Interpolated Continuous Optimisation Problems with Tunable Landscape Features

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ABSTRACT

In this paper, we introduce a new class of optimisation problems with tunable landscape features called Interpolated Continuous Optimisation Problems (ICOPs). ICOPs are defined by a search space, a set of solutions called seeds at selected positions, and their fitnesses. The rest of the fitness landscape is interpolated from the seeds using the inverse distance weighting interpolation function. We show that by evolving the position and the fitness of the seeds, we can generate extreme problems with respect to different fitness landscape measures.

CCS CONCEPTS

•Computing methodologies → Heuristic function construction; Continuous space search;

KEYWORDS

continuous optimisation, numerical optimisation, benchmark functions, metrics, differential evolution, fitness distance correlation, information landscape, dispersion metric

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1 INTRODUCTION

Benchmark suites are commonly used by the EA community to evaluate the comparative performance of proposed algorithms. In continuous optimisation, the CEC 2005 [10] and BBOB 2009 [3] benchmark sets are well-established standards.

The aim of any benchmark suite should be to provide a comprehensive range of challenging problems. From this perspective, the quality of a benchmark set may be assessed in terms of a range of problem characteristics including landscape features. In [2], the authors measure a broad range of features of the CEC 2005 and BBOB 2009 and show that some are under-represented or not covered in these sets. It is desirable that coverage by a benchmark set of problem features should be controllable *a priori* as opposed to only discoverable *post hoc*.

Generating benchmark problems in a guided way has received a growing attention in the past decade. For instance, in [5], the authors use genetic programming to evolve continuous optimisation problems. In [11], multimodal optimisation problems are obtained by combining several randomly distributed peaks.

In [7], the authors propose a new class of problems in binary space. The problems are defined using a small set of candidate solutions called seeds with the rest of the fitness landscape obtained by interpolating the fitness of those seeds using the Hamming metric. Using a technique based on spanning trees they are able to set the fitness of the seeds in such a way as to control the difficulty of the resulting problems for a hill-climber algorithm.

Here, we extend this idea to continuous spaces to introduce interpolated continuous optimisation problems (ICOPs). ICOPs are defined by a set of solutions called seeds and their assigned fitness from which the rest of the fitness landscape is interpolated. In this work we show that, by evolving the position and the fitness of the seeds, we can vary the values of different landscape difficulty measures. This provides us with a method of generating continuous problems that are tuned to take specific values on a variety of landscape measures.

2 INTERPOLATED CONTINUOUS OPTIMISATION PROBLEMS

ICOPs are defined by the following elements:

(1) **A search space** Ω: a set, whose elements we refer to as *solutions*, that defines the optimisation problem domain. For continuous problems, this will be a (subset of) real space of chosen dimension. In this paper we choose our search spaces to be the ℓ-dimensional cubes: $\Omega = [0,1]^\ell$
(2) **A distance function**, $d(x,y) : \Omega \times \Omega \to \mathbb{R}$ defining the distance between two solutions x and y. The pair (Ω, d) with these definitions is a *metric space*. A natural choice of distance function for continuous search spaces is Euclidean distance.
(3) **A set of seeds** $S \subset \Omega$: a set of distinct solutions with an assigned fitness. In general, S will be non-empty and finite. The solutions in S and their assigned fitnesses will define the entire optimisation problem by interpolation.
(4) **An interpolation function** $f_S : \Omega \to \mathbb{R}$: in this paper we use the inverse distance weighting method, originally defined by Shepard [8] for use in spatial analysis. Assuming the seed set S contains N seeds, labelled $s_1, \dots, s_N$, and with

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assigned fitnesses $u_1, ..., u_N$ respectively, we define for any solution $x \in \Omega$:

$$f_S(x) = \begin{cases} \frac{\sum_{i=1}^{N} \frac{u_i}{d(x,x_i)^p}}{\sum_{i=1}^{N} \frac{1}{d(x,x_i)^p}} & \text{if } d(x, x_i) \neq 0 \text{ for all } i \\ u_i, & \text{if } d(x, x_i) = 0 \text{ for some } i \end{cases} \quad (1)$$

where p is a positive real number called the *power parameter*. Higher values of p increase the relative influence of nearby seed solutions on the interpolated value. $f_S(x)$ is differentiable with respect to x provided $d(x, x_i)$ is derivable for each x_i.

(5) **An optimisation objective**: we chose minimisation of f_S as the objective, which is consistent with most benchmarks used in continuous optimisation. By construction, global minima will occur precisely at those solutions x_i in S that take minimum value u_i.

It is important to note that this class of problems is generalisable over a wide range of representations, in fact any search space that is a metric space.

3 EVOLVING ICOPS

The fitness landscape of ICOPs is defined by the choice of the seeds and their fitnesses. In this section, we evolve using a simple differential evolution (DE) [9] this set of seeds (fitnesses and positions in Ω) to create problems that seperatly maximise and minimise the following landscape measures:

- *Fitness Distance Correlation* (FDC [4]: The correlation between a set of fitness/distance pairs after calculating the distance to the global optimum for each of a random sample of points resulting in a value between -1 and 1. A low FDC indicates deceptive problems.
- *Information Landscape* (IL) [1] : The distance between the optimal IL (the sphere function in our case) and the IL of the problem from a random sample resulting in a value between 0 and 1. IL measures the searchability of the problem.
- *Dispersion Metric* (DM) [6] : measures the global topology of the fitness landscape. Negative values indicate the presence of multiple funnels in the landscape.

We evolved ICOPs over 4 dimensions $\ell = \{2, 3, 5, 10\}$, using seeds sets of size $N = \{10, 50, 100\}$ The average (over 25 runs) maximised and minimised measures obtained after 10000 function evaluations can be seen in Table 1.

We can see that the problems generated can nearly span the full range of each of these measures. We can obtain hard problems *i.e.* highly deceptive (FDC close to -1) with a low searchability (IL close to 1) or multiple funnels (negative DM) as well as easy problems (the opposite in each measure). It is also interesting to note that for those three measures, a small number of seeds ($N = 10$) is enough to minimise and maximise those measures.

4 CONCLUSION

In this paper, we presented a new class of tunable continuous optimisation problems called ICOP. These problems are constructed from a set of seeds with predefined fitness while the rest of the search space is interpolated from those seeds. ICOPs can be evolved to obtain easy and hard problems with respect to given landscape measures. Further objective of this research is to create an extensive set of benchmark problems covering a wider range fitness landscape measures to improve evaluation and comparison of existing and new algorithms.

Table 1: Minimisation and maximisation results for FDC, IL and DM

Dim	Seeds	Objective	FDC	IL	DM
2	10	max	0.997	0.994	0.366
	50	max	0.998	0.993	0.362
	100	max	0.995	0.854	0.362
	10	min	-0.995	0.101	-0.239
	50	min	-0.998	0.082	-0.235
	100	min	-0.575	0.132	-0.122
3	10	max	0.996	0.992	0.378
	50	max	0.997	0.988	0.371
	100	max	0.725	0.876	0.371
	10	min	-0.994	0.145	-0.241
	50	min	-0.996	0.135	-0.253
	100	min	-0.511	0.456	-0.118
5	10	max	0.996	0.993	0.371
	50	max	0.993	0.988	0.361
	100	max	0.823	0.858	0.361
	10	min	-0.995	0.136	-0.287
	50	min	-0.994	0.147	-0.146
	100	min	-0.68	0.411	-0.085
10	10	max	0.995	0.993	0.349
	50	max	0.992	0.985	0.344
	100	max	0.784	0.98	0.338
	10	min	-0.995	0.154	-0.291
	50	min	-0.988	0.156	-0.158
	100	min	-0.671	0.443	-0.219

REFERENCES

[1] Yossi Borenstein and Riccardo Poli. 2005. Information landscapes. In *Proceedings of the 7th annual conference on genetic and evolutionary computation*. ACM, 1515–1522.

[2] Robert W Garden and Andries P Engelbrecht. 2014. Analysis and classification of optimisation benchmark functions and benchmark suites. In *2014 IEEE Congress on Evolutionary computation*. IEEE, 1641–1649.

[3] Nikolaus Hansen, Anne Auger, Olaf Mersmann, Tea Tusar, and Dimo Brockhoff. 2016. COCO: A Platform for Comparing Continuous Optimizers in a Black-Box Setting. *ArXiv e-prints* (March 2016). arXiv:cs.AI/1603.08785

[4] Terry Jones and Stephanie Forrest. 1995. Fitness distance correlation as a measure of problem difficulty for genetic algorithms. In *Proceedings of the 6th international conference on genetic algorithms*. ACM, 184fi?!–192.

[5] William B. Langdon and Riccardo Poli. 2007. Evolving Problems to Learn About Particle Swarm Optimizers and Other Search Algorithms. *IEEE Transactions on Evolutionary Computation* 11, 5 (Oct 2007), 561–578.

[6] Monte Lunacek and Darrell Whitley. 2006. The dispersion metric and the CMA evolution strategy. In *Proceedings of the 8th annual conference on genetic and evolutionary computation*. ACM, 477–484.

[7] John A. W. McCall, Lee A. Christie, and Alexander E. I. Brownlee. 2015. Generating easy and hard problems using the proximate optimality principle. In *Proceedings of the companion publication of the 2015 annual conference on genetic and evolutionary computation*. ACM, 767–768.

[8] Donald Shepard. 1968. A two-dimensional interpolation function for irregularly-spaced data. In *Proceedings of the 23rd ACM national conference*. ACM, 517–524.

[9] Rainer Storn and Kenneth Price. 1997. Differential Evolution – A Simple and Efficient Heuristic for global Optimization over Continuous Spaces. *Journal of Global Optimization* 11, 4 (1997), 341–359.

[10] Ponnuthurai N. Suganthan, Nikolaus Hansen, J J Liang, Kalyanmoy Deb, Ying-Ping Chen, Anne Auger, and Santosh Tiwari. 2005. *Problem Definitions and Evaluation Criteria for the CEC 2005 Special Session on Real-Parameter Optimization*. Technical Report.

[11] Simon Wessing, Mike Preuss, and Gunter Rudolph. 2013. Niching by multiobjectivization with neighbor information: Trade-offs and benefits. In *2013 IEEE Congress on Evolutionary Computation*. 103–110.