Local Search Move Strategies within MOEA/D

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ABSTRACT

Local search (LS) is at the cornerstone of many advanced heuristics for single-objective combinatorial optimization. In particular, the *move strategy*, allowing to iteratively explore neighboring solutions, is a key ingredient in the design of an efficient local search. Although LS has been the subject of some interesting investigations dedicated to multi-objective optimization, new research opportunities arise with respect to novel multi-objective search paradigms. In particular, the successful MOEA/D algorithm is a decomposition-based framework which has been intensively applied to continuous problems. However, only scarce studies exist in the combinatorial case. In this paper, we are interested in the design of cooperative scalarizing local search approaches for decomposition-based multi-objective combinatorial optimization. For this purpose, we elaborate multiple move strategies taking part in the MOEA/D replacement flow. We there-by provide some preliminary results eliciting the impact of these strategy of the final population and more importantly on the anytime performance.

1. INTRODUCTION

The study conducted in this paper is at the crossroad of single and multi-objective combinatorial optimization. On the one hand, in single-objective optimization, one is given a problem for which a solution optimizing one single objective function has to be identified. Considering difficult optimization problems, randomized search heuristics have played an important role in deriving high-quality algorithmic solutions. Among many others, local search (LS) heuristics [1] refer to algorithms where a solution is improved in an iterative search process by performing little perturbation on its vicinity. A common ingredient being at the basis of this class of algorithms is the so-called *neighborhood exploration* and *move strategy*. In fact, the two basic components of LS are: (i) the definition of at least one neighborhood relation or structure, providing for every single solution a set of neighboring solutions that can be derived by performing little changes on the variables, and (ii) the setting of the move strategy, that is how to explore those neighboring solutions and how to guide the search process when iteratively moving from one solution to another neighboring one. The definition of a neighborhood structure is in general problem- or representation-specific, which makes it of less interest in the context of this paper. However, the move strategy can be designed in a more generic manner and is intrinsic to a particular LS heuristic. The specification and the combination of these two components are in general a cornerstone in the design of advanced single-objective LS algorithms.

On the other hand, LS is not restricted to single-objective problems and can be applied to multi-objective optimization problem as well [3, 4]. Let us recall that, in multi-objective optimization, one is given a problem for which a whole set of solutions, optimizing simultaneously two or more objective functions, is to be computed. Although LS-inspired components have been already considered in multi-objective optimization, there still exists room for new research investigations in order to design novel effective and efficient LS-based multi-objective search algorithms. In particular, the so-called MOEA/D (multi-objective evolutionary algorithm based on decomposition) [5] is an aggregation-based framework whose algorithmic components were mostly designed and investigated for continuous problems. The purpose of this paper lies specifically in studying the impact of different LS move strategies when designed within the MOEA/D framework.

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2. INCORPORATING LS IN MOEA/D

MOEA/D transforms a multi-objective problem into several single objective sub-problems obtained by using a scalarizing function and solved iteratively. First, a solution x is selected at random from the neighboring sub-problems of i,

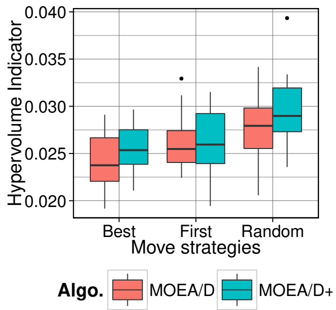

Figure 1: Relative (high budget) performance. Lower is better.

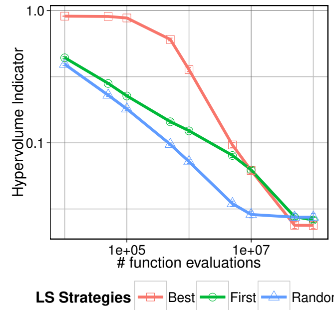

Figure 2: Relative anytime performance. Lower is better. Notice the log scale.

and we consider to use it a starting point for local search. In this research, we consider one neighborhood structure and three main move strategies to search for improving solutions, namely, Best, First, and Random. In the Best strategy, all solutions that can be reached from x are considered and all improving ones with respect to any neighboring sub-problem in MOEA/D are recorded. In the First strategy, the exploration of the neighborhood of solution x stops as soon as an improving solution with respect to any neighboring sub-problem is found. This solution is then recorded. In the Random strategy, one solution is selected at random among all the solution that can be reached from the local search neighborhood structure. Then the replacement can take place within MOEA/D. More precisely, among the improving solutions recorded in the previous stage, we shall decide which one(s) will enter the population. This is done according to two strategies: either the neighboring sub-problems of the current sub-problem i are traversed in a random order and an improving solution is assigned in a random order, or the best solution from the possibly improving ones is assigned to every sub-problem.

3. PRELIMINARY RESULTS

We consider the multi-objective euclidian traveling salesman problem as a case study [4] with 100 cities. We use the well known 2-opt neighborhood structure in LS. We consider the conventional variant of MOEA/D [5] with a population size and a number of directions of 100, a neighborhood of size 20, and the Chebychev scalarizing function to define the sub-problems. We also consider the variant described in [2] and denoted MOEA/D$^+$ where two additional parameters are used, namely, the umber of copies $nr = 2$ and the neighborhood selection probability $\delta = 0.1$. The stopping condition is 10^8 function evaluations. Each algorithm is executed for 20 independent runs. We use the hypervolume relative deviation and the additive epsilon indicators [6]. The reference point is set to the worst objective-value obtained over all approximations, and the reference set is the best-found approximation over all tested configurations.

First, we analyze the quality of the obtained approximation at termination, that is, after our stopping condition is satisfied, which is a relatively high budget of 10^8 calls of the evaluation function. In Fig. 1, we can see that the Best move strategies is performing better with respect to the hypervolume indicator than the First and Random strategies especially with conventional MOEA/D. Actually, no significant difference can be observed between MOEA/D and MOEA/D$^+$, and no significant difference can be reported between Best and First for MOEA/D (according to a Mann-Whitney non-parametric statistical test with a p-value of 0.05).

Nevertheless, when taking a more close look at the anytime time performance, that is at the quality of the approximation with different budgets, we basically find that the relative performance of the considered variants is deeply impacted. This is illustrated in Fig. 2 using the relative hypervolume indicator (for which a lower value is better) and MOEA/D (The same holds for the epsilon indicator and MOEA/D$^+$, which is omitted due to space restriction). We attribute this to the fact that random sampling from the neighborhood structure allows to find improving solutions easily at the early stages of the search, whereas the Best strategy will loose much budget exploring the whole neighborhood structure. However, as the quality of population is getting improved, finding improving solutions requires to explicitly explore the whole neighborhood structure.

These preliminary results suggest that future investigations should consider hybrid local search move strategy mixing the Best and the Random strategy. It would be in fact interesting to design and to analyze adaptive strategies allowing to detect when to dynamically switch from one strategy to the other one.

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