Evolutionary Computation and Cryptology

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Abstract

- Evolutionary Computation (EC) has been used with great success on various real-world problems. One domain abundant with difficult problems is cryptology.
- This tutorial will first give a brief introduction to cryptology intended for general audience.
- We concentrate on several topics from cryptography that are successfully tackled up to now with EC and discuss why those topics are suitable to apply EC. However, care must be taken since there exists a number of problems that seem to be impossible to solve with EC and one needs to realize the limitations of the heuristics.
- We discuss the choice of appropriate EC techniques for various problems and evaluate on the importance of that choice. Furthermore, we will discuss the gap between the cryptographic community and EC community and what does that mean for the results.
- To conclude, we present a number of topics we consider to be a strong research choice that can have a real-world impact. In that part, we give a special attention to cryptographic problems where cryptographic community successfully applied EC, but where those problems remained out of the focus of EC community. This tutorial will also present some live demos of EC in action when dealing with cryptographic problems.

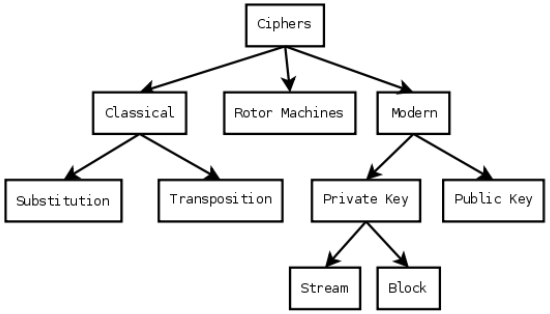
Contents

Introduction to Cryptology

Introduction to Cryptology

- **Cryptology** (from Greek words kryptos which means hidden and logos which means word) is the scientific study of cryptography and cryptanalysis.
- We can trace the origins of cryptology in an art form to the ancient Egypt.
- **Cryptography** is a science (and art) of secret writing with the goal of hiding the meaning of a message. In modern cryptography, it is not only important to achieve confidentiality, but also authentication, non-repudiation and data integrity among other goals.
- **Cryptanalysis** is a science of analyzing ciphers in order to find weaknesses in them.

Introduction to Cryptography



Taxonomy of ciphers

Classical Ciphers

- Transposition ciphers are such ciphers where the order of characters is shuffled around.
- Substitution ciphers are ciphers where each character in the alphabet is substituted with another character in the alphabet.
- Enigma machine is a mechanical rotor device that is comprised from several rotors that dynamically substitute the plaintext in accordance to the rotor position.
- Today, easy to cryptanalyze.
- We do not consider them here, but we give several references.
- Scytale, Caesar cipher, non-standard hieroglyphs, etc.

Modern Ciphers

- In 1940s Shannon published his paper on the design principles of block ciphers.
- Important milestones happened in 1970s.
- The design of the DES cipher, the introduction of public key cryptography.
- Modern cryptography has much more emphasize on definitions and proofs, although there are many primitives used today that do not have rigorous proofs.
- Informally, we distinguish classical from the modern cryptography on a basis that modern cryptography has a more scientific approach.

Basic notions

- Sender is a person who is sending a message. The most famous sender in cryptography is Alice.
- Receiver is a person who is receiving a message. The most famous message receiver in cryptography is Bob.
- Adversary is a malicious entity whose aim is to prevent the users of a cryptosystem from achieving their goals. Popular names are Eve in the case of passive adversaries and Mallory when talking about active adversaries.
- Cryptographic primitive is a part of a cryptographic tool used to provide information security, i.e., a low-level cryptographic algorithm that is frequently used.
- Cryptographic algorithm (cipher) is a mathematical function used for encryption, decryption, key establishment, authentication, etc.

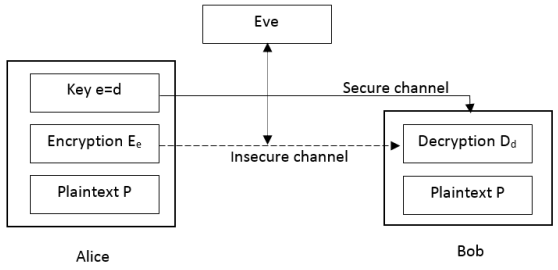
Basic notions

- Plaintext P or message is the information that the sender wishes to transmit to the receiver.
- Ciphertext C is the result of an encryption performed on plaintext using a cryptographic algorithm.
- Encryption is a process of applying a transformation E to the plaintext P. After that transformation, only an authorized party should be able to read the message, i.e., $E(P) = C$.
- Decryption is a process of applying a transformation D to the ciphertext C, i.e., $D(C) = P$.

Symmetric-key Cryptography

- Also known as private key cryptography.
- Symmetric-key cryptography uses the same key to encrypt/decrypt or to compute/verify the data.
- Assume that Alice and Bob want to exchange some message and they want it to remain secret, i.e., that no one else can read it.
- They have only an insecure channel to communicate through. Alice could encrypt her message and send it encrypted over an insecure channel to Bob. If Bob has the same key as Alice, he can then decrypt and read the message.
- Eve cannot decrypt the message if she does not know the key.

Symmetric-key Cryptography



Secure two party communication

Block Ciphers

- Block ciphers operate on blocks of fixed length of data with an unvarying transformation that is specified by the key.
- A block cipher with a given key should be indistinguishable from a random permutation by an adversary not knowing the key.
- Claude Shannon stated that computationally secure cryptosystem should follow confusion and diffusion principles.
- Confusion – the ciphertext statistics should depend on the plaintext statistics in a manner too complicated to be exploited by the cryptanalyst.
- Diffusion - each digit of the plaintext and each digit of the secret key should influence many digits of the ciphertext.
- DES, AES, MARS, PRESENT, etc.

Stream Ciphers

- They should behave as pseudorandom number generators (PRNGs).
- Most of the stream encryption schemes encrypt message bits by adding encryption bits modulo two.
- Historically looking, linear feedback shift registers (LFSRs) were used extensively, in order to produce pseudorandom numbers.
- An LFSR is a shift register whose input bit is a linear function of its previous state. Those bit positions that affect the next state are called taps.
- To add the nonlinearity (and therefore improve the security) one option is to add some nonlinear element, where a Boolean function represents a common choice.

Implementation Attacks

- All attacks that do not aim at the weaknesses of the algorithm itself, but on the actual implementations on cryptographic devices.
- Sources: power, sound, light, electromagnetic radiation, etc.
- Implementation attacks are among the most powerful known attacks against cryptographic devices.
- Common types of implementation attacks are side channel attacks and fault injection attacks.
- Side channel attacks are passive and non-invasive attacks.
- Fault injection attacks are active attacks since they enforce the target to work outside the nominal operation range.

Public Key Cryptography

- In symmetric key cryptography, both parties need to know the key before the communication in order to establish the secure channel.
- However, the problem is how to exchange that key if there exists no secure channel.
- One option is to use public key cryptography.
- Also called asymmetric cryptography.
- Here, there exist two keys: private and public key.
- To encrypt, one uses the public key, but to decrypt one needs to know the private key.

Public Key Cryptography

- Public key cryptography relies on difficult problems in mathematics, like integer factorization, discrete logarithm problem, knapsack problem, etc.
- RSA, Diffie-Hellman, ECC,…
- For public key cryptography, the are only a few papers where authors use evolutionary computation and the results are not spectacular.
- However, this is to be expected: it is much more difficult to design some cryptographic primitive here or to attack a system with evolutionary computation.

On the Evolutionary Computation

On the Evolutionary Computation

- Research area within computer science that draws inspiration from the process of natural evolution.
- Evolutionary algorithms are population based metaheuristic optimization methods that use biology inspired mechanisms like selection, crossover or survival of the fittest.
- Genetic Algorithm (GA), Holland, 1975.
- Tree based Genetic Programming (GP), Koza, 1992.
- Cartesian Genetic Programming (CGP), Miller, 1999.
- Evolution Strategy (ES), Rechenberg, Schwefel, 1970s.
- NSGA-II, Deb, 2002.

Examples of Applications

Basics

- How to solve hard problems in cryptology?
- Problems need to be hard (to be worthwhile), but not too difficult (to be impossible to solve).
- Plenitude of problems and possible methods to solve them.
- Care needs to be taken that one does not select too difficult problems.
- Often, evolutionary computation is not used to provide the final solutions, but instead to help us to improve the results of some other technique.

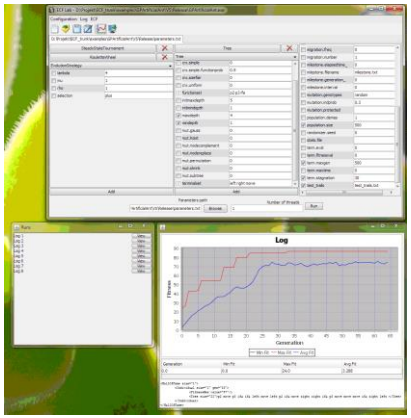
Evolutionary Computation Framework

- ECF is a C++ framework intended for application of any type of evolutionary computation.
- Developed by Evolutionary Computation group from Faculty of Electrical Engineering and Computing, Zagreb, Croatia:

  http://gp.zemris.fer.hr/
- Details about projects concerning evolutionary computation and cryptology:

  http://evocrypt.zemris.fer.hr/

Evolutionary Computation Framework



ECF GUI

Evolutionary Computation Framework

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Available algorithms

Boolean functions

- The easiest problem to start.
- There exists a natural mapping between the truth table representation of Boolean functions and representation of solutions in EC.
- Boolean functions are important cryptographic primitive and often used in stream ciphers as the source of nonlinearity.

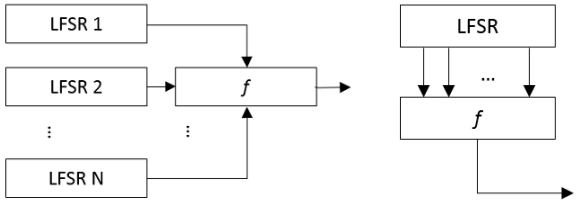
Truth table input		Truth table output
x_1	x_0	y
0	0	0
0	1	1
1	0	1
1	1	0

Boolean function with 2 inputs

Boolean functions

- To be used in cryptography, a Boolean function needs to fulfill a number of cryptographic properties.
- To be used in filter generators: balancedness, high nonlinearity, high algebraic degree, high algebraic immunity, high fast algebraic immunity.
- To be used in combiner generators additionally is required a good value of correlation immunity.
- To be used as a part of the side-channel attack countermeasure it needs to have low Hamming weight and high correlation immunity.
- To be of practical importance, it should have at least 13 inputs.
- Three options: algebraic constructions, random search, and heuristics.

Boolean functions



Combiner generator

Filter generator

Boolean functions, scenario 1

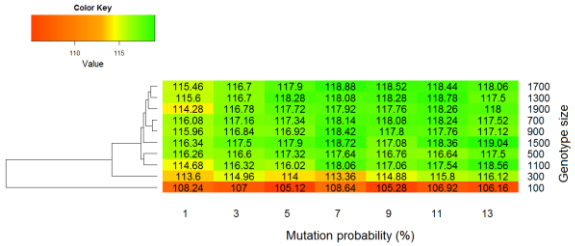
- Evolving Boolean functions that are to be used in combiner/filter generators.
- We are interested in a number of properties, where some of those properties are conflicting.
- Search space size is 2^(2^n).
- Representing solutions in the truth table form requires string of bits of length 2^n.
- Already for a Boolean function with 8 inputs, the search space size is 2^(256).

Boolean functions, scenario 1

- How to write fitness function?
- As a single objective with the weight factors, or a multiple stage fitness function, multi-objective approach or even many-objective approach.
- For Boolean functions up to 8 inputs, most of the EC techniques give good results.
- However, the best results are obtained with GP and CGP.
- Results comparable with algebraic constructions.
- The simplest problems seem to be either:
  - Evolving bent function (those that are not balanced, but with maximum nonlinearity)
  - Evolving balanced functions with high nonlinearity.

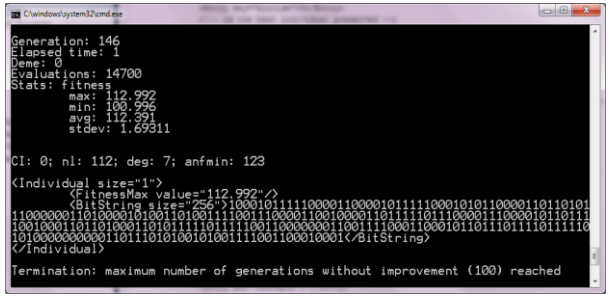
Boolean functions, scenario 1

- It seems that the genotype plays much larger role than the choice of the fitness function.



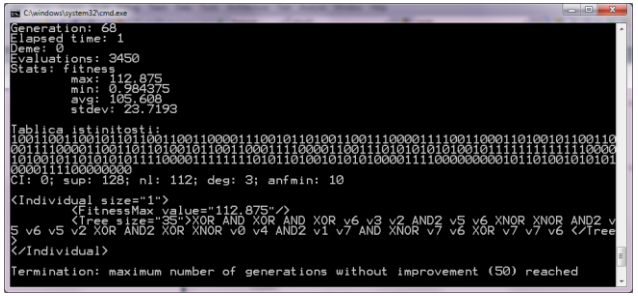
Average values, CGP, bent Boolean functions with 8 inputs

Boolean functions, scenario 1



GA, bitstring representation
Boolean function with 8 inputs

Boolean functions, scenario 1



GP, Boolean function with 8 inputs

Boolean functions, scenario 2

- Here we aim to evolve Boolean functions that have as small as possible Hamming weight and high correlation immunity in order to reduce the masking cost when used as a side-channel countermeasure.
- Masking consists in changing randomly the representation of the key to deceive the attacker.
- Example: if each bit k_i, $1 < i < n$ of a key k is masked with a random bit m_i, then an attacker could probe k_i *XOR* m_i.
- Provided m_i is uniformly distributed, the knowledge of k_i *XOR* m_i does not disclose any information on bit k_i.
- Since most of the algebraic constructions aim to find balanced Boolean functions, they are not appropriate for this problem.

Boolean functions, scenario 2

- Masking can be summarized as the problem of finding Boolean functions whose support is the masks' set, with the two following constraints:
  - small Hamming weight, for implementation reasons, and
  - high correlation immunity t to resist an attacker with multiple ($< t$) probes.
- There is a trade-off which motivates the research for low Hamming weight high correlation immunity Boolean functions.
- Interesting problem since we know the best possible values, but we do not know actual functions reaching those values.

Boolean functions, scenario 2

- Up to recently, there were several values of practical interest unknown.
- Attempts with SAT solvers did not resulted in success even after more than one month of calculation.
- For CGP and GP, this problem seems to be trivial.
- Optimal results sometimes achieved even in less than 1 hour.
- However, there are combinations of parameters as well as function sizes that seem more difficult for EC.

Boolean functions, scenario 2

t \ n	11	12	13	14	15	16
2	16	16	32	64	128	256
3	32	32	32	64	128	256
4	**128**	256	256	256	2048	4096
5	**256**	**256**	512	1024	2048	4096
6	512	**1024**	1024	2048	4096	4096
7	1024	1024	**2048**	4096	8192	8192
8	1024	2048	4096	8192	8192	16384
9	1024	2048	4096	8192	16384	16384
10	1024	2048	4096	8192	16384	32768
11		2048	4096	8192	16384	32768
12			4096	8192	163840	32768
13				8192	16384	32768
14					16384	32768
15						32768

Best obtained results with CGP and GP

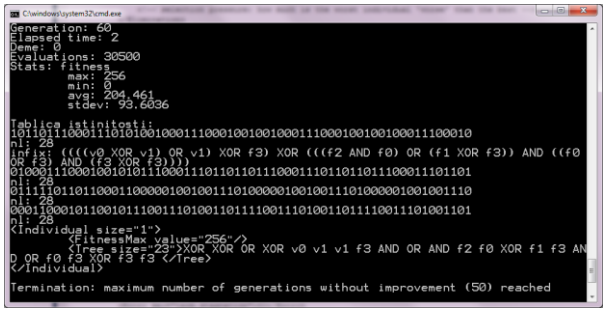
Boolean functions, scenario 3

- Previous results show that EC can be used to evolve Boolean functions of various sizes and properties.
- However, it is to be expected that after some size, the results will become worse and the evaluation process long.
- For instance, if we consider the algebraic immunity and fast algebraic immunity properties. To calculate those two properties can easily take several hours for a Boolean functions with e.g. 16 inputs.
- Therefore, at least for now, those properties were never included in the evaluation process for larger sizes of Boolean functions.
- The problem seems difficult to circumvent since it is actually a problem with the way of calculating the properties.

Boolean functions, scenario 3

- We already discussed there are several techniques how to generate Boolean functions.
- The question is can we combine several techniques.
- For instance, could we use evolutionary computation to evolve algebraic constructions?
- If yes, then we need just to show that our construction results in Boolean functions with good properties and that it holds for any size of Boolean functions.
- We evolve secondary algebraic constructions that result in bent Boolean functions.

Boolean functions, scenario 3



GP secondary construction

Boolean functions

- Possible challenges:
  - Finding balanced Boolean function with 8 inputs that have nonlinearity 118.
  - Use EC to evolve primary algebraic constructions.
  - Evolve Boolean functions to be used in combiner/filter generators where parameters are also algebraic immunity and fast algebraic immunity.
  - Use different, previously not investigated unique representations of Boolean functions.
  - Investigate many-objective optimization.
  - Quaternary Boolean functions.

S-boxes

- Natural extension from the Boolean function case.
- S-boxes (Substitution Boxes) are also called vectorial Boolean functions.
- Often used in block ciphers as a source of nonlinearity.
- However, this problem is much more difficult!
- S-box of dimension *mxn* has *n* output Boolean functions, but for the most of the properties we need to check all linear combinations of those functions.

S-boxes

Truth table input		Truth table output		LUT input	LUT output
x_1	x_0	y_1	y_0	x	y
0	0	0	1	0	1
0	1	1	0	1	2
1	0	1	1	2	3
1	1	0	0	3	0

Truth table input		Linear combinations			Walsh-Hadamard spectrum		
x_1	x_0	y_1	y_0	$y_1 \oplus y_0$	y_1	y_0	$y_1 \oplus y_0$
0	0	0	1	1	0	0	0
0	1	1	0	1	0	-4	0
1	0	1	1	0	0	0	-4
1	1	0	0	0	4	0	0

2x2 S-box example

S-boxes

- For an S-box of dimension nxm there are in total 2^(m*2^n) S-boxes.
- When n = m, some search space sizes of practical interest are:

n	4	5	6	7	8
#	2^{64}	2^{160}	2^{384}	2^{896}	$2^{2\,048}$

- Several options how to represent solutions.
- Again as in the Boolean function case, there are three design options: algebraic constructions, random search, and heuristics.

S-boxes, scenario 1

- When representing S-boxes with their truth tables (i.e., bitstring representation as with Boolean functions) we see the problem is very difficult.
- Indeed, already balancedness property requires that all columns of an S-box are balanced (that is, have the same number of zeros and ones), but also all linear combinations needs to be balanced.
- Still, this approach works for sizes ~4x4 where there are 15 linear combinations we need to consider.
- However, for larger sizes, it is almost impossible to obtain even balanced solution with bitstring representation.
- Therefore, we do not consider such representation anymore.

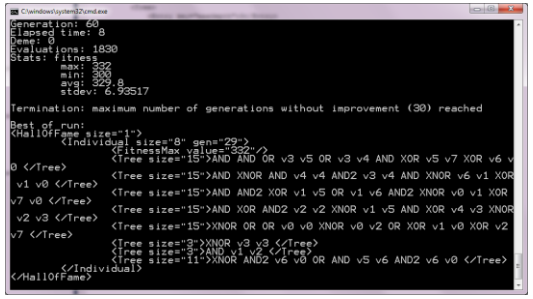
S-boxes, scenario 1

- It is also possible to use CGP and GP with the permutation encoding:

Algorithm 1 Translate to permutation encoding.

Require: i = 0, m = input_size, n = output_size
  for all values $i < 2^m$ **do**
    balanced[i] = i
    for j = n-1; j != 0; j– **do**
      evolved[i] = evolved[i] + truth_table[i][j]*(2^j)
    end for
  end for
  SORT balanced[] array using evolved[] array as key
  for all values $i < 2^m$ **do**
    for j = n-1; j != 0; j– **do**
      truth_table[i][j] = (balanced[i] * 2^j) & 0x01
    end for
  end for

S-boxes, scenario 1



GP solution of 8x8 S-box

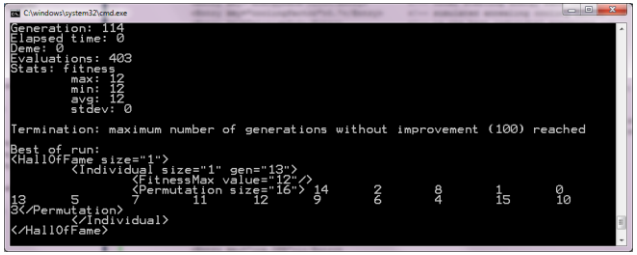
S-boxes, scenario 2

- We can represent S-boxes as permutations, i.e., all values between 0 and 2^n -1 (where n is the dimension of the S-box).
- Then, the S-box is always bijective and we do not need to concern with the balancedness property.
- Similar as with Boolean functions, there are many properties of interest when evolving S-boxes (besides the balancedness): high nonlinearity, low δ-uniformity, high algebraic degree, etc.
- For dimensions up to 4x4, permutation encoding gives optimal results (bijective solutions with maximal nonlinearity and minimal δ-uniformity).
- However, for instance for 8x8, algebraic construction can give nonlinearity of 112 and δ-uniformity of 4.

S-boxes, scenario 2

- Random search will result in nonlinearity up to 98 and nonlinearity down to 10.
- Heuristics, and EC more precisely with permutation encoding can go up to 104 nonlinearity and δ-uniformity of 8.
- The question is then whether there is any sense to use heuristics if such methods cannot compete with algebraic constructions.
- However, it turns out there are properties that algebraic constructions do not consider. For instance, properties related with the side-channel resistance will usually have poor values if S-boxes are constructed with algebraic constructions.
- Therefore, the task is to evolve S-boxes that have good side channel resistance while maintaining other properties optimal.

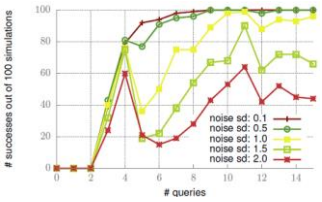
S-boxes, scenario 2



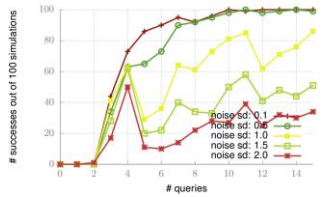
Permutation encoding of 4x4 S-box

S-boxes, scenario 2

- Additional problem is that such properties are conflicting with the nonlinearity property and there must be a trade-off.



PRESENT S-box

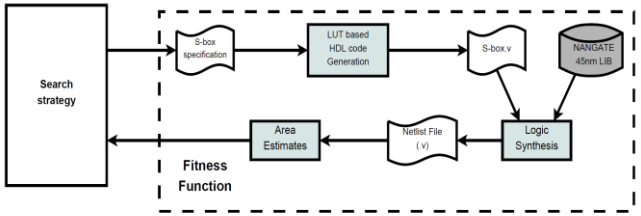


Evolved S-box

S-boxes, scenario 3

- Besides the properties related with the side-channel attacks, we are also interested in implementation properties like power, area, and latency.
- Again, algebraic constructions do not consider such properties but we can evolve S-boxes with good cryptographic properties that are hardware-friendly.
- Naturally, there exist the same problem as before: we do not want that cryptographic properties deteriorate too much.
- In this scenario, we require that our evolution framework can communicate with the framework that does the implementation properties analysis.

S-boxes, scenario 3



Evaluation setup when evolving S-boxes with good implementation properties

S-boxes, scenario 3

- However, as said, EC does not cope good with larger sizes of S-boxes and therefore our previous technique is expensive from the cryptographic perspective.
- To circumvent the problem, we can evolve affine transformation of an S-box.
- Affine transformation will change implementation properties, but leave cryptographic properties intact:

$$S_a(x) = B\big(S_b(A(x)\ XOR\ a)\big)\ XOR\ b.$$

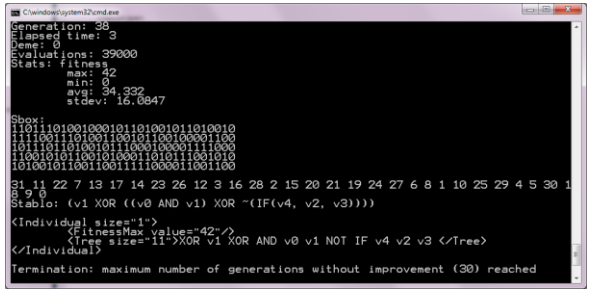
- Here, A and B are invertible nxn matrices in GF(2) and a and b are constants.

S-boxes, scenario 4

- Evolve S-boxes in a form of cellular automata (CA) rules.
- Such representation is also used in practice (Keccak cipher).
- It is possible to find many rules that result in good S-boxes.

S-box size	Min	Max	Th. max	Avg	Std dev	Var
4×4	16	16	**16**	16	0	0
5×5	38	42	**42**	41.73	1.01	0.99
6×6	70	84	**84**	80.47	4.72	21.52
7×7	140	182	**182**	155.07	8.86	75.92
8×8	258	312	**364**	281.87	13.86	185.85

S-boxes, scenario 4

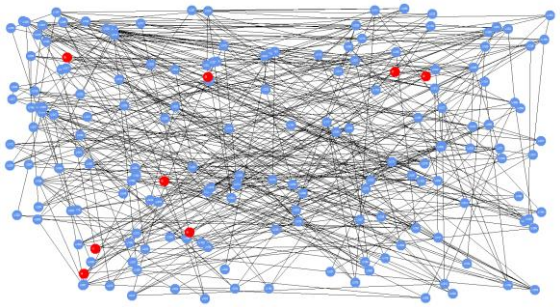


Evolved CA rule for the 5x5 S-box

S-boxes, scenario 5

- Adding flip flops in order to increase the throughput of combinatorial circuit.
- Applications beyond cryptography.
- Depending on the number of elements (cells) the problem can be extremely difficult.
- Current results show we are able to improve the throughput by almost 100%.
- Naturally, this causes the increase of area.

S-boxes, scenario 5



AES S-box realized in the tower field representation

S-boxes

- Possible challenges:
  - Evolve S-box of size 8x8 that has nonlinearity 112.
  - Use new representations of solutions.
  - Improve the efficiency of EC with the bitstring representation.
  - Consider S-box representations in a form of equations.
  - Find general rules for CA and S-boxes.

Addition chains

- Consider modular exponentiation; find the (unique) integer B ∈ [1,…, p-1] that satisfies:

$$B = A^c \bmod p.$$

- Several ways how to calculate this.
- Naïve way, multiply c times.
- Use addition chain.
- An addition chain for the exponent c of length l is a sequence V of positive integers $v_0 = 1, \ldots, v_l = c$, such that for each $i > 1$, $v_i = v_j + v_k$ for some j and k with $0 \leq j \leq k < i$.

Addition chains

- The length of the addition chain defines the number of multiplications required for computing the exponentiation.
- The aim is to find the shortest addition chain for a given exponent c.
- Example:
- Binary method: write 60 in binary: 111100; replace “1” with “DA” and “0” with “D”; cross out the first “DA” on the left; “DADADADD”, calculate:

$$1 \rightarrow 2 \rightarrow 3 \rightarrow 6 \rightarrow 7 \rightarrow 14 \rightarrow 15 \rightarrow 30 \rightarrow 60.$$

- Addition chain (7 operations):

A^1; A^2 = A^1 * A^1; A^4 = A^2 * A^2; A^6 = A^4 * A^2; A^12 = A^6 * A^6;A^24 = A^12 * A^12; A^30 = A^24 * A^6; A^60 = A^30 * A^30.

Addition chains

- The problem of finding the shortest addition chain for a given exponent is of great relevance in cryptography.
- However, the problem is believed to be NP-hard.
- There is no single algorithm that can be used for any exponent.
- Still, the best solutions are obtained by pen and paper method.
- Huge numbers so exhaustive search is impossible.
- Heuristics should be able to help.
- There exist many types of chains. Here we are interested in ascending addition chains.

Addition chains

- The values in the ascending addition chain have the property that they are the sum of two values appearing previously in the chain.
- Types of steps in the addition chain:
  - Doubling step; when $j = k = i - 1$. This step always gives the maximal possible value at the position i.
  - Star step; when j but not necessarily k equals $i - 1$.
  - Small step; when $log_2(a_i) = log_2(a_{i-1})$.
  - Standard step; when $a_i = a_j + a_k$ where $i > j > k$.
- A star chain is a chain that involves only star operations.

Addition chains

- For this problem we need custom crossover and mutation operator, as well as the repair mechanism.

Algorithm 1 Crossover operator.

Require: Exponent $exp > 0$, *Parent addition chains* P_1, P_2
  $rand = random(3, exp - 1)$
  for all i such that $0 \leq i \leq rand$ **do**
    $e_i = P_{1_i}$
  end for
  for all i such that $rand \leq i + 1 \leq n$ **do**
    $FindLowestPair(P_2, i, pair_1, pair_2)$
    $e_i = e_{pair_1} + e_{pair_2}$
  end for
  $RepairChain(e, exp)$
  return $e = e_0, e_1, ..., e_n$

Addition chains

Algorithm 2 Mutation operator.

Require: Exponent $exp > 0, e = e_0, e_1, ..., e_n$
  $rand = random(2, exp - 1)$
  $rand_2 = random(0, 1)$
  if $rand_2$ **then**
    $e_{rand} = e_{rand-1} + e_{rand-2}$
  else
    $rand_3 = random(2, rand - 1)$
    $e_{rand} = e_{rand-1} + e_{rand_3}$
  end if
  $RepairChain(e, exp)$
  return $e = e_0, e_1, ..., e_n$

Addition chains

r	c(r)	Binary	Optimized window	GA Min	GA Avg	GA Stdev
26	1 176 431	33	31	26	26.18	1.27
27	2 211 837	36	32	27	27.18	1.68
28	4 169 527	37	32	28	28.18	0.38
29	7 624 319	36	33	29	30.16	0.71
30	14 143 037	38	34	30	30.92	0.6
31	25 450 463	38	35	31	32.62	0.66
32	46 444 543	42	36	32	33.5	0.54
33	89 209 343	42	38	33	34.46	0.81
34	155 691 199	42	39	34	35.44	1.03
35	298 695 487	46	41	35	35.67	0.74
36	550 040 063	45	41	36	37.96	0.83
37	994 660 991	46	42	37	38.76	1.47
38	1 886 023 151	48	42	38	40.28	1.21
39	3 502 562 143	48	43	39	41.36	1.19
40	6 490 123 999	52	45	41	41.77	0.63

GA results, part 1

Addition chains

Exponent	$log_2(n)$	$\nu(n)$	Binary	Window	Optimized win.	GA Min	GA Avg	GA Stdev
$2^{37}-3$	36	35	71	57	51	43	45.32	0.99
$2^{47}-3$	46	45	91	69	63	54	56.25	1.11
$2^{57}-3$	56	55	111	82	76	64	64.9	0.87
$2^{67}-3$	66	65	131	94	88	73	73.2	0.43
$2^{77}-3$	76	75	151	107	101	85	85.4	0.51
$2^{87}-3$	86	85	171	119	113	97	104.3	3.56
$2^{97}-3$	96	95	191	132	126	106	107.2	0.91
$2^{107}-3$	106	105	211	144	138	115	115.71	0.75
$2^{117}-3$	116	115	231	157	151	126	126.6	0.89
$2^{127}-3$	126	125	251	169	163	136	136.8	0.83

GA results, part 2

Addition chains

- For most of the values we find the optimal one (or what is the current best).
- Out of all tested numbers, only *2^127 - 3* has practical importance.
- We find chain of 136 steps, also done by expert by hand.
- Human-competitive?
- We believe so, on average we need 10 mins, pen and paper requires a lot of experience and will last at least 1 hour.
- Plenty of numbers to investigate.
- Some more realistic numbers are *2^255 - 21* and *2^252 + 27742317777372353535851937790883648491*.

Addition chains

```
        avg: 15.0167
        stdev: 1.57628

Generation: 52
Elapsed time: 1
Deme: 0
Evaluations: 15900
Stats: fitness
        max: 21
        min: 14
        avg: 14.99
        stdev: 1.55498

0/0 0/1 2/2 3/3 3/4 5/5 5/6 7/7 8/8 9/9 9/10 10/11 3/12 0/13
<Individual size="1">
        <FitnessMin value="14"/>
        <Chain size="15">1 2 3 6 12 18 36 54 108 216 432 648 1080 1086 1087 </Chain>
</Individual>

Termination: maximum number of generations without improvement (50) reached

Best of run:
<HallOfFame size="1">
        <Individual size="1" gen="1">
                <FitnessMin value="14"/>
                <Chain size="15">1 2 3 6 12 18 36 54 108 216 432 648 1080 1086 1087 </Chain>
        </Individual>
</HallOfFame>
```

Evolved addition chain for 1087 value

Addition chains

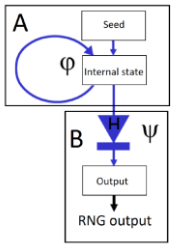
- Possible challenges:
  - Improve the speed of the algorithm.
  - Look for optimal chains for even larger numbers.
  - Differentiate between multiplication and squaring steps.
  - Analyze the structure of numbers with regards to the EC performance.
  - Support special structures of numbers.
  - Explore different types of chains.

Pseudorandom number generators

- In cryptography, random number generators (RNGs) play an important role.
- Most of the time, we need true random number generators (TRNGs), but still there are applications where pseudorandom number generators (PRNGs) are enough.
- TRNG is a device for which the output values depend on some unpredictable source that produces entropy.
- PRNGs represent mechanisms that produce random numbers by performing a deterministic algorithm on a randomly selected seed
- One example is masking for the side channel resistance.

Pseudorandom number generators

- We want to find extremely fast and small PRNGs that pass all NIST tests.
- We can use GP and CGP to evolve PRNGs.



Model of a PRNG

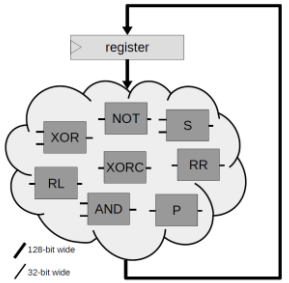
Pseudorandom number generators

- We evolve PRNGs that have *n* inputs and 1 output (GP) or *m* outputs (CGP).
- All variables are 32-bit integer values.
- Function set are function that are fast and small when implemented in hardware (shift, rotate, permute, and logical operations XOR, NOT, AND).
- Here, obvious advantage of CGP over GP is that GP needs to iterate *m* times to produce the same size of the output as CGP produces in a single iteration.

Pseudorandom number generators

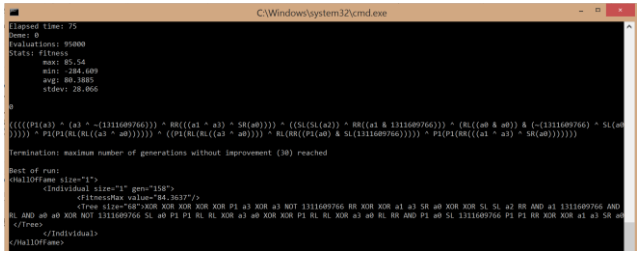
- Fitness function needs to be simple, yet powerful enough to drive our search.
- We use approximate entropy test from the NIST statistical test suite as a fitness function.
- After the evolution process is over, our parser automatically takes the best individual and outputs it as a C source code.
- That source code is then used to produce 10 million bits that are then evaluated with the NIST statistical suite.
- We cannot use whole test suite in the evolution since it would be too slow.
- Our current fitness function consists of 130 evaluations of the approximate entropy function.

Pseudorandom number generators



Structure of evolved PRNGs

Pseudorandom number generators



Example of evolved PRNG

Pseudorandom number generators

```
uint GP(uint a0, uint a1, uint a2, uint a3) {
    uint x0 = (const >> 1) | (const << 31);
    uint x1 = a2 & a3;
    uint x2 = x1 ^ x0;
    uint x3 = p1(a3);
    uint x4 = p1(x3);
    uint x5 = x4 ^ x2;
    uint x6 = (a0 << 1) | (a0 >> 31);
    uint x7 = p1(x6);
    uint x8 = (x7 << 1) | (x7 >> 31);
    uint x9 = x8 ^ x5;
    uint x10 = (a0 >> 1) | (a0 << 31);
    uint x11 = a0 ^ a3;
    uint x12 = x11 ^ x10;
    uint x13 = p1(x12);
    uint x14 = a3 & a1;
    uint x15 = (x14 >> 1) | (x14 << 31);
    uint x16 = (a0 >> 1) | (a0 << 31);
    uint x17 = a3 ^ x16;
    uint x18 = x17 ^ x15;
    uint x19 = x18 ^ x13;
    uint x20 = x19 ^ x9;
    return x20;
}
```

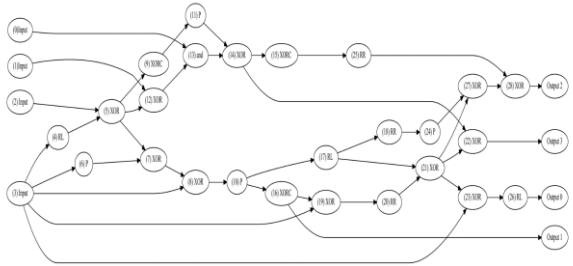
GP evolved PRNG

Pseudorandom number generators

```
void CGP(uint x0,uint x1,uint x2,uint x3,
uint* z0,uint* z1,uint* z2,uint* z3) {
    uint y4 = x0 & x1;
    uint y5 = x2 ^ x3;
    uint y6 = (y5 >> 1) | (y5 << 31);
    uint y7 = p1(y6);
    uint y8 = x3 ^ y7;
    uint y9 = p1(y8);
    uint y10 = y6 ^ y9;
    uint y11 = (y9 << 1) | (y9 >> 31);
    uint y12 = const;
    uint y13 = p1(y10);
    uint y14 = y12 ^ y11;
    uint y15 = y12 ^ y13;
    uint y16 = (y15 >> 1) | (y15 << 31);
    uint y17 = y10 ^ y16;
    uint y18 = p1(y17);
    uint y19 = y18 >> 1;
    uint y20 = y18 ^ y4;
    uint y21 = p1(y20);
    uint y22 = y18 ^ y21;
    uint y23 = p1(y18);
    uint y24 = y19 ^ y18;
    uint y25 = y23 ^ y19;
    uint y26 = y22 ^ y14;
    *z0 = y18;
    *z1 = y25;
    *z2 = y26;
    *z3 = y24;
}
```

CGP evolved PRNG

Pseudorandom number generators

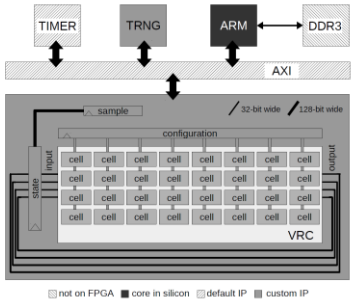


CGP solution

Pseudorandom number generators

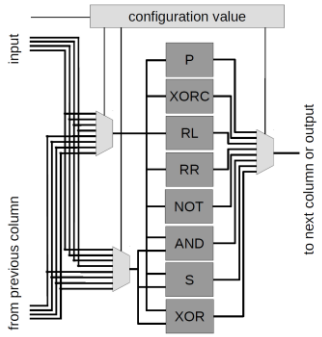
- The same technique can be used to produce PRNGs on-the-fly.
- Then, we can use evolvable hardware that constantly updates the PRNG part.
- In order to ensure that our designs always use all terminals, we penalize solutions that do not have all inputs.
- Maximal throughput on ASIC 117 Gb/s and for FPGA 66 Gb/s.
- Here, GP and CGP are used to evolve only the update functions, but EC can be also used to evolve the non-invertible function.

Pseudorandom number generators



Evolvable hardware setting

Pseudorandom number generators



Virtual reconfigurable circuit cell

Pseudorandom number generators

- Possible challenges:
  - Improve the fitness function and consequently the evaluation process.
  - Add to the fitness function also consideration about the size and speed of specific functions (platform dependent).
  - Experiment with different sizes of the update function as well as different terminal sets.
  - Improve the efficiency of the evolvable hardware scenario.

Fault injection

- A fault injection (FI) attack is successful if after exposing the device to a specially crafted external interference, it shows an unexpected behavior exploitable by the attacker.
- Insertion of signals has to be precisely tuned for the fault injection to succeed.
- Finding the correct parameters for a successful FI can be considered as a search problem where one aims to find, within a minimum time, the parameter configurations which result in a successful fault injection.
- The search space is typically too large to perform an exhaustive search.
- Use heuristics to find search space parameters that lead to successful attack.

Fault injection

- Voltage switching, three parameters: glitch length, glitch voltage, and glitch offset.
- Two scenarios:
  - Finding faults in a minimal number of measurements.
  - Characterizing the parameter space, again in a minimal number of measurements.
- FI testing equipment can output only verdict classes that correspond to successful measurements.
- Attacking the PIN mechanism.

Fault injection

```
for (i = 0; i < 4; i++)
{
    if (pin[i] == input[i])
        ok_digits++;
}
if (ok_digits == 4)  //LOCATION FOR ATTACK
    respond_code(0x00, SW_NO_ERROR_msb, SW_NO_ERROR_lsb);
else
    respond_code(0x00, 0x69, 0x85);
```

PIN authentication mechanism

Fault injection

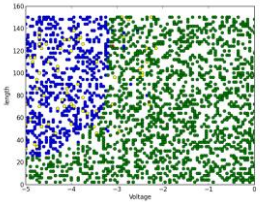
- Several possible classes for classifying a single measurement:
  - NORMAL: smart card behaves as expected and the glitch is ignored
  - RESET: smart card resets as a result of the glitch
  - MUTE: smart card stops all communication as a result of the glitch
  - INCONCLUSIVE: smart card responds in a way that cannot be classified in any other class
  - SUCCESS: smart card response is a specific, predetermined value that does not happen under normal operation

Fault injection

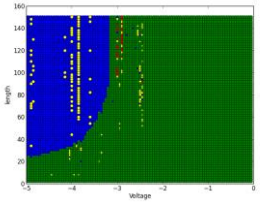
Input: crx_count = 0, mut_count = 0
1: **repeat**
2: select first parent
3: **if** first parent of SUCCESS class **then**
4: try to find matching second parent
5: **else**
6: try to find second parent of different class
7: **end if**
8: perform crossover (depending on parent classes)
9: copy child to new generation
10: crx_count = crx_count + 1
11: **until** $p_c * N <$ crx_count
12: **repeat**
13: select random individuals for tournament
14: copy best of tournament to new generation
15: mut_count = mut_count + 1
16: **until** $(1 - p_c) * N <$ mut_count
17: perform mutation on new generation with probability p_m
18: evaluate population

Custom GA

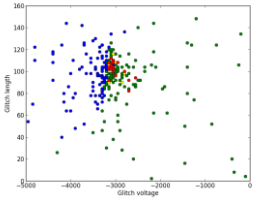
Fault injection



Random, 2500 measurements

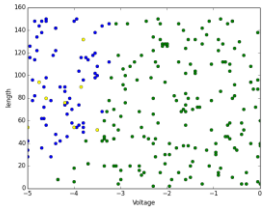


Exhaustive, 7500 measurements

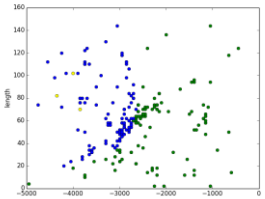


GA + LS, 250 measurements

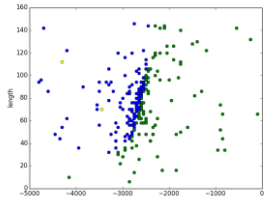
Fault injection



Random, 250 measurements



GA, 250 measurements



GA + LS, 250 measurements

Fault injection

- Possible challenges:
  - Working with more relevant parameters.
  - Attacking cards with countermeasures.
  - Switching to other sources of attacks.
  - Making the search algorithm more powerful.

Final Remarks, Perspectives and Conclusions

Final Remarks

- All the examples presented here are available from SVN repository:

  http://evocrypt.zemris.fer.hr/

- In all the experiments we use Evolutionary Computation Framework (ECF) that can be downloaded from:

  http://ecf.zemris.fer.hr/

- For updated version of slides as well as for the further references, please check:

  http://www.evocrypt.com/

Perspectives

- Stepping outside of the cryptology area and considering security area there are many more interesting problems:
  - Malware detection.
  - Intrusion detection.
  - Automatic code improvement.
  - Spam detection.
  - Etc.
- We also need to step outside the EC area and consider other heuristic techniques.

Perspectives

- We presented here only a handful of applications, there are many more options.
- Even for each of the applications, there is a plethora of options still to try:
  - New algorithms.
  - Representations.
  - Fitness functions.
  - Combinations of parameters.
  - Etc.
- The results obtained up to now are good, but there is still much room for improvement.

Conclusions

- Up to now, EC proved to be successful in cryptology:
  - When there exist no other, specialized approaches.
  - To rapidly check whether some concept (e.g. formula) is correct.
  - To assess the quality of some other method.
  - To produce ``good-enough" solutions.
  - To produce novel and human-competitive solutions (solutions produced by EC that can rival the best solutions created by humans).

Conclusions

- Heuristic methods are not a magic solvers.
- They require knowledge and experience if to be used correctly.
- Nice problems, both from the practical perspective, but also as benchmarks.
- If there are others, specialized algorithms, EC rarely can beat them.
- Besides EC, there are other techniques that also proved to be efficient.
- Currently, there is a large interest in machine learning methods in cryptography.
- Necessary collaboration between the optimization and cryptology community.

Conclusions

- Without proper collaboration, for optimization community cryptology problems are just something to be solved, but without consideration on the constraints and the quality of the obtained solutions.
- For cryptographic community, EC techniques are just a tool that are used without understanding.
- Without good understanding the problem and the tool to be used, it is hard to expect nice results.
- Thank you for your attention.

Questions?

Instructor

- Stjepan Picek is currently a postdoctoral researcher in the Computer Security and Industrial Cryptography (COSIC) group at KU Leuven, Belgium.
- In July 2015, he completed his PhD at Radboud University Nijmegen, The Netherlands and Faculty of Electrical Engineering and Computing, Zagreb, Croatia (double doctorate).
- His primary research interests are cryptography, evolutionary computation, and machine learning.

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