Solving Order/Degree Problems by Using EDA-GK

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ABSTRACT

The[1] Estimation of Distribution Algorithms with Graph Kernels called EDA-GK is an extension of the Estimation of Distribution Algorithms to cope with graph-related problems. The individuals of the EDA-GK are represented by graphs. In this paper, the EDA-GK is applied to solve for the Order/Degree problems, which are an NP-hard problem and are a benchmark problem in graph theory studies. Experimental results on several problem instances on the Order/Degree problems show the effectiveness of the EDA-GK.

CCS CONCEPTS

• **Theory of Computation → Design and analysis of algorithms**; *Mathematical optimization*; Discrete optimization; Network optimization

KEYWORDS

Estimation of Distribution Algorithms, graph kernel, Order/ Degree problems

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1 INTRODUCTION

The EDA-GK [1] is an extension of the Estimation of Distribution Algorithms [2] to cope with graph-related problems. The EDA-GK employs graph kernels to estimate the distribution of graphs, i.e., individuals. In this paper, we tackle to solve the Order/Degree problems by using the EDA-GK. The Order/Degree problems are a sort of difficulties in graph theory domain: to find the smallest diameter of graphs of given order and maximum degree. Moreover, graphs with the smallest diameter and the minimum average shortest path length (ASPL) are regarded as optimal solutions of the problem instance.

2 ORDER/DEGREE PROBLEMS

A graph G is defined by $G = (V, E)$, where V and E denote a set of nodes and a set of edges, respectively. An edge is associated with two nodes. The degree $\delta(v)$ of the node $v \in V$ in a graph is defined by the number of edges which are associated with the node v. The shortest path length between two nodes $v_1, v_2 \in V$ is defined as the least number of edges from the node v_1 to the node v_2. The diameter $k(G)$ of graph G is the greatest number of the shortest path length among possible combinations of two nodes $v_1, v_2 \in V$ in the graph G. Moreover, the average shortest path length $l(G)$ of graph G is calculated by averaging the shortest path length over all the possible combinations of two edges.

For given the number of nodes n and the degree d of graphs, the Order/Degree problems are defined as follows:

$$\begin{aligned} &\min_{G=(V,E)} \quad k(G) \\ &\text{subject to} \quad |V| = n, \\ &\qquad\qquad\quad |\delta(v)| = d \quad {}^{\forall} v \in V \end{aligned} \tag{1}$$

The Order/Degree problems [3] can be solved by finding a graph with the minimum value of the average shortest path length of graph G with the least diameter if the number of nodes and the degree of each node are assigned in advance. In the case of undirected graphs, these problems are known as NP-hard problems. A competition for solving the Order/Degree problems is held [4].

Although these problems are just a benchmark problems of graph theory communities, the optimal solution of these problems can be used in a practical computer network design. For instance, suppose that there are 16 network hubs with three network ports for connecting other network hubs. Such network design can be regarded as the Order/Degree problems of (16 nodes and 3 degrees). The solution of the Order/Degree problems is to find out a network topology with the least ASPL and with minimum diameter.

3 EDA-GK

The EDA-GK is an extension of the Estimation of Distribution Algorithms, which can cope with graphs. Hence, the individuals of the EDA-GK are represented by graphs. The procedure of algorithms is similar to the one of the EDAs: Selection of better

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individuals, Estimation of the probability distribution of the selected individuals, and Sampling new individuals from the estimated probability distributions. The kernel density estimation is adopted to estimate the probability distribution of graphs. Moreover, the graph kernel [5,6] is used in the kernel density estimation in order to cope with graphs.

4 EXPERIMENTAL RESULTS

The fitness function for solving the order degree problems is defined by the following equation:

$$f(G) = k(G)\times w + l(G), \tag{2}$$

where $k(G)$ and $l(G)$ indicate the diameter and the average shortest path distance of graph G, respectively. The parameter w denotes a weight: This solution should have the least diameter $k(G)$, and should be of the minimum average shortest path length $l(G)$ among graphs with the least diameter. The parameter w is set to be 10 in this paper. This fitness function is a minimized one.

We examined two sorts of algorithms: the EDA-GK and the local search method. The population size of the EDA-GK is set to be 100. The number of generations is 3000. Hence, 300,000 fitness evaluations are executed for a single run. The shortest path distance kernel is employed for the EDA-GK. For the local search method, we employ iterated local search, where the neighborhood is defined by a 2-opt operation. The number of total iterations for the local search method is the same as the number of fitness evaluations in the EDA-GK, i.e., 300,000. If the local search method reaches to a local optimum, the search point of the local search method is reinitialized.

Table 1 shows experimental results on four problem instances of the Order/Degree problems which are examined in the competition [4]. The problem instances of the Order/Degree problems can be distinguished by two variables, the number of nodes $|V|$ and the degree $|\delta(v)|$ of all the nodes in a graph. In the table, (36, 3) means the number of nodes is 36, and the degree of all the nodes is 3. For each problem instance, 50 runs are carried out.

In the case of problem instance (36, 3) and (64, 4), the local search method outperforms the EDA-GK regarding the average fitness. The reason of this is that the neighborhood size of these problem instances is not so large: 1431 and 8128, respectively. The best fitness of the problem instance (36, 3) for both the EDA-GK and the local search method is the same. For other problem instance, i.e., in the case of larger neighborhood size, the EDA-GK outperforms the local search method. The neighborhood size of problem instances (64, 8) and (96, 3) is 32640 and 10296, respectively.

Table 2 summarizes the theoretical lower bound for each problem instance in Table 1. The right row of the table shows the fitness value of the lower bound if such solution existed. For some problem instance, the experimental results in Table 1 is closed to the lower bound.

In comparison with the results by the participants of the competition [4], the results in this paper are competitive, e.g., ranked from the 2nd to the 5th, but are not the first prize.

Table 1: Experimental Results on four problem instances of the Order/Degree problem: average value over 50 runs

Prob. Instance ($\|V\|$, $\delta(v)$)	EDA-GK	Local Search
(36, 3)	52.47(43.07)	47.07(43.07)
(64, 4)	42.92(42.91)	42.91(42.90)
(64, 8)	32.01(31.00)	32.10(32.09)
(96,3)	72.52(64.31)	74.12(64.32)

Table 2: Theoretical lower bound for each problem instance in Table 1

Prob. Instance ($\|V\|$, $\delta(v)$)	LB of ($k(G)$, $l(G)$)	Fitness if existed
(36, 3)	(4, 3.06)	43.06
(64, 4)	(4, 2.86)	42.86
(64, 8)	(2, 1.87)	21.87
(96,3)	(6, 4.20)	64.20

5 CONCLUSIONS

In this paper, the EDA-GK, the Estimation of Distribution Algorithms with Graph Kernels, is applied to solve for the Order/Degree problems. We examined four problem instances of the Order/Degree problems, which are listed on the competition site [4]. In comparison with the local search method, the EDA-GK outperforms the local search method if the neighborhood size is large.

Future works are summarized as follows: Experiments on a variety of problem instances are needed even if such problem instances are adopted in the competition. Comparison with other evolutionary algorithms should be carried out. Memetic algorithms of the EDA-GK can be constituted by incorporating the idea in the top-ranked algorithms of the competition into the local search operation.

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