Visualization in Multiobjective Optimization

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Draft version

The final version will be available at
http://dis.ijs.si/tea/research.htm

Contents

Motivation

Motivation

Numerous visualization methods exist, but are not well known in the community

- Visualization of approximation sets
- Visualization of EAF values and differences

No evaluation methodology for visualization methods

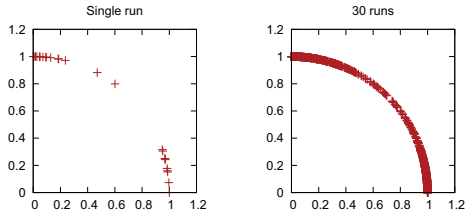
- We propose benchmark approximation sets

Introduction

Introduction

Multiobjective optimization

- More than a single objective → Pareto front
- Multiobjective evolutionary algorithms (stochastic)
- Single run → single approximation set
- Multiple runs → multiple approximation sets → EAFs



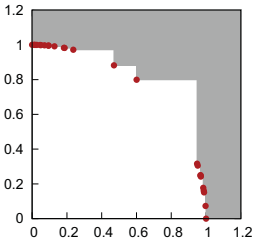
Introduction

Weak dominance relation

$$\mathbf{a} \preceq \mathbf{b} \Leftrightarrow a_j \leq b_j \text{ for all objectives } j$$

Goal-attainment

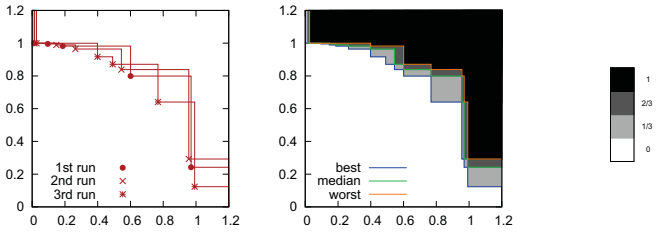
- Approximation set A
- Objective vector $\mathbf{z}$ is attained by A when exists $\mathbf{a} \in A$ so that $\mathbf{a} \preceq \mathbf{z}$



Introduction

Empirical attainment function (EAF)

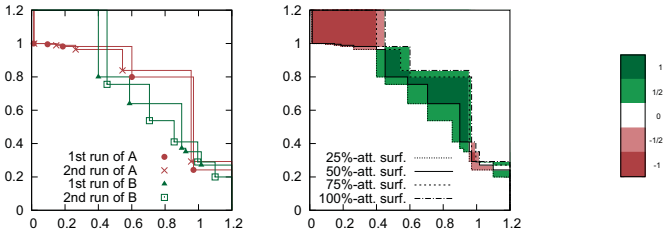
- Algorithm $\mathcal{A}$, approximation sets $A_1, A_2, \ldots, A_r$
- EAF of $\mathbf{z}$ is the frequency of attaining $\mathbf{z}$ by $A_1, A_2, \ldots, A_r$
- Summary (or k %-) attainment surfaces



Introduction

EAF differences

- Algorithm $\mathcal{A}$, approximation sets $A_1, A_2, \ldots, A_r$
- Algorithm $\mathcal{B}$, approximation sets $B_1, B_2, \ldots, B_r$
- Visualize differences between EAF values



Introduction

Two distinct tasks

- Visualization of approximation sets
  - A few approximation sets at a time
  - Compare single runs of algorithms
- Visualization of EAFs
  - Numerous approximation sets at a time
  - Compare multiple runs of algorithms

Visualization of approximation sets

Visualization of approximation sets

Desired properties of visualization methods

- Preservation of the
  - dominance relation
  - front shape
  - objective range
  - distribution of vectors
- Robustness
- Handling of large sets
- Simultaneous visualization of multiple sets
- Scalability
- Simplicity

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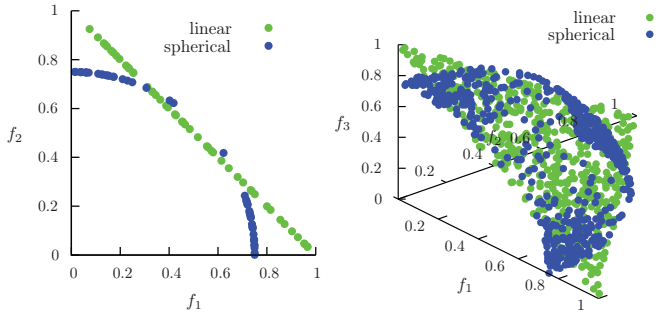
Benchmark approximation sets



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General methods

- Scatter plot matrix
- Bubble chart
- Radial coordinate visualization
- Parallel coordinates
- Heatmaps
- Sammon mapping
- Neuroscale
- Self-organizing maps
- Principal component analysis
- Isomap

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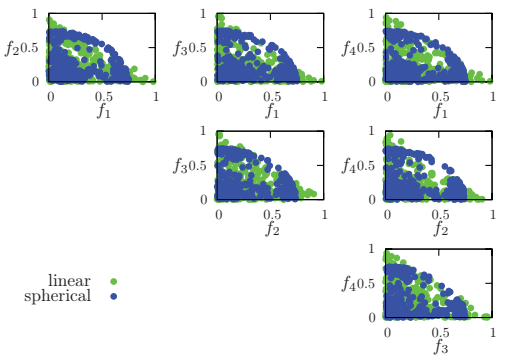
Scatter plot matrix



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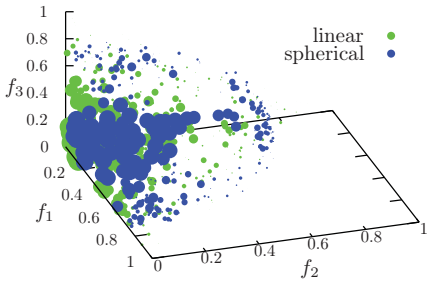
Bubble chart



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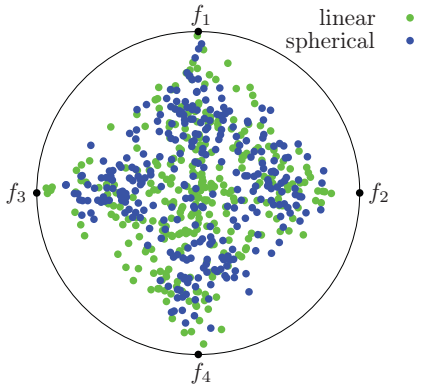
Radial coordinate visualization



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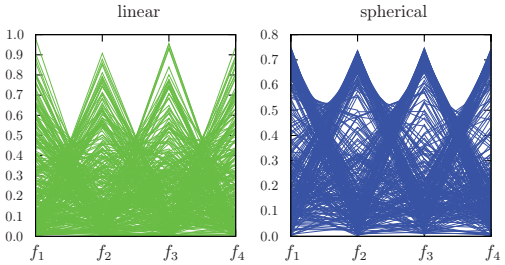
Parallel coordinates



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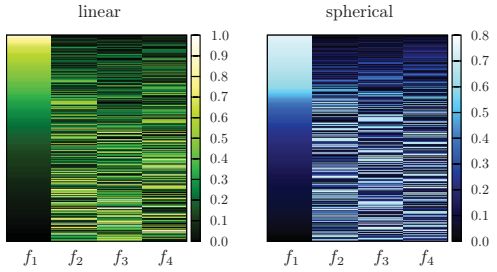
Heatmaps



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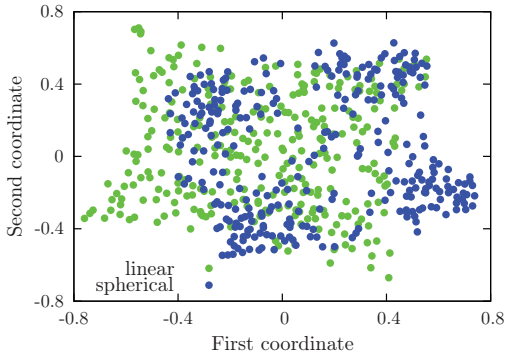
Sammon mapping



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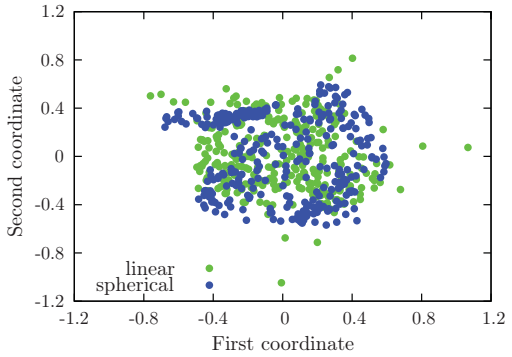
Neuroscale



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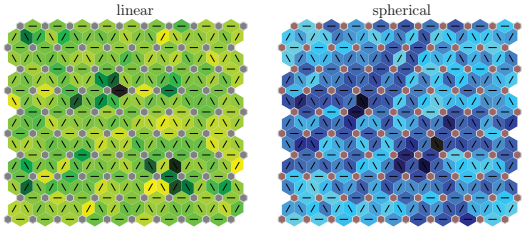
Self-organizing maps



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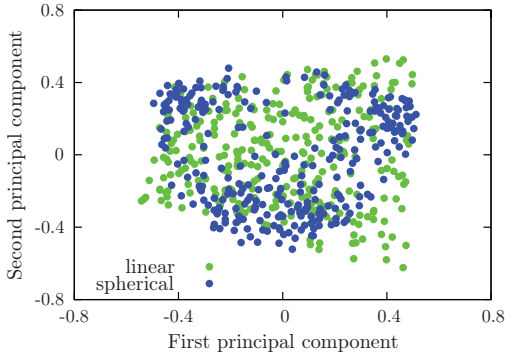
Principal component analysis



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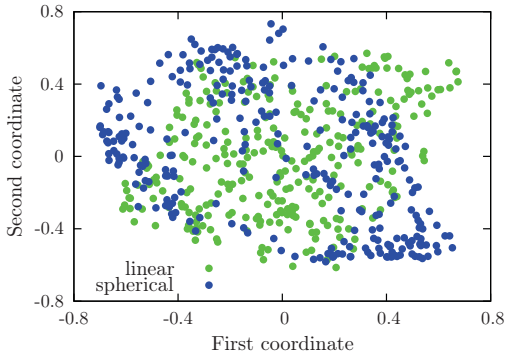
Isomap



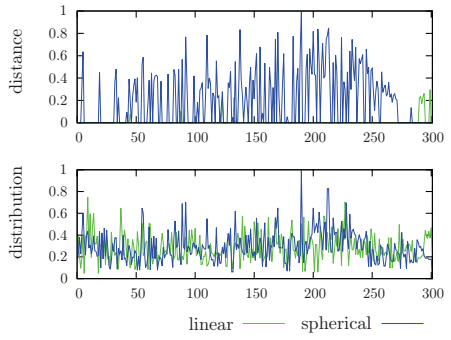
Summary of general methods

Method	Preservation[a] of the dominance relation	Preservation[a] of the front shape	Preservation[a] of the objective range	Preservation[a] of the distribution of vectors	Robustness	Handling of large sets	Simultaneous visualization	Scalability	Simplicity
Scatter plot matrix	×	≈	✓	≈	✓	≈[b]	✓	×	✓
Bubble chart	×	≈	✓	≈	✓	≈[b]	✓	×	✓
Radial coord. visual.	×	×	×	≈	✓	≈[b]	✓	✓	✓
Parallel coordinates	≈[c]	×	✓	≈[c]	✓	×	×	✓	✓
Heatmaps	×	×	✓	×	✓	×	×	✓	✓
Sammon mapping	×	×	×	✓	≈	≈[b]	✓	✓	×
Neuroscale	×	×	×	×	≈	≈[b]	✓	✓	×
Self organizing maps	×	×	×	×	≈	✓	×	✓	×
Principal comp. anal.	×	×	×	×	≈	≈[b]	✓	✓	×
Isomap	×	×	×	≈	≈	≈[b]	✓	✓	×

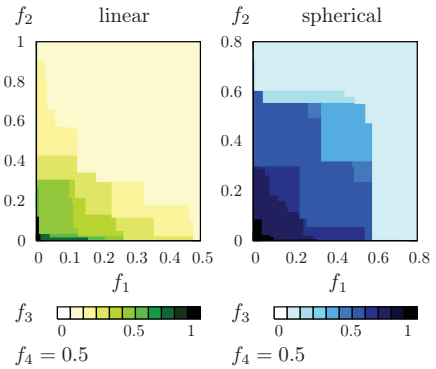
Specific methods

- Distance and distribution charts
- Interactive decision maps
- Hyper-space diagonal counting
- Two-stage mapping
- Level diagrams
- Hyper-radial visualization
- Pareto shells
- Seriated heatmaps
- Multidimensional scaling
- Prosections

Distance and distribution charts



Interactive decision maps



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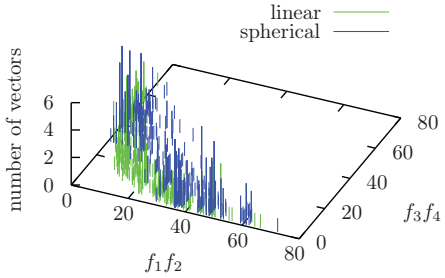
Hyper-space diagonal counting



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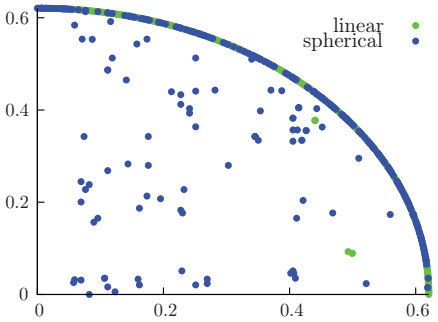
Two-stage mapping



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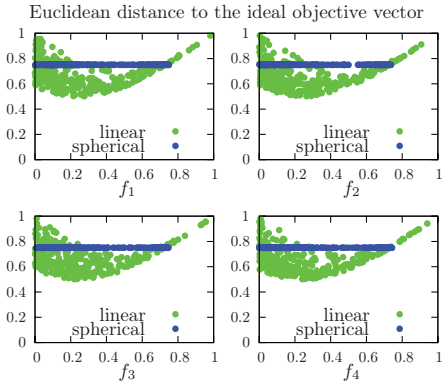
Level diagrams



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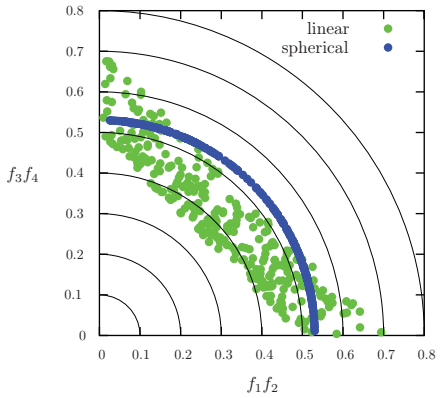
Hyper-radial visualization



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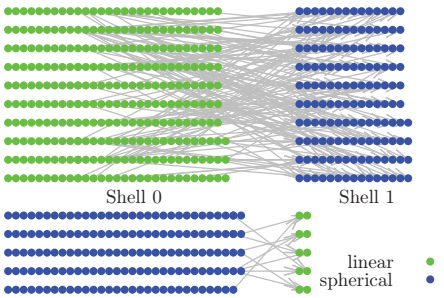
Pareto shells



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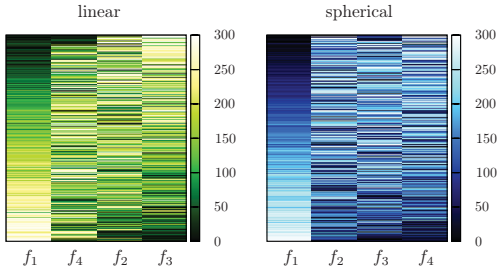
Seriated heatmaps



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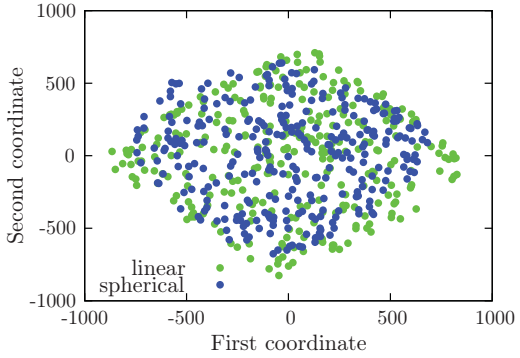
Multidimensional scaling



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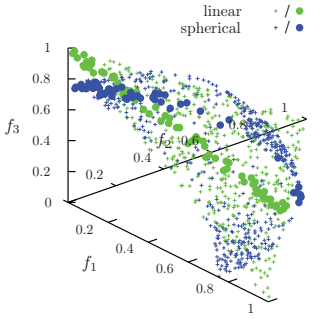
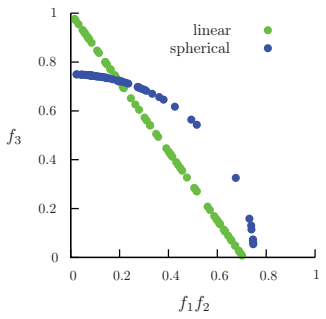
Prosections $3D \to 2D$

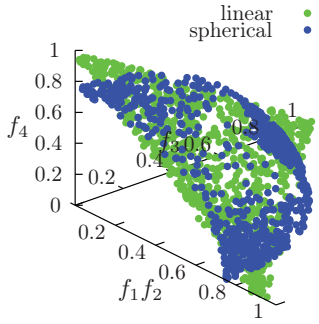


Before prosection (vectors in the section are emphasized)

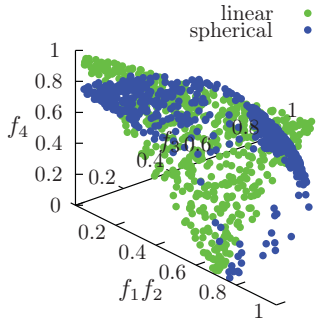


After prosection
$3\mathrm{D}(\mathbf{0}, f_1 f_2, 45°, 0.05)$

Prosections $4D \to 3D$

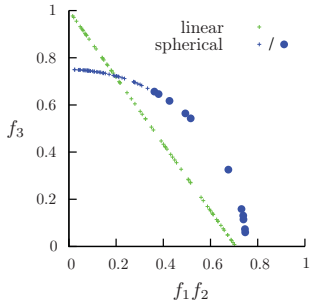


$4\mathrm{D}(\mathbf{0}, f_1 f_2, 5°, 0.05)$

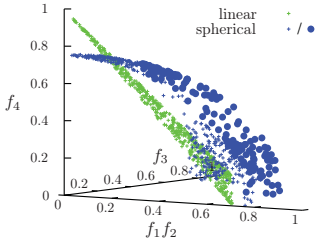


$4\mathrm{D}(\mathbf{0}, f_1 f_2, 45°, 0.05)$

Prosections – preserving the dominance relation

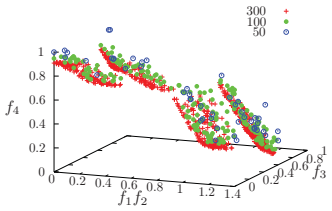


$3\mathrm{D}(\mathbf{0}, f_1 f_2, 45°, 0.05)$

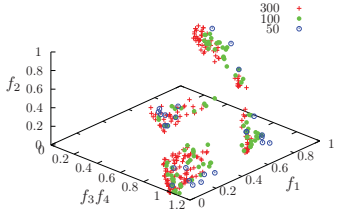


$4\mathrm{D}(\mathbf{0}, f_1 f_2, 45°, 0.05)$

Prosections – visualizing the progress of a MOEA



$4\mathrm{D}(\mathbf{0}, f_1 f_2, 45°, 0.05)$



$4\mathrm{D}(\mathbf{0}, f_3 f_4, 45°, 0.05)$

Summary of specific methods

Method	Preservation[a] of the				Robustness	Handling of large sets	Simultaneous visualization	Scalability	Simplicity
	dominance relation	front shape	objective range	distribution of vectors					
Distan. and distr. ch.	≈	×	×	×	✓	×	✓	≈	×
Interactive dec. maps	×	≈	✓	≈	✓	✓	×	×	✓
Hyper-sp. diag. count.	×	×	×	≈	✓	✓	✓	≈	×
Two-stage mapping	≈	×	×	×	×	≈[b]	✓	≈	×
Level diagrams	×	≈	✓	×	✓	≈[b]	✓	✓	✓
Hyper-radial visual.	×	≈	✓	×	✓	≈[b]	✓	✓	✓
Pareto shells	✓	×	×	×	×	×	✓	✓	✓
Seriated heatmaps	×	×	×	×	≈	×	×	✓	×
Multidim. scaling	×	×	×	×	×	≈[b]	✓	✓	×
Prosections	✓	✓	≈	✓	✓	✓	✓	×	≈

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Visualization of EAF values and differences

Visualization of EAF values and differences

2D EAFs

- Visualization of exact EAF values
- Visualization of exact EAF differences

3D EAFs

- Visualization of exact EAF values
- Visualization of exact EAF differences
- Visualization of approximated EAF values
- Visualization of approximated EAF differences

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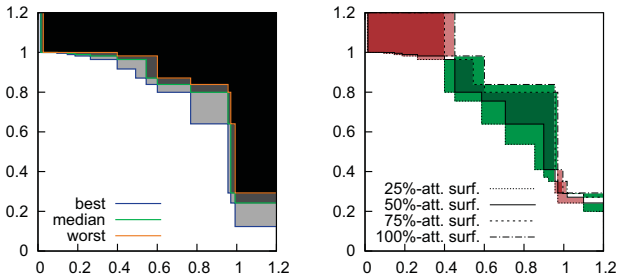
Visualization of exact 2D EAF values and differences



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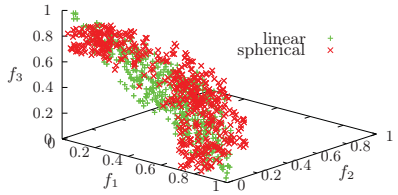
Visualization of exact 3D EAF values and differences

Sets of approximation sets

- 5 linear approximation sets with a uniform distribution of vectors
- 5 spherical approximation sets with a nonuniform distribution of vectors



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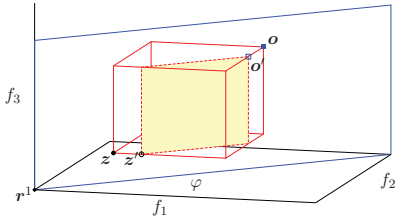
Exact 3D EAF values and differences

Slicing

- Visualize cuboids intersecting the slicing plane
- Several slicing planes are possible



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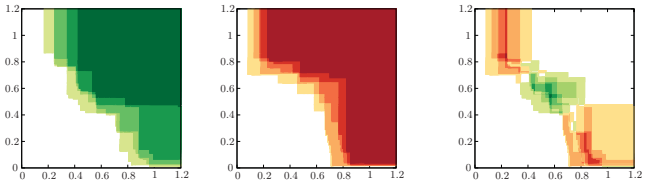
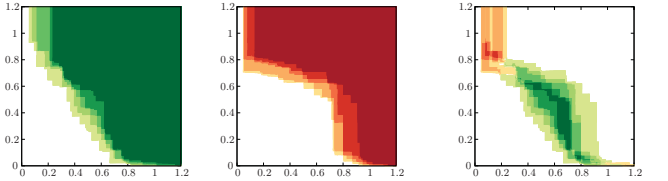
Exact 3D EAF values and differences

Slicing



Slice of α_5^{L} at $\varphi = 5°$ | Slice of α_5^{S} at $\varphi = 5°$ | Slice of $\delta_5^{\mathsf{L}-\mathsf{S}}$ and $\delta_5^{\mathsf{S}-\mathsf{L}}$ at $\varphi = 5°$



Slice of α_5^{L} at $\varphi = 45°$ | Slice of α_5^{S} at $\varphi = 45°$ | Slice of $\delta_5^{\mathsf{L}-\mathsf{S}}$ and $\delta_5^{\mathsf{S}-\mathsf{L}}$ at $\varphi = 45°$

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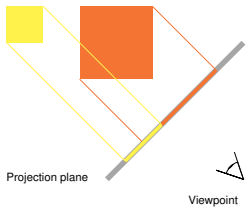
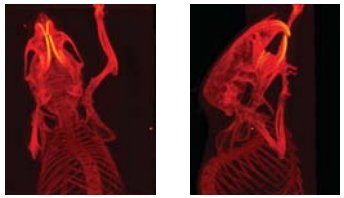
Exact 3D EAF differences

Maximum intensity projection

- Volume rendering method for spatial data represented by voxels
- Simple and efficient
- No sense of depth, cannot distinguish between front and back





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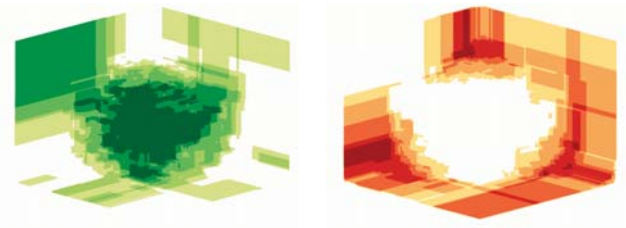
Exact 3D EAF differences

Maximum intensity projection

- Suitable for visualizing EAF differences (focus on large differences)
- Need to plot a large number of cuboids
- Sorting with regard to EAF differences



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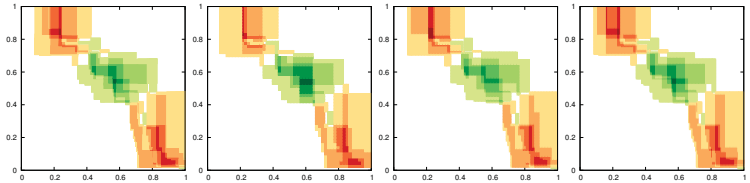
The approximated case

Discretization into voxels

- Discretization of cuboids
- Discretization from the space of EAF values/differences



Exact 64^3 voxels 128^3 voxels 256^3 voxels

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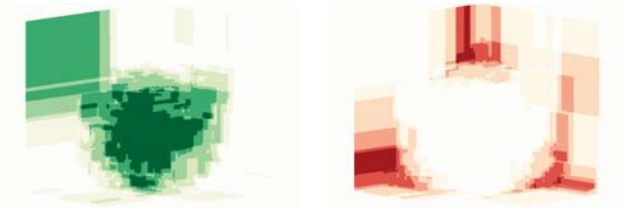
Approximated 3D EAF differences

Maximum intensity projection

- Plots produced using Voreen
- Some loss of information



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Approximated 3D EAF values and differences

Direct volume rendering

- Volume rendering method for spatial data represented by voxels
- A transfer function assigns color and opacity to voxel values
- Enables to see “inside the volume”
- Requires the definition of the transfer function

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Approximated 3D EAF differences

Direct volume rendering

DVR of $\delta_5^{L-S} = 1/5$

DVR of $\delta_5^{L-S} = 2/5$

DVR of $\delta_5^{L-S} = 3/5$

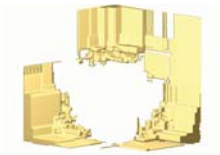
DVR of $\delta_5^{L-S} = 4/5$

DVR of $\delta_5^{L-S} = 5/5$

DVR of $\delta_5^{L-S} \in \{1/5, 5/5\}$

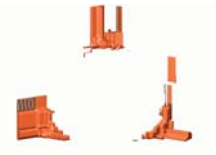
Approximated 3D EAF differences

Direct volume rendering

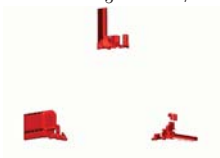


DVR of $\delta_5^{L-S} = 1/5$

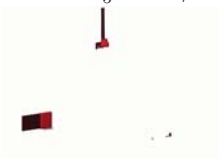
DVR of $\delta_5^{L-S} = 2/5$



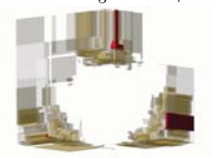
DVR of $\delta_5^{L-S} = 3/5$



DVR of $\delta_5^{L-S} = 4/5$



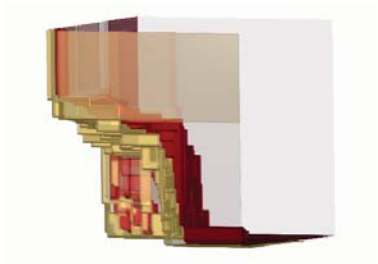
DVR of $\delta_5^{L-S} = 5/5$



DVR of $\delta_5^{L-S} \in \{1/5, 5/5\}$

Approximated 3D EAF values

Direct volume rendering



Summary

Summary – visualization of approximation sets

General methods

- Scatter plot matrix
- Bubble chart
- Radial coordinate visualization
- Parallel coordinates
- Heatmaps
- Sammon mapping
- Neuroscale
- Self-organizing maps
- Principal component analysis
- Isomap

Specific methods

- Distance and distribution charts
- Interactive decision maps
- Hyper-space diagonal counting
- Two-stage mapping
- Level diagrams
- Hyper-radial visualization
- Pareto shells
- Seriated heatmaps
- Multidimensional scaling
- Prosections

Summary – visualization of EAFs

2D case

Plot EAF values and differences

Exact 3D case

EAF values

- Slicing

EAF differences

- Slicing
- MIP

Approximated 3D case

EAF values

- Slicing
- DVR

EAF differences

- Slicing
- MIP
- DVR

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