Utilization of Infeasible Solutions in MOEA/D for Solving Constrained Many-objective Optimization Problems

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ABSTRACT

For solving constrained many-objective optimization problems, this work proposes CMOEA/D-DM introducing the Directed Mating utilizing useful infeasible solutions having better scalarizing function values than feasible ones in the MOEA/D algorithm framework. Experimental results on knapsack problems with 3-7 objectives and 2-6 constraints shows that the proposed CMOEA/D-DM achieves higher search performance than the conventional CMOEA/D.

CCS CONCEPTS

• **Computing methodologies** → **Optimization algorithms**; • **Theory of computation** → Evolutionary algorithms;

KEYWORDS

MOEA/D, constraint-handling, many-objective optimization

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1 INTRODUCTION

MOEA/D [1] is known as an effective algorithm for solving many-objective optimization problems. MOEA/D is able to emphasize the selection pressure of solutions by specifying approximating parts on the Pareto front with uniformly distributed weight vectors even in many-objective optimization. For solving problems with constraints, Constraint-MOEA/D (CMOEA/D) was also proposed [2]. Since CMOEA/D only keeps best solution for each weight vector, generated infeasible solutions are just discarded after a feasible solution is founded. However, useful infeasible solutions having better objective values than feasible ones exist, and they can be utilized to enhance the solution search. The directed mating utilizing useful infeasible solutions as parents was proposed with a dominance-based algorithm [3], and its effectiveness has been shown on several multi-objective test problems.

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To utilize useful infeasible solutions in the decomposition-based algorithm which is effective for many-objective optimization, in this work, we propose Constrained MOEA/D with the Directed Mating (CMOEA/D-DM) which introduces useful infeasible solution archive and the directed mating utilizing them as parents. The search performance of the proposed CMOEA/D-DM is compared with CMOEA/D on knapsack problems with 3-7 objectives and 2-6 constraints.

2 CONVENTIONAL CMOEA/D

CMOEA/D [2] is a variant of MOEA/D and involves a constraint handling for solving constrained multi-objective problems. CMOEA/D employs N sets of weight vector $\boldsymbol{\lambda}^i$ and solution $\boldsymbol{x}^i$ ($i = 1, 2, \ldots, N$) and simultaneously optimizes N kinds of scalarizing functions $g(\boldsymbol{x}^i|\boldsymbol{\lambda}^i)$ ($i = 1, 2, \ldots, N$) to approximate the Pareto front. To generate an offspring $\boldsymbol{y}$, CMOEA/D focuses on a weight $\boldsymbol{\lambda}^i$, selects parents from T-neighbor solutions of $\boldsymbol{\lambda}^i$, and applies genetic operators to them. Then, CMOEA/D tries to replace existing solutions paired with T-neighbors of the focused $\boldsymbol{\lambda}^i$ with the generated offspring $\boldsymbol{y}$. If any of the following conditions are true, an existing solution $\boldsymbol{x}^j$ is replaced with $\boldsymbol{y}$.

1. $\boldsymbol{x}^j$ and $\boldsymbol{y}$ are feasible, and $g(\boldsymbol{y}|\boldsymbol{\lambda}^j)$ is better than $g(\boldsymbol{x}^j|\boldsymbol{\lambda}^j)$.
2. $\boldsymbol{x}^j$ is infeasible and $\boldsymbol{y}$ is feasible.
3. $\boldsymbol{x}^j$ and $\boldsymbol{y}$ are infeasible, and $\boldsymbol{x}^j$ has a higher value of the sum of constraint violation values than $\boldsymbol{y}$.

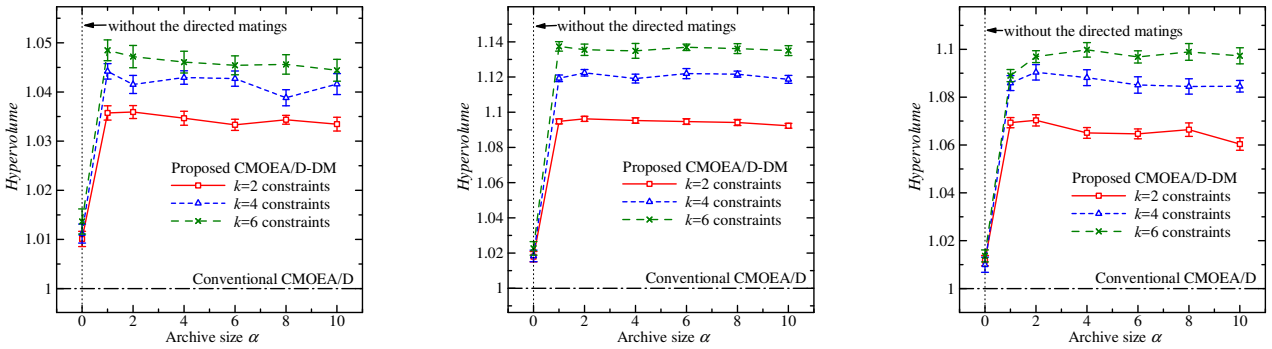
Since CMOEA/D compares infeasible solutions based only on the constraint violation values without considering their scalarizing function values, scalarizing function values of their offspring would not be good even if feasible offspring are obtained. Also, infeasible solutions having better objective values than feasible ones have useful variable information for the search, however, CMOEA/D just discards infeasible solutions after once a feasible solution is founded to each weight. Consequently, their useful variable information cannot be utilized for the search.

3 PROPOSED CMOEA/D-DM

The proposed CMOEA/D-DM introduces a novel solution replacement criterion in the MOEA/D framework and archives of useful infeasible solutions having better scalarizing function values than feasible ones for each weight.

3.1 Solution Replacement Criterion

CMOEA/D-DM tries to replace existing solutions of T-neighbors of the focused $\boldsymbol{\lambda}^i$ with the generated offspring $\boldsymbol{y}$. If any of the following conditions are true, an existing $\boldsymbol{x}^j$ is replaced with $\boldsymbol{y}$.



(a) $m = 3$ objectives (b) $m = 5$ objectives (c) $m = 7$ objectives

Figure 1: Results of *HV* at the final generation

1. $\boldsymbol{y}$ dominates $\boldsymbol{x}^j$ in constraint violation values.
2. $\boldsymbol{y}$ and $\boldsymbol{x}^j$ are non-dominated in constraint violation values but $g(\boldsymbol{y}|\boldsymbol{\lambda}^j)$ is better than $g(\boldsymbol{x}^j|\boldsymbol{\lambda}^j)$.

When two infeasible solutions are compared, the proposed criterion uses their dominance relation on constraint violation values [3] and scalarizing function values while the conventional CMOEA/D uses only the sum of constraint violation values. Therefore, the proposed replacement criterion works to obtain feasible solutions having good scalarizing function values.

3.2 Directed Mating with Archives

The proposed CMOEA/D-DM utilizes useful infeasible solutions having better scalarizing function values as parents and stores them in archive $\mathcal{A}^i$ for each weight $\boldsymbol{\lambda}^i$ $(i = 1, 2, \ldots, N)$.

The archives $\mathcal{A}^i$ $(i = 1, 2, \ldots, N)$ are empty at the initialization. To generate one offspring, CMOEA/D-DM focuses on a weight $\boldsymbol{\lambda}^i$ and selects its paired solution $\boldsymbol{x}^i$ as the first parent. If $\boldsymbol{x}^i$ is feasible and $|\mathcal{A}^i| > 0$, the second parent is randomly selected from $\mathcal{A}^i$. Otherwise, the second parent is randomly selected from T-neighbor solutions of the focused $\boldsymbol{\lambda}^i$. After an offspring $\boldsymbol{y}$ is obtained, the proposed CMOEA/D-DM replaces existing solutions of T-neighbors of the focused $\boldsymbol{\lambda}^i$ with $\boldsymbol{y}$ according to the aforementioned solution replacement criterion. Also, $\boldsymbol{y}$ is added to each archive $\mathcal{A}^j$ if an existing $\boldsymbol{x}^j$ is feasible, $\boldsymbol{y}$ is infeasible and $g(\boldsymbol{y}|\boldsymbol{\lambda}^j)$ is better than $g(\boldsymbol{x}^j|\boldsymbol{\lambda}^j)$. The maximum archive size is limited by a parameter α. When $|\mathcal{A}^j|$ exceeds α by adding $\boldsymbol{y}$, the most inferior solution in $\mathcal{A}^j$ according to the proposed replacement criterion is eliminated.

4 RESULTS AND DISCUSSION

We compare the proposed CMOEA/D-DM and the conventional CMOEA/D on the knapsack problem with $m = \{3, 5, 7\}$ objectives, $k = \{2, 4, 6\}$ constraints, feasibility ratio $\phi = 0.5$ and $n = 500$ items. Both algorithms use Tchebycheff function as g, the uniform crossover with the ratio 1.0, and bit-flip mutation with the ratio $1/n$. The parameters of the algorithms are set as follows: the population size $N = 210$, the neighbor size $T = 20$, and 10^4 generations as the termination condition. As a performance metric, Hypervolume (*HV*) with the reference point $\boldsymbol{r} = (0.0, \ldots, 0.0)$ is employed. The higher *HV*, the better search performance. Experimental results are shown by the average (mean) values of 30 runs.

Fig. 1 shows results of *HV* values at the final generation. The values are normalized by one obtained by CMOEA/D. The error bars indicate 95% confidence intervals. Note that CMOEA/D-DM with $\alpha = 0$ indicates that the algorithm without using the directed mating and infeasible archives.

First, we can see that CMOEA/D-DM without the directed mating ($\alpha = 0$) shows higher *HV* values than CMOEA/D in all problems used in this work. It reveals that the effectiveness of the proposed solution replacement criterion. Next, it can be seen that the proposed CMOEA/D-DM significantly improves *HV* by introducing the directed mating ($\alpha \geq 1$). It reveals that the archives of infeasible solutions and the directed mating in the proposed CMOEA/D-DM further improve search performance. In problems with $m = 3$ and $k = \{4, 6\}$, and $m = 5$ and $k = 6$, $\alpha = 1$ achieves the highest *HV* since only the best useful infeasible solution is maintained in $\mathcal{A}^i$ and becomes the second parent. However, it has a drawback that the same infeasible solution frequently becomes the second parent, it causes a search performance deterioration. Therefore, in other problems, $\alpha = 2$ or 4 is appropriate. As α is increased from the appropriate value, *HV* tends to be deteriorated since inferior solutions are also maintained in $\mathcal{A}$ although a large α brings the diversity of parents. Although the appropriate α maximizing *HV* is different on each problem, α seems insensitive to the search performance.

5 CONCLUSIONS

This work proposed CMOEA/D-DM which utilizes useful infeasible solutions having better scalarizing function values than feasible ones as parents in the decomposition based MOEA/D. The experimental results showed the proposed CMOEA/D-DM outperforms the conventional CMOEA/D by utilizing useful infeasible solutions and their archives. As a future work, we will verify the effects of CMOEA/D-DM on continuous many-objective problems.

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