Community Structure Detection for the Functional Connectivity Networks of the Brain

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Abstract. The community structure detection problem in weighted networks is tackled with a new approach based on game theory and extremal optimization, called Weighted Nash Extremal Optimization. This method approximates the Nash equilibria of a game in which nodes, as players, chose their community by maximizing their payoffs. After performing numerical experiments on synthetic networks, the new method is used to analyze functional connectivity networks of the brain in order to reveal possible connections between different brain regions. Results show that the proposed approach may be used to find biomedically relevant knowledge about brain functionality.

Keywords: Community structure · Weighted networks · Game theory · Brain functional connectivity networks

1 Introduction

During the last years, more and more computational methods for community structure detection focus on dealing with very large datasets [8], while small sets with more challenging structures are often ignored. However, many applications require 'sensible' algorithms that can reveal the inner structure in networks for which the architecture is not obvious and for which there is no available information about the real structure. An example of such networks are the functional connectivity networks of the brain, usually constructed from raw fMRI data. A growing interest in brain research is reflected by recent American and European large scale research projects that are dedicated to study the brain and its disorders[1]. As the expected impact of these projects may be compared to that of the

[1] Such projects include the BRAIN Initiative (http://www.braininitiative.nih.gov/, April, 2016) and the European Human Brain Project (https://www.humanbrainproject.eu/, April, 2016).

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celebrated Human Genome Project, we anticipate an increased need for methods that allow exploratory analysis and predictions based on datasets describing the dynamics of the brain, such as fMRI data. As different parts of the brain work in collaboration, community detection has a high potential to reveal relevant information about brain functionality which may contribute to better understanding the mechanisms of the brain and brain disorders.

In this context we are proposing exploring such networks with a new game theoretic tool that uses the concept of Nash equilibria within an extremal optimization algorithm to identify possible communities. We show that this method is capable to identify inner network connections that are not grasped by other methods.

2 Weighted Nash Extremal Optimization

The community structure detection problem consists in finding groups of nodes in a network that are more linked to each other than to the rest of the network [4]. In spite of the fact that there are many computational approaches to this problem, there still does not exist a formal definition for the community structure that is universally accepted to encompass the simple description from above. In this paper we explore the use of the Nash equilibrium concept from game theory as a possible characterization for the community structure with non-overlapping nodes in weighted, undirected, networks.

2.1 The Community Structure Detection Game

Consider a weighted graph $G = (V, E)$ where V is the set of nodes, $V = \{i\}_{i=\overline{1,n}}$, and E the set of edges. Let $W = \{w_{ij}\}_{i,j \in V}$ be the set of weights w_{ij} associated to each edge $e_{ij} = (i, j)$ from E. In this work we will consider positive weights.

Let game $\Gamma = (N, S, U)$ be composed of:

- the set of players $N = V$, i.e. each node is the network G is a player in game Γ;
- the set of strategy profiles $S = S_1 \times S_2 \times \ldots \times S_n$, where $\times$ represents the cartesian product, and S_i is the set of strategies of player i. In Γ, S_i represents communities in G, i.e. each node has to chose a community; an element $s \in S$ is called a strategy profile having the form $s = (s_1, s_2, \ldots, s_n)$, where s_i represents the community chosen by player i.
- the payoff functions $U = \{u_i\}_{i \in N}$, where $u_i : S \to \mathbb{R}$, computed as the contribution of a node to its community [16]:

$$u_i(s_1, s_2, \ldots, s_n) = f(s_i) - f(s_i \backslash \{i\}), \tag{1}$$

where

$$f(C) = \frac{\sum_{i,j \in C} w_{ij}}{\sum_{i \in C, j \in V} w_{ij}} \tag{2}$$

is the fitness of community C. Thus, the payoff of a node i depends on its strategy, as well as the strategies of the other nodes that have chosen the same community as i, and the nodes that did not choose the community of i.

A strategy profile s^* is a Nash Equilibrium if $u_i(s_i, s^*_{-i}) \leq u_i(s^*_i)$, $\forall i \in N$ and $\forall s_i \in S_i$, where $(s_i, s^*_{-i}) = (s^*_1, \dots, s_i, \dots, s^*_n)$ is the strategy profile in which all players chose their strategies from s^*, except player i that chooses s_i. A Nash equilibrium (NE) of game Γ is a partition over the set of nodes $N = V$ such that no node can increase its payoff by unilateral deviation. We can consider this also as an alternate definition for the community structure of a network; to test this hypothesis we use numerical experiments performed on benchmarks with known community structures.

NEs of a game can be computed with heuristic methods by using the Nash ascendancy relation between strategy profiles [9] that counts the number $t_N(s,q)$ of players that can improve their payoffs by unilateral deviation from one strategy profile s to q:

$$t_N(s,q) = card\{i \in N | u_i(s) < u_i(q_i, s_{-i}), q_i \neq s_i\}. \tag{3}$$

Strategy s is better in Nash sense than strategy q (or strategy s Nash ascends strategy q) if $t_N(s,q) < t_N(q,s)$. A strategy profile s^* is non-dominated with respect to the Nash ascendancy relation if $\nexists q \in S$ such that q Nash ascends s. It is known that the set of Nash non-dominated solutions is equal to the set of Nash equilibria of the game [9].

2.2 Method

The community detection problem in unweighted networks has been previously approached by an extremal optimization algorithm based on the game theoretic approach described above [16]. Another similar extremal optimization approach that maximizes the modularity function [12] can be found in [10]. In this paper we present a new extremal optimization variant, called Weighed Nash Extremal Optimization (W-NEO), designed to capture the community structure in weighted networks.

An extremal optimization (EO) algorithm [2] typically uses one individual $s = (s_1, \dots, s_n)$ to search the space and preserves each iteration the best solution found up to that moment, s_{best}. A fitness value is assigned to each component s_i in s, $i = \overline{1,n}$. Each iteration, the component s_j having the worst fitness value is randomly re-initialized. If the new individual is better than s_{best}, it will replace it. If not, the search continues from the new value of s.

W-NEO extends EO by evolving a population of pairs (s, s_{best}) that search independently for the Nash equilibria of game Γ by using the Nash ascendancy relation.

Encoding. Individuals s (and s_{best}) are represented as strategy profiles of game Γ, i.e. integer vectors; a component i represents the community of node i. Communities are numbered from 0 to a maximum value n_{comm}. The value of n_{comm} differs between EO pairs (s, s_{best}), it is set at the beginning of the search, and

Algorithm 1. W-NEO step

1: For current configuration s evaluate $u_i(s)$, the payoff function corresponding to each node $i \in \{1, \ldots, n\}$.
2: find the k worst components in s and replace them with a random value;
3: **if** (s ***Nash ascends*** s_{best}) **then**
4: set $s_{best} := s$.
5: **end if**

Algorithm 2. *Weighted Nash Extremal Optimization*

1: Randomly initialize and evaluate *popsize* pairs of configurations (s, s_{best}).
2: Compute k_{Nash};
3: Set $k_1 = k_{Nash}$;
4: **repeat**
5: Update $k = \min\{k_{Nash}, [k_1 + 2\frac{nr.it}{MaxGen}(1 - k_1)]\}$;[2]
6: Apply a W-NEO step on each (s, s_{best}) pair;
7: Update k_{Nash};
8: **until** the maximum number of generation is reached;
9: Return s_{best} with highest fitness Φ.

[2] *nr.it* is the iteration number, and $[\cdot]$ represents the integer part.

takes values between a minimum and maximum expected number of communities c_{min} and c_{max}.

Fitness Assignment. For each individual, the payoff functions u_i, $i = \overline{1, n}$ (1) are computed and used to compare nodes within a W-NEO iteration. To compare s and s_{best}, a different fitness function, Φ, is used, computed as:

$$\Phi(s) = \sum_{i=1}^{n} u_i(s) \cdot w_i^{(in)}, \tag{4}$$

where $w_i^{(in)}$ is the sum of the weights of the links node i has with other nodes in its community. W-NEO uses fitness Φ as an alternative to the modularity function [12].

Extremal Optimization. Several EO variants proposed for the community structure problem in unweighted networks extend the typical EO by modifying more than one node during an iteration. This number, denoted by k, can be fixed, or set adaptively. In the first half of the search, Noisy EO [10] linearly decreases the value of k from a given value to 1, whereas in its second phase, k is kept constant. MNEO [16] decreases k exponentially throughout the search. The recommended initial value for k is 10 % of the number of nodes, which is a parameter for both methods.

W-NEO uses an adaptive mechanism to update k values by combining the linear decrease of NoisyEO with the $t_N(s, s_{best})$ operator used by the Nash ascendancy relation (3). Each iteration, the number k_{Nash} is computed as the maximum value of $t_N(s, s_{best})$ in that iteration. The number k of nodes changed in one iteration is computed as the minimum value between k_{Nash} and the one corresponding to NoisyEO. The first k_{Nash} value, denoted by k_1, is computed immediately after the initialization of the population and it is used to set the initial value in the equation that decreases k linearly. Thus, W-NEO does not need a parameter for the initial value of k. The outline of W-NEO is presented in Algorithm 2 and a W-NEO step is detailed in Algorithm 1.

Parameters. W-NEO uses the following parameters:

- Population size - $popsize$;
- Maximum number of generations - $MaxGen$;
- Expected minimum and maximum number of communities.

2.3 Numerical Experiments - Synthetic Benchmarks

The performance of W-NEO is tested on a set of synthetic benchmarks and compared with the results obtained by other methods that compute the community structure in weighted networks.

Benchmark. The LFR benchmark [5] is used to evaluate the performance of W-NEO in a first phase. Three sets of networks and corresponding community structures were generated[2], with parameters presented in Table 1. The most important parameters are μ, representing the ratio of links a node has outside its community. A μ value of 0.5 indicates that the node has an equal number of links in its community and outside, and a μ value of 0.6 that the node has more links outside than inside. The μ_w parameter is similar, taking weights into account. $\mu_w = 0.6$ means that the sum of weights of the links the node has in its community is 0.4 of the total strength of that node.

Table 1. LFR benchmarks. 30 networks were generated for each μ and μ_w value. κ is the average node degree, κ_{max} is the maximum degree, and τ_1 and τ_2 are the minus exponents for the degree sequence and for the community size distribution, respectively.

Name	N	κ	κ_{max}	τ_1	τ_2	μ	μ_w	Comm. size
LFR 128	128	20	50	2	1	0.3,0.4,0.5,0.6	0.1–0.6	[10,50]
LFR 1000 S	1000	20	50	2	1	0.3,0.4,0.5,0.6	0.1–0.6	[10,50]
LFR 1000 B	1000	20	50	2	1	0.3,0.4,0.5,0.6	0.1–0.6	[20,100]

The most challenging sets in this benchmark are the small ones (128 nodes), with μ and μ_w values above 0.4, having the least well defined structures. The bigger networks may seem more challenging because of their size, but they all present a well defined community structure even for $\mu, \mu_w = 0.5$, because of the greater number of communities in which the outside links of a node can be distributed, making the difference between the number of links inside its

[2] By using the code available at https://sites.google.com/site/andrealancichinetti/software, accessed May, 2015.

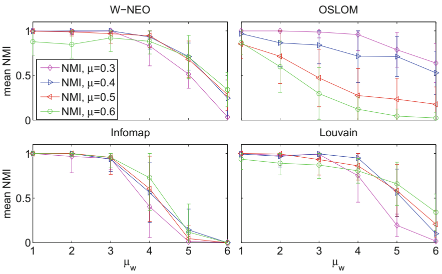

Fig. 1. Average NMI values for the LFR sets with 128 nodes. Wilcoxon sign-rank tests results are presented in Table 2

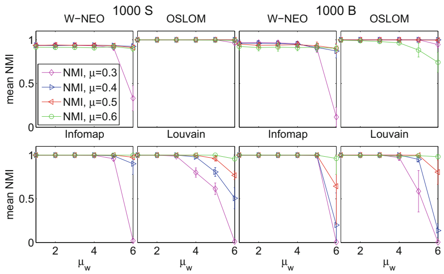

Fig. 2. Average NMI values for the 1000 nodes sets. Wilcoxon sign-rank tests results are presented in Table 2.

community and the number of links in any other community bigger than in the case of networks with 128 nodes and smaller number of communities.

Performance Evaluation. Results are evaluated by using the normalized mutual information indicator (NMI) [6]. A NMI of 1 indicates identical community structures. When two different community structures are compared to the real structure, the one having the higher NMI value is considered better.

Comparisons with Other Methods. The results obtained by W-NEO are compared with those obtained by three state of art methods: Oslom [7], Infomap [14], and Louvain [1]. Differences in median NMI values obtained by each method for each set of 30 networks are evaluated by using the Wilcoxon sign-rank test with a confidence level of 0.05.

Parameter Settings. W-NEO parameters are: population size, minimum and maximum expected number of communities, and maximum number of generations. Considering that (s, s_{best}) pairs evolve independently, the effect of size of the of the population is the usual one, in this case using a larger population being equivalent with performing multiple independent runs with smaller populations. The expected number of communities influences the results in a similar

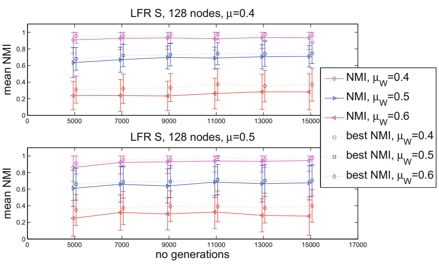

Fig. 3. NMI values of W-NEO for the LFR 128 nodes set, and different $MaxGen$ values.

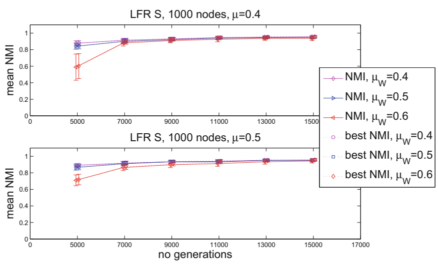

Fig. 4. NMI values of W-NEO for two LFR sets with 1000 nodes, and different $MaxGen$ values.

Table 2. Wilcoxon sign -rank test results. A • indicates that the corresponding method provided the best results. If there are more methods with results that are not statistically different from the best one, they are also marked with a •.

		128 nodes				1000 nodes S				1000 nodes B			
μ	μ_W	W -NEO	Oslom	Infomap	Louvain	W -NEO	Oslom	Infomap	Louvain	W -NEO	Oslom	Infomap	Louvain
0.3	0.1	•	•	•	•	-	•	•	•	-	•	•	•
	0.2	•	•	•	•	-	•	•	•	-	•	•	•
	0.3	•	•	-	•	-	•	•	-	-	•	•	•
	0.4	-	•	-	-	-	•	•	-	-	•	•	-
	0.5	-	•	-	-	-	•	-	-	-	•	-	-
	0.6	-	•	-	-	-	•	-	-	-	•	-	-
0.4	0.1	•	-	•	•	-	•	•	•	-	•	•	•
	0.2	•	-	•	•	-	•	•	•	-	•	•	•
	0.3	•	-	-	•	-	•	•	•	-	•	•	•
	0.4	•	-	-	•	-	•	•	-	-	•	•	•
	0.5	•	•	-	-	-	•	-	-	-	•	•	-
	0.6	-	•	-	-	-	•	-	-	-	•	-	-
0.5	0.1	•	-	•	•	-	•	•	•	-	•	•	•
	0.2	•	-	•	•	-	•	•	•	-	•	•	•
	0.3	•	-	•	•	-	•	•	•	-	-	•	•
	0.4	•	-	-	-	-	•	•	•	-	-	•	•
	0.5	•	-	-	-	-	•	•	-	-	-	•	•
	0.6	•	-	-	-	-	•	-	-	-	-	•	•
0.6	0.1	-	-	•	-	-	•	•	•	-	•	-	-
	0.2	-	-	•	-	-	-	•	•	-	-	•	•
	0.3	•	-	•	-	-	-	•	-	-	-	•	•
	0.4	•	-	-	-	-	-	•	•	-	-	•	-
	0.5	•	-	-	•	-	-	•	•	-	-	•	-
	0.6	•	-	-	•	-	-	•	-	-	-	•	•

manner. For these numerical experiments, the minimum and maximum number of communities was set such that approx. 20 % of the population has assigned the real number of communities. The population size was set to 30. Because the maximum number of generations indirectly influences the results, as it related to the value of k (Algorithm 2), several values are tested for this parameter.

Results and Discussion. Numerical results obtained on the synthetic benchmarks are presented as error-bars in Figs. 1 and 2 ($MaxGen = 10\,000$). The results of the Wilcoxon sign-rank test are presented in Table 2. For the small networks, the results provided by W-NEO are in some cases the best compared with the other methods, and in most cases as good as the others. For the 1000 nodes sets, W-NEO results are statistically different than all the others, but with NMI values greater than 0.9 in almost all cases (Fig. 2).

W-NEO Parameters. Figures 3 and 4 illustrate the variation of average NMI values with the maximum number of generations. For each set two values are represented: the average NMI of the individuals having the best Φ value in each run and the average NMI of the individual with the best NMI in the final population. The small differences between the two values indicate the the function Φ can be considered as an efficient fitness function for assessing the quality of a community structure.

3 Brain Functional Connectivity Networks

In order to examine if W-NEO can detect communities of the brain, we used a public resting-state fMRI database from the 1000 Functional Connectomes Project, Addiction Connectome Preprocessed Initiative. In our study we used the MTA 1 dataset with the ANTS registered, no scrubbing, no global signal regression preprocessing pipeline.[3] The dataset contains 126 subjects' resting-state data, based on which, and an atlas of 90 functional regions of interest (ROI) [15], we calculated the Pearson correlation between the activities of the ROIs.[4] We calculated an "averaged" network, in which nodes correspond to ROIs, denoted as $r_1, r_2, \ldots$, and the weight of each connection $\{r_i, r_j\}$ is the average of the correlations between r_i and r_j over all the subjects. Only positive correlations with values above 0.35 were considered.

For each subject, information about cannabis usage and the childhood diagnosis for Attention Deficit Hyperactivity Disorder (ADHD) is available. Therefore, additionally to the "averaged" network, we considered four disjoint groups of subjects: (A) the healthy subjects (no cannabis usage, no ADHD), (B) cannabis users without ADHD, (C) ADHD patients who do not use cannabis, and (D) subjects with childhood diagnoses of ADHD who regularly use cannabis. For each of these groups, we obtained a network of ROIs. In each of these networks, we calculated the weight of the connection $\{r_i, r_j\}$ as the average of the correlations between r_i and r_j for the subjects belonging to the group.

From the community structure detection point of view, the brain functional connectivity networks proved to be challenging; performing multiple runs with the four algorithms led to different results for each run and each algorithm, with Oslom, Infomap and Louvain finding structures with maximum 3 communities. However, by setting the values for the minimum and maximum number of communities to 10 and 20, W-NEO provides structures with more communities that can be further analyzed.

Thus, after performing 30 independent runs ($MaxGen = 3000$) for each network, the resulting community structures were aggregated in the following manner: each node was placed in the same community with the node with which it was placed in the same community most of the times in the 30 runs. If there are several such nodes, one of them is selected at random. Because the resulting community structure contained many communities formed only by two nodes, a further step consisted in uniting the communities having the smallest fitness values with those with which they have the strongest link. The strength of the link between two communities is computed as the ratio between the sum of weights of the links that connect the communities and the number of nodes that link them. Communities are merged until their number equals the recommendation of the domain experts, i.e. 14.

[3] See http://fcon_1000.projects.nitrc.org/indi/ACPI/html/ for details.

[4] One ROI (Basal Ganglia 4) did not include meaningful measurement for any of the 126 subjects, therefore we ignored this ROI in the subsequent analysis.

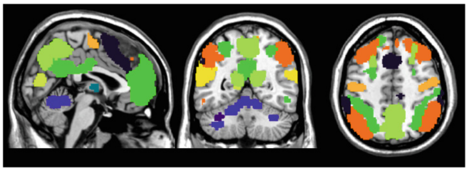

Fig. 5. Community structure of the averaged whole brain functional connectivity network.

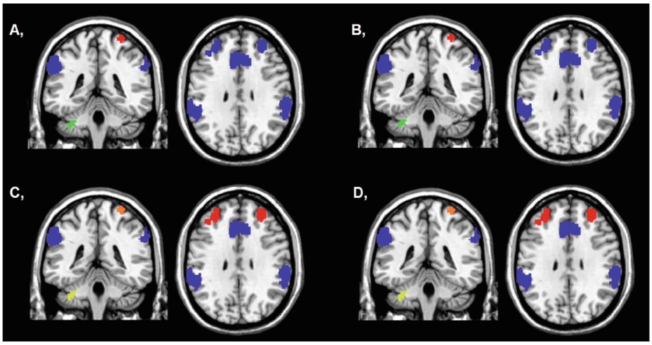

Fig. 6. Community structure of the anterior and posterior salience network in case of (A) healthy subjects, (B) cannabis users without ADHD, (C) subjects with childhood diagnoses of ADHD who does not use cannabis, (D) subjects with childhood diagnoses of ADHD who regularly use cannabis.

Results. The detected community structures (Figs. 5 and 6) are consistent with domain knowledge and they illustrate that the proposed community detection approach may be applicable to discover new insights about brain functionality and brain disorders. In particular, we examined the structure of two large communities of brain regions, the so called default mode network[5] (anterior and posterior default mode networks), and the salience network (anterior and posterior salience networks).

The role of the default mode network (DMN) in drug addiction has been shown by several studies [11,13]. In our community structures we found that the DMN is more intact (more ROIs are in the same community) in non-addicted subjects. In healthy subjects, 13 ROIs of the DMN belong to the same community, whereas we observed 11 ROIs of the DMN to be highly connected in ADHD patients. In contrast, in case of cannabis addicts, both with and without ADHD, the DMN is decomposed to several smaller communities (with less than 7 ROIs).

[5] We note that in the brain research community, the phrases *default mode network* and *salience network* are used to refer to two specific sets of strongly interconnected regions of the brain. Therefore, the *default mode network* and the *salience network* are *communities* according to the terminology used throughout this paper.

The salience network has a critical role in attention, therefore it is expected to be related to ADHD [3]. In healthy subjects and cannabis addicts without ADHD, the salience networks were found to be intact, in particular 11 and 12 ROIs were observed within the same community. However, in subjects diagnosed with ADHD, the salience network's largest community has only 7 ROIs, see Fig. 6.

4 Conclusions

The analysis of brain functional connectivity networks from the community structure point of view can offer important information about the structure and functioning of the brain. The brain networks are relatively small, with very unclear structure, not detected by existing algorithms. In this paper we propose a game theoretic approach capable to identify strong connections in these networks and construct community structures that can offer relevant knowledge about the functioning of the brain.

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