A Bi-objective memetic algorithm proposal for solving the minimum sum coloring problem

[Extended Abstract]

Olfa Harrabi
Higher Institute Of Management Of Tunis, University of Tunis, Tunisia
olfa-harrabi@hotmail.com

Ezzeddine Fatnassi
Higher Institute Of Management Of Tunis, University of Tunis, Tunisia
ezzeddine.fatnassi@gmail.com

Hend Bouziri
ESSEC, University of Tunis. Tunisia
hend.bouziri@gmail.com

Jouhaina Chaouachi
IHEC Carthage Prsidence-2016 Tunis, Tunisia
siala.jouhaina@gmail.com

ABSTRACT

The minimum sum coloring problem (MSCP) is an extension of the graph coloring problem (GCP) which is known to be NP-hard. Due to its theoretical and practical relevance, MSCP attracts increasing attention. In this paper, a bi-objective memetic algorithm is proposed. The algorithm combines the use of an effective crossover operator and tabu search as a mutation operator. Experiments are performed on instances extracted from the second DIMACS and COLOR02 challenges. Obtained results of our algorithm improved upper bound values from the literature.

CCS CONCEPTS

•Applied computing → Operations research; •Computing methodologies → *Bio-inspired approaches;*

KEYWORDS

sum coloring problem ; chromatic sum ; bi-objective formulation ; memetic algorithms; VEGA algorithm ; tabu search.

ACM Reference format:
Olfa Harrabi, Ezzeddine Fatnassi, Hend Bouziri, and Jouhaina Chaouachi. 2017. A Bi-objective memetic algorithm proposal for solving the minimum sum coloring problem. In *Proceedings of GECCO '17 Companion, Berlin, Germany, July 15-19, 2017,* 2 pages.
DOI: http://dx.doi.org/10.1145/3067695.3082035

1 INTRODUCTION

The minimum sum coloring problem (MSCP) is an NP-hard problem introduced by Ewa Kubicka [3] in 1987. For our problem, we consider an undirected graph G=(V, E) where V is the set of $|V| = n$ vertices and E the set of $|E| = m$ edges. A **proper** (or valid) coloring of a graph is to associate a color to each vertex so that two connected vertices are colored differently. Let $c(x)$ be the color associated to the vertex x. The minimum sum coloring problem consists on finding a valid coloring so that the sum of colors is minimal. This sum is called the **chromatic sum** of G and denoted by $\sum(G) = min_c \sum_{v \in V} c(v)$. The **strength** s(G) of a graph G is the smallest number of colors over all optimal sum colorings of G. The **chromatic number** of a graph G is the minimum number χ for which G has a proper coloring. Since $\chi(G) \leq s(G)$, $\chi(G)$ is considered as a lower bound of s(G).

In the literature, several heuristic and metaheuristic approaches were proposed to tackle this problem. For an excellent bibliographic survey on the MSCP, see [2]. On the other hand, memetic algorithms are among the most powerful paradigms for solving NP-hard combinatorial optimization problems. In particular, they have been successfully applied to the tightly related Graph Coloring Problem [1]. In this paper, we are interested to solve a bi-objective problem related to the MSCP. The two objectives consist on minimizing the sum of coloring and the number of conflicting edges having its endpoints colored with the same colors. For the underlying problem, we treat both feasible and non feasible solutions. This feature gives to search process more "freedom" in order to discover more promising regions in the search space. In fact, the same concept was frequently adopted by many approaches in the literature. The proposed algorithm combines the use of a tabu search procedure and a genetic algorithm. A new crossover operator was adopted namely SPX. The tabu-search procedure is used as a mutation operator. Experimentations show the good quality of results on a set of benchmarks instances from the literature.

2 THE PROPOSED SOLUTION APPROACH

While minimizing the sum of colors, the problem aims to determine a partition of V in a minimum number of color classes $s(G)$ such no conflicting edges existed. From this point of view, we can consider a bi-objective MSCP problem which consists on optimizing the two objectives: the sum of colors used and the number of the conflicting edges.

Our problem can be formulated as the following bi-objective programming problem $minF(x) = (\sum_{i=1}^{k} c_i(x), p(x))$, where $\sum_{i=1}^{k} c_i$ corresponds to the sum of colors used and $p(x)$ is the number of the conflicting edges. If $p(x) = 0$, solution is then proper.

Consequently, we decided to study separately these two objectives and propose a bi-objective memetic algorithm MA-MSC. On the other hand, the VEGA (Vector Evaluated Genetic Algorithm)

[4] strategy consists on decomposing the initial population according to each objective functions we have. This fact may help us in treating each objective independently, and this was the reason for choosing the VEGA schema. Obtained solutions correspond to k-colorings which can be proper or not (k is not a fixed value). The algorithm is adapted to the sum coloring problem in the following way:

Algorithm 1 The algorithm of the memetic MA-MSC for MSCP (Input: A graph G, an integer k)

Output: k-coloring c^*, sum of colorings $\sum(c^*)$
Begin
while(number of iterations is not reached) **do**
  Random Population Initialization(P, N, k) /* Population P has N solutions using k colors */
  for $i \leftarrow 1$ to MaxGeneration **do**
    Evaluate the fitness of P ;
    Divide P into two sub-populations (P_1, P_2);
    Declining Process (P_1);
    $P \longleftarrow$ Combine the two sub-populations (P_1, P_2);
    $P^{'} \leftarrow$ Selection(P) // Select 2 parents for crossover
    $o \leftarrow$ SPX crossover $(P^{'})$ //Crossover to get an offspring solution
    $o \leftarrow$ Mutation(o) // Improve o with **One-Move operator**

    if o is the best **then**
    Population Updating (P, o)
    end if
    end for
  if(There is no proper solutions) **then** $(k \leftarrow k + 1)$
  return c^*
End

Based on Algorithm 1, we devise our population into two sub-populations based on our fitness values. Before proceeding to evolutionary steps, we define a **declining process** in order to enhance the fitness values of the sub-population having a minimum sum of colors. Firstly, we sort the color classes by decreasing cardinalities. Then, for each vertex $v \in V$ with color i, we try to move it to a different color class X_j , such that there is no adjacent vertex of v in X_j and $xj \geq xi$ (x_i and x_j are cardinalities of the the representative color classes). We propose as crossover, a new operator SPX (Sum Partition crossover). It consists on preserving, in the resulting child, the largest color sets of the parents. This crossover principle was already adopted by GPX operator in [1] to deal with the k-coloring problem. The mainly SPX contribution is on assigning to the largest color class the smallest possible color in the GPX offspring. During mutation, we used Tabu search as a mutation operator. The considered neighborhood is **One-Move operator** [2]. The size of the tabu-list was set to 10 [1]. Finally, the MaxGeneration is fixed to $10 * |V|$. This value was determined empirically.

[1]Smaller values may create cycling and larger values do not improve the procedure while increasing the computation time

Table 1: Results obtained using the MA-MSC algorithm.

Instances	MA-MSC		LB_{MA-MSC}
	gap(%)	ARPD(%)	gap(%)
DSJC125.1	0.267	0	0.211
DSJC125.5	0.821	0	0.405
DSJC125.9	0.484	0	0.320
DSJC250.1	0.718	0	0.385
DSJC250.9	0.934	0	0.480
Le450-15a	0.128	0	0.125
Le450-15b	0.124	0	0.127
Le450-15c	0.484	0	0.261
Le450-15d	0.342	0	0.232
Le450-25a	0.053	0	0.030
Le450-25b	0.018	0	0.010
Le450-25c	0.297	0.045	0.153
Le450-25d	0.236	0	0.174

3 EXPERIMENTAL RESULTS

Our MA-MSC approach was tested on a set of 42 well-known graphs: DIMACS and COLOR02 instances [2]. Like many memetic algorithms, we use a small population of 10 individuals [3]. To obtain our computational results, each instance is solved 30 times independently with different random seeds. Each run is stopped when the processing time reaches its timeout limit [4]. Note that for each tested instance, our proposed approach obtain both proper and non proper solutions. To highlight the efficiency of MA-MSC, the smallest obtained proper solutions are used. From the remaining configurations (non proper), lower bounds for our problem were found. The experimental results show that MA-MSC was able to minimize the sum of colors in comparison to the best obtained results from the literature. Also, the proposed bi-objective memetic algorithm have improved the chromatic lower bounds when non proper solutions were found.

4 CONCLUSION

The proposed algorithm is quite effective and has a robust behavior on all the considered instances. Currently, we are in the process of making extensive statistical comparisons between our algorithm and other approaches from the literature.

REFERENCES

[1] Philippe Galinier and Jin-Kao Hao. 1999. Hybrid evolutionary algorithms for graph coloring. *Journal of combinatorial optimization* 3, 4 (1999), 379–397.
[2] Yan Jin, Jean-Philippe Hamiez, and Jin-Kao Hao. 2016. Algorithms for the minimum sum coloring problem: a review. *Artificial Intelligence Review* (2016), 1–28.
[3] Ewa Kubicka and Allen J Schwenk. 1989. An introduction to chromatic sums. In *Proceedings of the 17th conference on ACM Annual Computer Science Conference*. ACM, 39–45.
[4] J David Schaffer. 1985. *Some experiments in machine learning using vector evaluated genetic algorithms*. Technical Report. Vanderbilt Univ., Nashville, TN (USA).

[2]The experiments were performed a personal laptop with an Intel i3 CPU 2.53 GHz and 3 GB of RAM.
[3]We set crossover rate to 0.9 and the mutation rate to 0.4
[4]The timeout limit is set to be five CPU hours except for some hard graphs where the number of nodes is great or equals to 300 for which a time limit of 10 hours is allowed