An Improved MOEA/D Utilizing Variation Angles for Multi-Objective Optimization

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ABSTRACT

This work proposes a decomposition-based multi-objective evolutionary algorithm utilizing variation angles among objective and weight vectors. The proposed algorithm introduces an angle-based proportional selection and dominance- and angle-based solution comparison criterion. Experimental results using WFG4 and WFG5 problems show that the proposed algorithm achieves better search performance than the conventional MOEA/D and MOEA/D-CRU.

CCS CONCEPTS

•**Computing methodologies → Optimization algorithms;**

KEYWORDS

Multi-objective optimization, MOEA/D

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1 INTRODUCTION

MOEA/D is known as a representative evolutionary algorithm for solving multi-objective optimization problems [1]. This work addresses two issues on the algorithm framework of MOEA/D. The first issue is the parent selection. MOEA/D focuses on a weight vector and randomly selects parents from limited solutions paired with T-neighbor weights of the focused weight. Consequently, variable information resources out of neighbor solutions cannot be utilized in the search. The search performance of MOEA/D would be improved by enhancing the availability of variable information resources maintained in the entire population for the search. For this issue, the proposed algorithm introduces an angle-based proportional selection. In this selection, all solutions in the population have the chance to become a parent in every mating, and the selection probability of each solution is determined by the variation angle between its paired weight and the focused weight. The second issue is the solution comparison criterion. MOEA/D-based algorithms compare solutions by using their scalarizing function values. Although there are several options in scalarizing functions, each of them has advantages and disadvantages. The weighted Tchebycheff function is the Pareto dominance compliant and parameter-less. PBI and the inverted PBI functions with an appropriate parameter achieve better search performance than the weighted Tchebycheff on several problems. However, their parameter tunings cannot be avoided. Hence, it is desirable to design a solution comparison criterion which is the Pareto dominance compliant, parameter-less and effective for the search. For this issue, the proposed algorithm introduces the dominance- and angle-based comparison criterion which is the Pareto dominance compliant and parameter-less. In this work, the search performance of the proposed algorithm is compared with the conventional MOEA/D [1] and MOEA/D-CRU [2] on WFG4 and WFG5 problems with two objectives.

2 DECOMPOSITION-BASED MOEAS

MOEA/D [1] decomposes a multi-objective optimization problem into a number of single-objective scalarizing function optimization problems with a set of weight vectors $\mathcal{L} = \{\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \dots, \boldsymbol{\lambda}^N\}$ and simultaneously optimizes them to approximate the Pareto front. Each weight $\boldsymbol{\lambda}^i$ is paired with one solution $\boldsymbol{x}^i$, and the set of solutions becomes the population $\mathcal{P} = \{\boldsymbol{x}^1, \boldsymbol{x}^2, \dots, \boldsymbol{x}^N\}$. To generate one offspring, MOEA/D focuses on a weight $\boldsymbol{\lambda}^i$. According to T-neighbor weight vector indices $\mathcal{B}_i = \{i_1, i_2, \dots, i_T\}$ of the focused weight $\boldsymbol{\lambda}^i$, MOEA/D randomly selects two parents from the limited solutions $\boldsymbol{x}^{i_1}, \boldsymbol{x}^{i_2}, \dots, \boldsymbol{x}^{i_T}$ and generates an offspring $\boldsymbol{y}$ by applying genetic operators. Then, MOEA/D tries to replace the existing solutions $\boldsymbol{x}^{i_1}, \boldsymbol{x}^{i_2}, \dots, \boldsymbol{x}^{i_T}$ with the newly generated offspring $\boldsymbol{y}$ based on their scalarizing function values.

MOEA/D-CRU (MOEA/D with Chain-Reaction Update) [2] employs an alternative solution update mechanism. MOEA/D-CRU adaptively determines the existing solution order tried to be replaced with each generated offspring based on its location in the objective space while MOEA/D predetermines the existing solution order tried to be updated before each offspring is generated and evaluated. Concretely, for each generated offspring $\boldsymbol{y}$, MOEA/D-CRU calculates the objective balance vector. Then, MOEA/D-CRU calculates Euclidean distances between the objective balance vector and each of all weight vectors $\mathcal{L} = \{\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \dots, \boldsymbol{\lambda}^N\}$ and tries to replace solutions in order of increasing distance of paired weight.

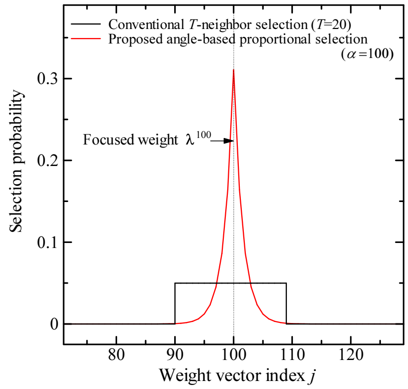
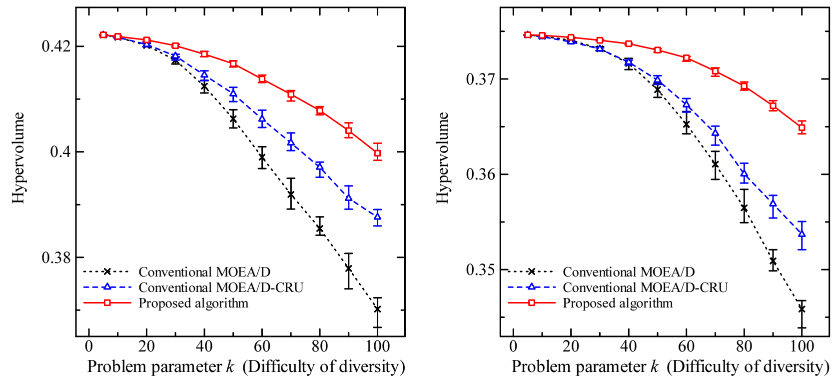


Figure 1: Selection probability



(a) WFG4 (a) WFG5

Figure 2: Hypervolume at the final generation

3 PROPOSED ALGORITHM

3.1 Angle-Based Proportional Selection

In the proposed algorithm, each solution in the population have the chance to become a parent for any focused weight, and the selection probability of each solution is biased by the angle between its paired weight and the focused weight. To generate an offspring, we focus on a weight vector and proportionally select two parents based on the angles with the focused weight. In this work, the angles are raised to the α-th power for the control of selection probabilities. **Fig. 1** shows the selection probability of each weight $\boldsymbol{\lambda}^j$ when $\boldsymbol{\lambda}^{100}$ is focused in a case with $m = 2$ objectives and the number of weights $N = 201$. The conventional T-neighbor selection [1] determines the neighbors by the number of weights T. Also, the selection probability in the neighbors is equivalent, and the selection probability out of the neighbors is zero. On the other hand, the proposed angle-based proportional selection does not have the border determining neighbors, and the selection probability is increased as the angle with the focused $\boldsymbol{\lambda}^{100}$ decreases.

3.2 Dominance- and Angle-Based Update

For a generated offspring $\boldsymbol{y}$, we sort all weight vectors in ascending order of their angles with the normalized objective vector of $\boldsymbol{y}$. The sorted weight order corresponds to the focusing weight order for the solution replacement. First, $\boldsymbol{y}$ tries to replace the solution $\boldsymbol{x}^i$ paired with the weight vector having the minimum angle with $\boldsymbol{y}$. If $\boldsymbol{y}$ dominates $\boldsymbol{x}^i$, $\boldsymbol{x}^i$ is replaced with $\boldsymbol{y}$. Also, if the angle between $\boldsymbol{y}$ and $\boldsymbol{\lambda}^i$ is smaller than the angle between $\boldsymbol{x}^i$ and $\boldsymbol{\lambda}^i$ even $\boldsymbol{y}$ and $\boldsymbol{\lambda}^i$ are non-dominated each other, $\boldsymbol{x}^i$ is replaced with $\boldsymbol{y}$. Next, when $\boldsymbol{y}$ tries to replace a solution $\boldsymbol{x}^i$ paired with a weight vector not having the minimum angle with $\boldsymbol{y}$, $\boldsymbol{x}^i$ is replaced with $\boldsymbol{y}$ if $\boldsymbol{x}^i$ does not dominate $\boldsymbol{y}$ and the angle between $\boldsymbol{y}$ and $\boldsymbol{\lambda}^i$ is smaller than the angle between $\boldsymbol{x}^i$ and $\boldsymbol{\lambda}^i$.

4 EXPERIMENTAL RESULTS

This work compares the search performances of the conventional MOEA/D [1], MOEA/D-CRU [2] and the proposed algorithm with $\alpha = 100$ on WFG4 and WFG5 problems with two objectives and different diversity difficulty parameters k. These three algorithms use the same population size $N = 201$. To generate offspring, we use SBX with a ratio 0.8 and a distribution index $\eta_c = 20$ and the polynomial mutation with a ratio $1/n$ and an index $\eta_m = 20$. The termination condition is set to totally 3,000 generations. The conventional MOEA/D and MOEA/D-CRU use the reciprocal weighted Tchebycheff scalarizing function [2] and the neighbor size $T = 20$. As a search performance metric, this work employs Hypervolume (HV). The higher HV, the better approximation performance of the Pareto front. We normalize objective values of solutions as $f_1'(\boldsymbol{x}) = f_1(\boldsymbol{x})/2$ and $f_2'(\boldsymbol{x}) = f_2(\boldsymbol{x})/4$, and calculate HV with the reference point $\boldsymbol{r} = (1.1, 1.1)$. For each algorithm, HV values are obtained by 50 independent runs.

Fig. 2 shows results of HV obtained by the three algorithms at the final generation. In these figures, each marker indicates median, error bars indicate the first and third quartiles of HV values, and the diversity difficulty k is varied on the horizontal axis. As a general tendency, we can see that HV decreases by increasing the diversity difficulty parameter k. Next, we can see that the conventional MOEA/D-CRU achieves higher HV than the conventional MOEA/D. Also, the proposed algorithm achieves higher HV than the conventional MOEA/D and MOEA/D-CRU, and the effectiveness of the proposed algorithm increases as the diversity difficulty k increases. These results reveal that the proposed angle-based proportional selection and the proposed dominance- and angle-based solution update contribute to improving the search performance on multi-objective optimization problems.

5 CONCLUSIONS

This work proposed an MOEA/D-based algorithm utilizing variation angles among objective and weight vectors. The proposed algorithm introduced the angle-based proportional selection and the dominance- and angle-based solution comparison criterion. The experimental results showed that the proposed algorithm achieved higher search performance than MOEA/D and MOEA/D-CRU on problems with high diversity difficulty.

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