Escher-like Tiling Design Using Hierarchical Optimization

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ABSTRACT

This paper proposes a method that generates a tileable shape similar to a given image. An analytical method have been proposed; however, it sometimes produced a shape that cannot be tiled, and its output shape affects even from small change on input image shape, resulting in requiring much trial-and-error of adjusting the input shape. The proposed method in this paper adopts a hierarchical approach, which allows it to use flexible objective functions and constraints. The proposed method uses the previous analytical method as the lower optimizer, and the upper layer optimizer based on Genetic Algorithm selects points constituting the input image to the lower optimizer and a tiling pattern suitable for the given image shape. Experimental results showed that the proposed method produces tileable shapes more similar to given images than the previous method while avoiding constraint violation.

CCS CONCEPTS

• **Mathematics of computing → Evolutionary algorithms;** • **Computing methodologies → Texturing;**

KEYWORDS

Tiling, hierarchical optimization, genetic algorithm

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1 INTRODUCTION

Tiling is an act of continuously covering the plane with finite types of figures, or tiles, so that there are no gaps and overlaps[2]. Kaplan, who has been inspired by Escher and his elegant works, has defined the problem of finding a tileable figure as close as possible to the given figure as *Escherization Problem* [2]:

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Problems ("ESCHERIZATION"): Given a closed plane figure $\boldsymbol{W}$ (the 'goal shape'), find a new closed figure $\boldsymbol{U}$ such that:

(1) $\boldsymbol{U}$ is as close as possible to $\boldsymbol{W}$; and
(2) copies of $\boldsymbol{U}$ fit together to form a tiling of the plane.

It is difficult to generate complex tileable figures by solving the Escherization problem due to tight restriction where some edges, or a set of line segments, should be shared between two adjacent figure with same shape.

An analytical method for generating tileable figures has been proposed [3]. This method should be forced to repeat selecting an appropriate tiling pattern for the input figure and adjusting of the position of vertices to obtain the favorable output without containing intersection between segments, called *self-intersection*. On the other hand, there have been studies for finding a tileable figure by the certain period of trial-and-error repetitions using meta-heuristics [2, 4, 5]. However, these modeled problems may be optimization problems hard to solve due to very large search space and the strong dependency between variables.

In this paper, we propose a method that generates a tileable figure close to the input figure by hierarchically-structured search. The proposed method uses the analytical method [3] as the lower optimization solver and Genetic Algorithms (GAs) as the upper one. We can thus design the objective function and constraint condition flexibly, enabling to exclude the candidate solution, in which the self-intersection is contained, by adopting the self-intersection constraint that cannot be handled in the conventional method [3]. In addition, the hierarchical model makes the search space narrow down properly, e.g., 120 real-valued variables in [2, 4, 5] to only 185 binary variables in the experiment of this paper, enabling efficiently finding a higher-quality solution.

2 THE PROPOSED METHOD

This paper proposes a method to solve Escherization problem using hierarchical approach to resolve drawbacks of the previous methods, i.e., the need for repetition of input figure adjustment in the analytical method [3] and low solution quality of the meta-heuristics-based methods [2, 4, 5]. The proposed method utilizes the analytical method as a lower layer optimizer, and a GA as upper layer optimizer. The upper optimization using the GA attempts to find the optimal set of vertices $\tilde{\boldsymbol{W}}$ and an appropriate tiling pattern, both of

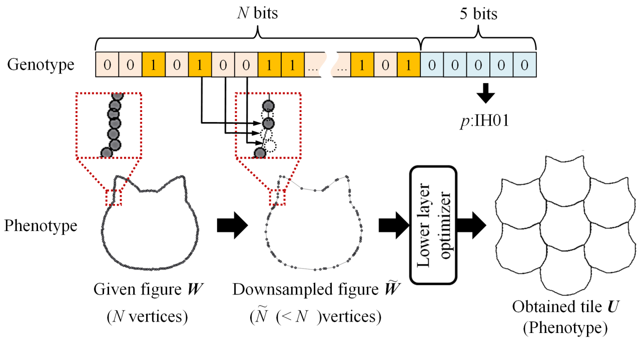


Figure 1: Chromosome representation.

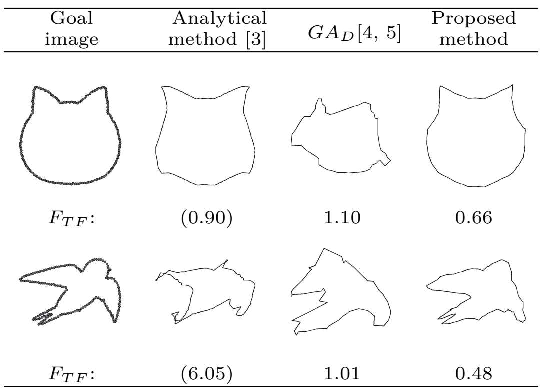
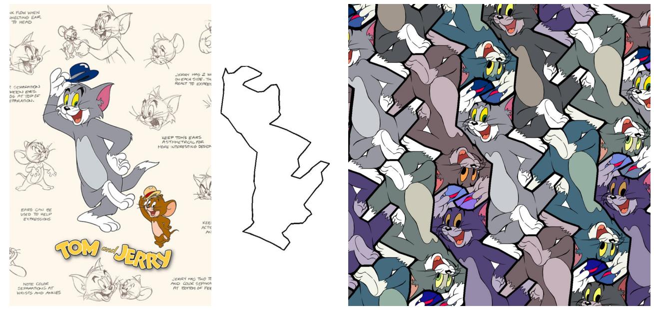


Figure 2: Comparison with previous methods.

which are input to the lower optimization using the analytical method. The upper layer optimizer evaluates outputs of the lower layer optimizer with objective functions and constraints that cannot be considered in the lower layer. The tiling pattern suitable to the input figure and the set of vertices $\tilde{\boldsymbol{W}}$ to produce a satisfactory tileable figure close to the input figure can be obtained simultaneously with performing the optimization only once.

Figure 1 shows the definition of genotype in GA in the upper layer. A chromosome is represented as a binary string which is composed of a mask pattern and a tiling pattern, whose length is $N+5$. The mask pattern represents vertices selected to form an input $\tilde{\boldsymbol{W}}$ to the lower optimizer oft of n vertices in $\boldsymbol{W}$ Therefore, the number of vertices is also automatically determined during optimization. A phenotye in the proposed method is a tileable shape $\boldsymbol{U}$ obtained by applying the lower layer optimizer to $\tilde{\boldsymbol{W}}$ with the selected tiling pattern p. Note that $\boldsymbol{U}$ always satisfies edge matching constraints but does not necessarily satisfy a self-intersection constraint.

Objective functions in the proposed method provides how close $\boldsymbol{U}$ and $\boldsymbol{W}$ are, and involves a penalty function for the self-intersection constraint in $\boldsymbol{U}$. the difference between two *turning functions* F_{TF} [1] is used to represent the distance between two polygons.



(a) Given image*. (b) Output. (c) Tiled result.

* TOM AND JERRY and all related characters and elements © & ™ Turner Entertainment Co. (s17)

Figure 3: Demonstrational result.

3 EVALUATION

Fig. 2 shows the comparison result of the proposed method with the analytical method [3], which were repeated 174 times while changing input point sets and the best was shown in the figure, and the previous method using a GA (GA_D) to directly modify a tile shape [4, 5] for two test shapes. The proposed method successfully found tiling shapes more similar to the given shapes than the other two methods.

Fig. 3 shows a result using the proposed method, where the proposed method successfully deformed the cat's shape so that it can be tiled without any overlap and gap. Note that the shape in Fig. 3(c) was slightly modified from the output by the proposed method shown in Fig. 3(b) by hand.

4 CONCLUSION

This paper proposes a method for Escher-like tiling design based on hierarchically-structured optimization, where the lower optimizer analytically generates a tileable shape while upper layer optimizer determines input of the lower optimizer and evaluates output of the lower optimizer. Experimental results showed that the proposed method successfully promotes the high performance of the analytical method while eliminating violaitons of self-intersection.

In future, we plan to modify multi-objective optimization so that the proposed method becomes effective for more diverse figures.

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