = x^3 - 0.2$



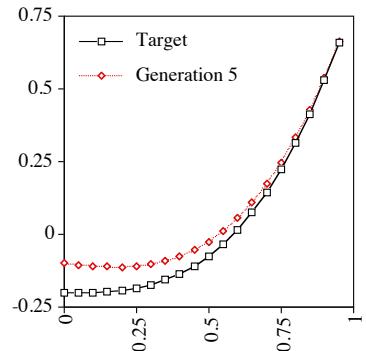
Best Program, Gen 0

(
(- (% (* 0.1
(* X X))
(- (% 0.1 0.1)
(* X X)))
0.1)



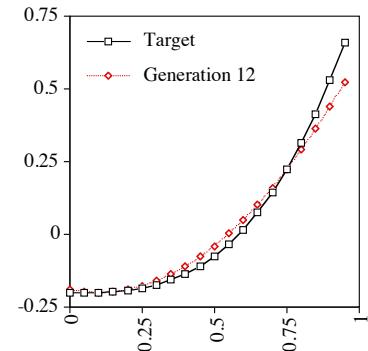
Best Program, Gen 5

(
(- (* (* (% X 0.1)
(* 0.1 X))
(- X
(% 0.1 X))))
0.1)



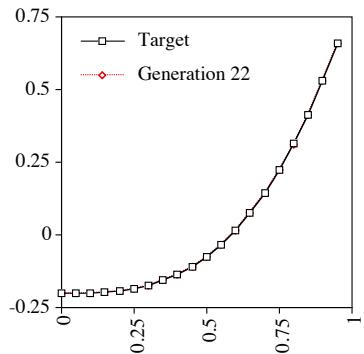
Best Program, Gen 12

(
(+ (- (- 0.1
(- 0.1
(- (* X X))
(+ 0.1
(- 0.1
(* 0.1
0.1))))))
(* X
(* (% 0.1
(% (* (* (- 0.1 0.1)
(+ X
(- 0.1 0.1))))
X)
(+ X (+ (- X 0.1)
(* X X))))
(* X X))



Best Program, Gen 22

```
(- (- (* X (* X X))  
     0.1)  
    0.1)
```



Expressiveness

- Turing machine tables
- Lambda calculus expressions
- Partial recursive functions
- Register machine programs
- Assembly language programs
- etc.

Evolvability

- The fact that a computation **can be expressed** in a formalism does **not** imply that it **will be produced** in by a human programmer **or** by evolution.
- Research program:
 1. Provide expressiveness
 2. Study/enhance evolvability

Data/Control Structure

- Data abstraction and organization
Data types, variables, data structures, name spaces, ...
- Control abstraction and organization
Conditionals, loops, modules, threads, ...

Evolving Structure (1)

- Specialize GP techniques to **directly** support human programming language abstractions
- Examples:
 1. Structured data via strongly typed GP
 2. Structured control via automatically defined functions

Strongly Typed GP (Montana)

- Primitives annotated with types
- Constrained code generation
- Constrained mutation and recombination

Automatically Defined Functions (Koza)

- All programs in the population have the same, pre-specified architecture
- Genetic operators respect that architecture
- Significant implementation costs
- Significant pre-specification
- Architecture-altering operations: more power and higher costs

Evolving Structure (2)

- Specialize GP techniques to **indirectly** support human programming language abstractions
- Map from unstructured genomes to programs in languages that support abstraction (e.g. via grammars)

Evolving Structure (3)

- Evolve programs in a minimal-syntax language that nonetheless supports a full range of data and control abstractions
- For example: orchestrate data flows via stacks, not via syntax
- Minimal syntax + maximal, flexible semantics
- **Push**

Push (1)

- Designed for program evolution
- Data flows via stacks, not syntax
- One stack per type:
integer, float, boolean, string, code, exec, vector, ...
- program → instruction | literal | (program*)
- Turing complete, with rich data and control structures

Push (2)

- Missing argument? NOOP
- Argument order: Generally reflect expected order from text or canonical usage
- PushGP is a GP system that evolves Push programs
- <http://pushlanguage.org>

Push (3)

- Implementations in C++, Clojure, Common Lisp, Java, Javascript, Python, Racket, Ruby, Scala, Scheme, Swift
- Examples in this presentation use Clojush, a Push/PushGP implementation in Clojure

Why Push?

- Expressive: data types, data structures, variables, conditionals, loops, recursion, modules, ...
- Elegant: minimal syntax and a simple, stack-based execution architecture
- Supports several forms of meta-evolution
- Evolvable? At minimum, supports investigation of relations between expressiveness and evolvability

Plush

Instruction	integer_eq	exec_dup	char_swap	integer_add	exec_if	
Close?	2	0	0	0	1	
Silence?	1	0	0	1	0	

- Linear genomes for Push programs
- Facilitates useful placement of code blocks
- Permits uniform linear genetic operators
- Allows for epigenetic hill-climbing

Push Program Execution

- Push the program onto the exec stack.
- While exec isn't empty, pop and **do** the top:
 - If it's an instruction, execute it.
 - If it's a literal, push it onto the right stack.
 - If it's a list, push its elements back onto the exec stack one at a time.

Instructions Implemented for Most Types

- <type>_dup
- <type>_empty
- <type>_eq
- <type>_flush
- <type>_pop
- <type>_rot
- <type>_shove
- <type>_stackdepth
- <type>_swap
- <type>_yank
- <type>_yankdup

Selected Integer Instructions

```
integer_add integer_dec integer_div  
integer_gt integer_fromstring integer_min  
integer_mult integer_rand
```

Selected Boolean Instructions

boolean and boolean xor boolean frominteger

Selected String Instructions

```
string_concat string_contains string_length  
string_removechar string_replacechar
```

Many More Types and Instructions

Selected Exec Instructions

Conditionals:

exec if exec when

General loops:

exec_do*while

“For” loops:

`exec_do*range` `exec_do*times`

Looping over structures:

`exec_do*vector_integer exec_string_iterate`

Combinators:

exec_k exec_y exec_s

```
;; https://github.com/lspector/Clojush/
=> (run-push '(1 2 integer_add) (make-push-state))

:exec ((1 2 integer_add))
:integer ()

:exec (1 2 integer_add)
:integer ()

:exec (2 integer_add)
:integer (1)

:exec (integer_add)
:integer (2 1)

:exec ()
:integer (3)
```

```
=> (run-push '(2 3 integer_mult 4.1 5.2 float_add
              true false boolean_or)
              (make-push-state))

:exec ()
:integer (6)
:float (9.3)
:boolean (true)
```

In other words

- Put 2×3 on the integer stack
- Put $4.1 + 5.2$ on the float stack
- Put $true \vee false$ on the boolean stack

```
=> (run-push '(2 boolean_and 4.1 true integer_div
              false 3 5.2 boolean_or integer_mult
              float_add)
              (make-push-state))

:exec ()
:integer (6)
:float (9.3)
:boolean (true)
```

Same as before, but

- Several operations (e.g., `boolean_and`) become NOOPs
- Interleaved operations

```
=> (run-push
  '(4.0 exec_dup (3.13 float_mult) 10.0 float_div)
  (make-push-state))

:exec ((4.0 exec_dup (3.13 float_mult) 10.0 float_div))
:float ()

:exec (4.0 exec_dup (3.13 float_mult) 10.0 float_div)
:float ()

:exec (exec_dup (3.13 float_mult) 10.0 float_div)
:float (4.0)

:exec((3.13 float_mult) (3.13 float_mult) 10.0 float_div)
:float (4.0)

...
:exec ()
:float (3.91876)
```

Computes $4.0 \times 3.13 \times 3.13 / 10.0$

```
=> (run-push '(1 8 exec_do*range integer_mult)
              (make-push-state))

:integer (40320)
```

Computes $8!$ in a fairly “human” way

```
=> (run-push '(code_quote
  (code_quote (integer_pop 1)
    code_quote (code_dup integer_dup
      1 integer_sub code_do
      integer_mult)
    integer_dup 2 integer_lt code_if)
  code_dup
  8
  code_do)
(make-push-state))

:code ((code_quote (integer_pop 1) code_quote (code_dup
  integer_dup 1 integer_sub code_do integer_mult)
  integer_dup 2 integer_lt code_if))
:integer (40320)
```

A less “obvious” evolved calculation of 8!

```
=> (run-push '(0 true exec_while
  (1 integer_add true))
  (make-push-state))

:exec (1 integer_add true exec_while (1 integer_add
  true))
:integer (199)
:termination :abnormal
```

- An infinite loop
- Terminated by eval limit
- Result taken from appropriate stack(s) upon termination

```
=> (run-push '(in1 in1 float_mult 3.141592 float_mult)
  (push-item 2.5 :input (make-push-state)))

:float (19.63495)
:input (2.5)
```

Computes the area of a circle with the given radius: $3.141592 \times \text{in1} \times \text{in1}$

Set up run for target $x^3 - 2x^2 - x$

```
(pushgp
  {:error-function
   (fn [program]
     (vec
       (for [input (range 10)]
         (let [output (-> (make-push-state)
                           (push-item input :input)
                           (run-push program)
                           (top-item :integer)))
              (if (number? output)
                  (Math/abs (float (- output
                                         (- (* input
                                                input
                                                input)
                                               (* 2 input input)
                                               input))))
                                         1000))))))
  :atom-generators (list (fn [] (lrand-int 10))
                         'in1
                         'integer_div
                         'integer_mult
                         'integer_add
                         'integer_sub)
  :population-size 100})
```

Auto-Simplification

- Loop:
 - Make it randomly simpler
 - If it's as good or better: keep it
 - Otherwise: revert
 - GECCO-2014 poster showed that this can efficiently and reliably reduce the size of the evolved programs
 - GECCO-2014 student paper explored its utility in a genetic operator

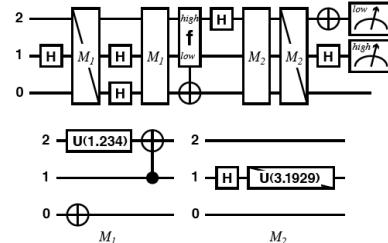
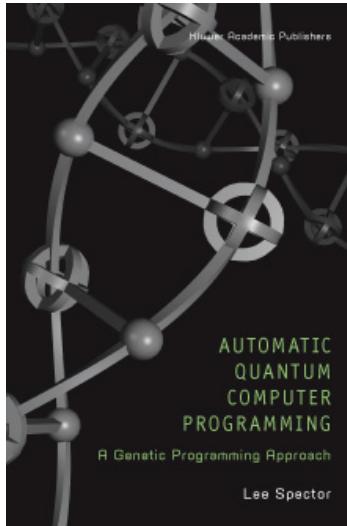
```
SUCCESS at generation 29
Successful program: (5 4 in1 integer_sub in1 integer_mult
integer_sub integer_div integer_mult 6 integer_sub integer_add
in1 5 integer_sub integer_add in1 5 integer_add integer_add
integer_mult in1 integer_mult)
Errors: [0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0]
Total error: 0.0
History: null
Size: 24
```

```
Auto-simplifying with starting size: 24
...
step: 1000
program: (5 4 in1 integer_sub in1 integer_mult integer_sub 6
integer_sub in1 5 integer_sub integer_add in1 5 integer_add
integer_add in1 integer_mult)
errors: [0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0]
total: 0.0
size: 20
```

DEMO

Problems Solved by PushGP in the GECCO-2005 Paper on Push3

- Reversing a list
- Factorial (many algorithms)
- Fibonacci (many algorithms)
- Parity (any size input)
- Exponentiation
- Sorting



**Humies 2004
GOLD MEDAL**

Figure 8.7. A gate array diagram for an evolved version of Grover's database search algorithm for a 4-item database. The full gate array is shown at the top, with M_1 and M_2 standing for the smaller gate arrays shown at the bottom. A diagonal line through a gate indicates that the matrix for the gate is transposed. The “ f ” gate is the oracle.

Genetic Programming for Finite Algebras

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**Humies 2008
GOLD MEDAL**

29 Software Synthesis Benchmarks

- Number IO, Small or Large, For Loop Index, Compare String Lengths, Double Letters, [Collatz Numbers](#), Replace Space with Newline, [String Differences](#), Even Squares, [Wallis Pi](#), String Lengths Backwards, Last Index of Zero, Vector Average, Count Odds, Mirror Image, [Super Anagrams](#), Sum of Squares, Vectors Summed, X-Word Lines, [Pig Latin](#), Negative to Zero, Scrabble Score, [Word Stats](#), Checksum, Digits, Grade, Median, Smallest, Syllables
- PushGP has solved all of these except for the ones in blue
- Presented in a GECCO-2015 GP track paper

Example: Checksum

Multiple types, looping, and code blocks

Simplified solution:

```
("Check sum is " print_string
  inl 64 exec_string_iterate (integer_fromchar integer_add)
  64 integer_mod
  \space integer_fromchar integer_add char_frominteger
  print_char)
```

Example: Checksum

Multiple types, looping, and code blocks

Simplified solution:

```
("Check sum is " print_string
  inl 64 exec_string_iterate (integer_fromchar integer_add)
  64 integer_mod
  \space integer_fromchar integer_add char_frominteger
  print_char)
```

First: Print out the header

Example: Checksum

Multiple types, looping, and code blocks

Simplified solution:

```
("Check sum is " print_string
  inl 64 exec_string_iterate (integer_fromchar integer_add)
  64 integer_mod
  \space integer_fromchar integer_add char_frominteger
  print_char)
```

Second: Convert each character to an integer, sum, and add to 64.

Example: Checksum

Multiple types, looping, and code blocks

Simplified solution:

```
("Check sum is " print_string
  inl 64 exec_string_iterate (integer_fromchar integer_add)
  64 integer_mod
  \space integer_fromchar integer_add char_frominteger
  print_char)
```

Third: Mod result by 64

Example: Checksum

Multiple types, looping, and code blocks

Simplified solution:

```
("Check sum is " print_string
  in1 64 exec_string_iterate (integer_fromchar integer_add)
  64 integer_mod
  \space integer_fromchar integer_add char_frominteger
  print_char)
```

Third: Add modulus result to 32 and convert to char

Example: Checksum

Multiple types, looping, and code blocks

Simplified solution:

```
("Check sum is " print_string
  in1 64 exec_string_iterate (integer_fromchar integer_add)
  64 integer_mod
  \space integer_fromchar integer_add char_frominteger
  print_char)
```

Fourth: Print resulting char

Example: Replace Space With Newline

Multiple types, looping, multiple tasks

Simplified solution:

```
(\space char_dup exec_dup in1
  \newline string_replacechar print_string
  string_removechar string_length)
```

Example: Replace Space With Newline

Multiple types, looping, multiple tasks

Simplified solution:

```
(\space char_dup exec_dup in1
  \newline string_replacechar print_string
  string_removechar string_length)
```

First: Duplicate space character and input string for use in both tasks

Example: Replace Space With Newline

Multiple types, looping, multiple tasks

Simplified solution:

```
(\space char_dup exec_dup in1
  \newline string_replacechar print_string
  string_removechar string_length)
```

Second: Replace spaces with newlines and print

Example: Replace Space With Newline

Multiple types, looping, multiple tasks

Simplified solution:

```
(\space char_dup exec_dup in1
  \newline string_replacechar print_string
  string_removechar string_length)
```

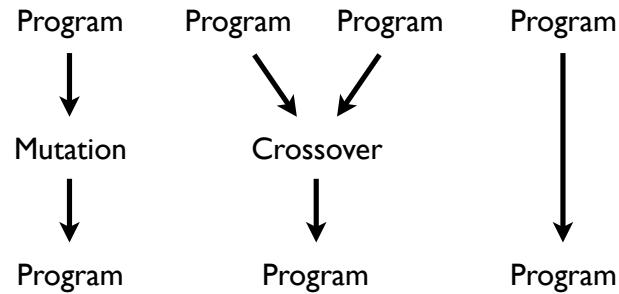
Third: Remove all spaces from second copy of input, and push length of result on integer stack for return

DEMO

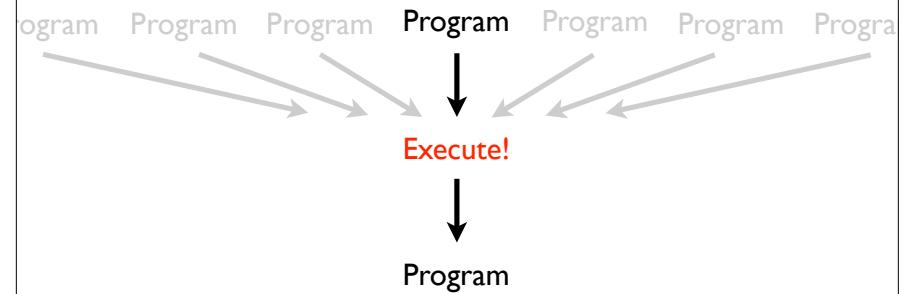
Autoconstructive Evolution

- Individual programs make their own children
- Hence they control their own mutation and recombination rates and methods
- The machinery of reproduction and diversification (i.e., the machinery of evolution) evolves
- In Push, experimentation with autoconstructive evolution is easy and natural

Variation in Genetic Programming



Autoconstruction



Autoconstruction

- It works!
- Evolved autoconstructive solutions to non-trivial problems (e.g., replace-space-with-newlines)
- Slower and less consistent than human engineered operators, but much more work to do
- More to explore

Expressiveness and Assessment

- Expressive languages ease representation of programs that over-fit training sets
- Expressive languages ease representation of programs that work only on subsets of training sets
- Lexicase selection may help: Select parents by starting with a pool of candidates and then filtering by performance on individual fitness cases, considered one at a time in random order

Evolving Modular Programs with Push

- Via code manipulation on the code or exec stacks
- Via naming and a name stack
- Via tags, inspired by Holland's work on arbitrary identifiers with inexact matching
- Evolvability challenges abound! The relationship between evolvability, robustness, and modularity and is complex
- Push facilitates experimentation in this space

Future Work

- Expression of variable scope and environments (implemented in Push, but not yet studied systematically)
- Expression of concurrency and parallelism
- Applications for which expressiveness is likely to be essential, e.g. complete software applications, agents in complex, dynamic environments

Conclusions

- GP in expressive languages may allow for the evolution of complex software
- Minimal-syntax languages can be expressive, and GP systems that evolve programs in such languages can be unusually simple and powerful
- Push has produced significant successes and provides a rich framework for experimentation
- <http://pushlanguage.org>

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