Weighted Optimization Framework for Large-scale Multi-objective Optimization

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ABSTRACT

In this work we introduce a new method for solving multi-objective optimization problems that involve a large number of decision variables. The proposed *Weighted Optimization Framework* (WOF) relies on variable grouping and weighting to transform the original optimization problem and is designed as a generic method that can be used with any population-based algorithm. Our experiments use the WFG benchmark problems with 2 and 3 objectives and 1000 variables. Using WOF on two well-known algorithms (NSGA-II and SMPSO), we show that our method can significantly improve their performance on all of the test problems.

Keywords

Multi-objective optimization; Large-scale Optimization; Metaheuristics; Many-variable optimization; Weighting

1. INTRODUCTION

In the area of multi-objective optimization a growing interest in so called *large-scale optimization* can be observed [6]. The performance of classic metaheuristic algorithms often deteriorates when the dimension of the decision space increases. This work focuses on multi-objective optimization problems that involve a large number of decision variables (many-variable problems). In the recent years, a large variety of many-variable optimizers have been developed, most of which involve cooperative coevolution (CC). A major inspiration for the presented WOF was the article by Yang et al. [9]. They applied a CC variant using a special weighting scheme to apply and optimize a weight to each CC subcomponent, i.e. apply the same weight value to every variable in the same subcomponent. The same principle of using CC with weights has also been applied in [5, 8]. However, in the mentioned works only single-objective problems have been handled. In the area of multi-objective CC, good performance was reported in [1] for the ZDT [10] problems, combining CC with differential evolution. The performance for more complicated benchmark problems like the WFG toolkit [3] has not been tested. Iorio and Li [4] combined the concept of CC with the NSGA-II algorithm. However, they tested only small instances of 10 and 30 variables and focused on the ZDT problems. In this work we propose the *Weighted Optimization Framework* (WOF) which is based on variable grouping and optimizing weight variables for each group. We show that the WOF can significantly enhance the performance of population-based metaheuristic algorithms for many-variable multi-objective optimization problems.

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2. PROPOSED METHOD

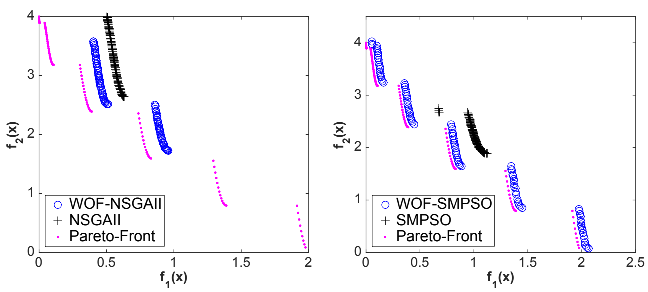
In contrast to the CC-based studies mentioned above, our proposed method is extending the concept of weighting the variables from [9] to the case of multiple objectives. In WOF, instead of optimizing the decision vector $\vec{x}$, for any fixed real values of $\vec{x}$ we optimize a smaller vector $\vec{w}$ ($|\vec{w}| \leq |\vec{x}|$) to approximate an optimal solution. For a fixed solution $\vec{x}'$, the variables are grouped together and a weight w_i is applied to each group. This process can be seen in Eq. 1.

$$\vec{f}(\vec{w} \odot \vec{x}) = \vec{f}(\underbrace{w_1 x_1, ..., w_1 x_l}_{w_1}, ..., \underbrace{w_k x_{n-l+1}, ..., w_k x_n}_{w_k}) \quad (1)$$

The correct choice of $\vec{x}'$ is crucial for the success of the optimization, as well as an appropriate grouping scheme. In WOF, we alternate two different phases of optimization. (1) The original problem is optimized with an arbitrary algorithm for a fixed number of function evaluations. (2) A number of q different solutions $\vec{x}'_i$ ($i = 1, 2, \ldots, q$) are drawn from the current population. For every $\vec{x}'_i$, the problem is optimized by using the reduced vector $\vec{w}$ as the new decision variables. The alternate optimization of variables and weights will take place during the first 50% of the total function evaluations, after which a normal optimization process is resumed. Different grouping strategies might be applied in WOF. This work uses a rather naive approach which groups variables together based on their absolute values. The selection of the used $\vec{x}'_i$ is based on a diversity indicator.

Table 1: Average hypervolume values and standard errors (50 runs) for 3-objective experiments.

	WOF-NSGAII	NSGAII	WOF-SMPSO	SMPSO
WFG1	**0.3462*** (± 1.39e-03)	0.0706 (± 4.44e-04)	**0.3808*** (± 1.18e-03)	0.3486 (± 4.87e-04)
WFG2	**0.5259*** (± 6.88e-03)	0.3619 (± 4.94e-03)	**0.8228*** (± 2.10e-03)	0.4433 (± 3.40e-03)
WFG3	**0.4304*** (± 1.43e-03)	0.2619 (± 8.19e-04)	**0.5020*** (± 1.20e-03)	0.2791 (± 7.37e-04)
WFG4	**0.3721*** (± 4.26e-03)	0.1283 (± 4.09e-04)	**0.4782*** (± 2.42e-03)	0.2109 (± 7.26e-04)
WFG5	**0.2497*** (± 3.09e-03)	0.1052 (± 3.58e-04)	**0.3613*** (± 3.31e-03)	0.1593 (± 5.44e-04)
WFG6	**0.3529*** (± 1.10e-02)	0.1200 (± 5.53e-04)	**0.5779*** (± 8.41e-04)	0.4391 (± 6.43e-04)
WFG7	**0.3743*** (± 2.51e-03)	0.1530 (± 4.62e-04)	**0.4489*** (± 1.89e-03)	0.2143 (± 5.20e-04)
WFG8	**0.2716*** (± 2.80e-03)	0.1480 (± 3.72e-04)	**0.4353*** (± 6.40e-03)	0.1928 (± 1.62e-03)
WFG9	**0.3426*** (± 7.58e-03)	0.1747 (± 8.50e-04)	**0.4754*** (± 3.54e-03)	0.3724 (± 3.53e-03)



(a) (WOF-) NSGA-II (b) (WOF-) SMPSO

Figure 1: WFG2 problem with 1000 variables.

3. EXPERIMENTS

Due to page limitations, we report the experiments without going into details about parameter settings. As WOF is intended to work well for large-scale problems, we examine its ability for 1000-variable instances of the WFG test problems 1 - 9 [3] (2 and 3 objectives). We use the NSGA-II [2] and SMPSO [7] algorithms and compare each of them with their WOF-enhanced version respectively. Average hypervolume values and standard errors (3 objectives) are given in Table 1. Bold indicates superior performance, underlined indicates overall best performance. An asterisk indicates statistical significance ($p < 0.001$). We can observe that both WOF algorithms significantly outperformed the original algorithms. Both convergence and diversity of the obtained solution were improved significantly. The WOF-SMPSO algorithm performed best among all four methods. The same results were obtained for the 2-objective problems as exemplary shown in Figs. 1 and 2 (Runs shown are closest to the median hypervolume). The same superior performance of the the WOF algorithms was observed for the IGD indicator.

4. ACKNOWLEDGMENTS

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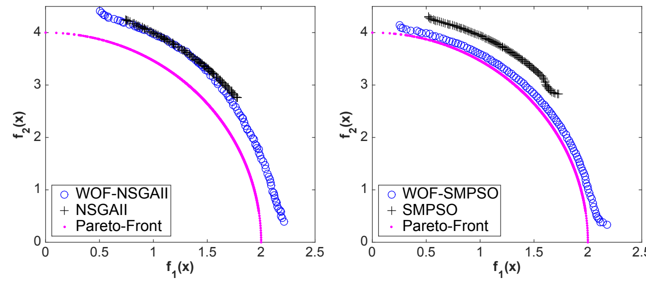

(a) (WOF-) NSGA-II (b) (WOF-) SMPSO

Figure 2: WFG7 problem with 1000 variables.

5. REFERENCES

[1] L. M. Antonio and C. A. Coello Coello. Use of Cooperative Coevolution for Solving Large Scale Multiobjective Optimization Problems. In *2013 IEEE Congress on Evolutionary Computation*, pages 2758–2765, 2013.

[2] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan. A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation*, 6(2):182–197, 2002.

[3] S. Huband, P. Hingston, L. Barone, and L. While. A Review of Multiobjective Test Problems and a Scalable Test Problem Toolkit. *Evolutionary Computation, IEEE Transactions on*, 10(5):477–506, 2006.

[4] A. W. Iorio and X. Li. A Cooperative Coevolutionary Multiobjective Algorithm Using Non-dominated Sorting. In *Genetic and Evolutionary Computation–GECCO 2004*, pages 537–548. Springer, 2004.

[5] X. Li and X. Yao. Tackling High Dimensional Nonseparable Optimization Problems By Cooperatively Coevolving Particle Swarms. In *2009 IEEE Congress on Evolutionary Computation*, pages 1546–1553. IEEE, may 2009.

[6] S. Mahdavi, M. E. Shiri, and S. Rahnamayan. Metaheuristics in large-scale global continues optimization: A survey. *Information Sciences*, 295:407–428, 2015.

[7] A. J. Nebro, J. J. Durillo, J. Garcia-Nieto, C. A. Coello Coello, F. Luna, and E. Alba. SMPSO: A New PSO-based Metaheuristic for Multi-objective Optimization. In *Computational intelligence in miulti-criteria decision-making, 2009. Mcdm '09. IEEE symposium on*, pages 66–73. IEEE, 2009.

[8] Z. Yang, K. Tang, and X. Yao. Differential Evolution for High-Dimensional Function Optimization. In *2007 IEEE Congress on Evolutionary Computation, CEC 2007*, pages 3523–3530, 2007.

[9] Z. Yang, K. Tang, and X. Yao. Large scale evolutionary optimization using cooperative coevolution. *Information Sciences*, 178(15):2985–2999, 2008.

[10] E. Zitzler, K. Deb, and L. Thiele. Comparison of Multiobjective Evolutionary Algorithms: Empirical Results. *Evol. Comput.*, 8(2):173–195, 2000.