GP-Based Motion Control Design for the Double-Integrator System Subject to Velocity Constraint

Ollin Peñaloza-Mejía
Instituto Tecnológico de Sonora
Cd. Obregón, Sonora, México 85000
ollin.penaloza@itson.edu.mx

Eddie Clemente
Instituto Tecnológico de Ensenada
Ensenada, BC, México 22780
eclemente@ite.edu.mx

Marlen Meza-Sánchez
CONACYT – TecNM – ITE
Ensenada, BC, México 22780
immezasa@conacyt.mx

Cynthia B. Pérez
Instituto Tecnológico de Sonora
Cd. Obregón, Sonora, México 85000
cynthia.perez@itson.edu.mx

Francisco Chávez
University of Extremadura
Mérida, Spain 06800
fchavez@unex.es

ABSTRACT

The motion control problem for the double-integrator system subject to velocity constraint is addressed. A novel methodology, which consists of a two-stage process and regards a trade-off between natural and learned behaviors to develop a family of analytic controllers, is proposed. To this end, firstly, a natural behavior is designed to achieve asymptotic tracking of a desired continuous trajectory by using a Control-Theory approach. Secondly, learned behaviors are discovered by using a Genetic Programming approach to synthesize an analytic controller to ensure a bounded velocity of the system. The integration of these approaches allows the system to exhibit a good tracking performance while keeping the velocity bounded to a desired value, freely set by the user. Simulation results are provided to illustrate the effectiveness of the proposal, and a comparison with a traditional *Control-Theory-Based* solution is also given and discussed.

KEYWORDS

Motion control design, genetic programming, double-integrator system, bounded velocity

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1 INTRODUCTION

The double-integrator system is one of the most fundamental plants used in control theory due to its wide applications in engineering (see e.g. [1, 2]). Its dynamics is described by the second-order differential equation

$$\ddot{q}(t) = u(t), \tag{1}$$

where $q(t), \dot{q}(t), \ddot{q}(t) \in \mathbb{R}$ is the position, velocity and acceleration, respectively, and $u(t) \in \mathbb{R}$ is the control input to be designed to impose a desired behavior. The dynamics (1) describes the motion of a single-degree-of-freedom rigid body with unit moment of inertia and simplified models of mechanical systems, commonly used in Robotics and other fields (see e.g. [3–5]). The motion control problem for this system include asymptotic convergence to a given reference. Though this problem has been solved from the control theory approach, the input and the velocity, in general, can take any value, which in practice could remain saturated because the systems have physic limits. However, the operation of systems under saturated conditions for long periods of time has proved to cause damage of parts, deterioration of performance, and even instability. In this work, the integration of both, Control Theory (CT) and Genetic Programming (GP) approaches, is proposed to build up a family of controllers valid for the double-integrator system, to achieve asymptotic tracking of a desired continuous trajectory while bounding the system velocity (to a desired value). The main contributions of this paper are summarized as follows.

- A methodology for learning analytic control functions based on genetic programming.
- A *natural behavior* is generated for a double-integrator system by means of a classical CT-based controller. Next, such behavior is complemented, by implementing a GP approach, upon that the system *learns* how-to accomplish a previously established performance considering velocity constraints.

Then, our control objective is described as the design of the control input $u(t)$ such that the following two performance conditions are achieved. First, the position $q(t)$ asymptotically converges to a desired trajectory $q_d(t)$; that is

$$\lim_{t \to \infty} \left(q_d(t) - q(t) \right) = 0. \tag{2}$$

Second, the velocity $\dot{q}(t)$ remains within a boundary layer defined by the desired velocity bound $\dot{q}_{max}$; that is

$$|\dot{q}(t)| \leq \dot{q}_{max}, \quad \forall \quad t \geq 0. \tag{3}$$

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Table 1: Some solutions discovered by the evolutionary process and control laws used for performance comparison.

Solution	Mathematical expression	Fitness
u_{GP_1}	$10*(\dot{q}_d(t)-\dot{q}(t))+sinh(sinh(3*(\dot{q}_d(t)-\dot{q}(t))))$	0.94
u_{GP_2}	$2*\dot{q}_d(t)-\dot{q}(t)+sinh(3*(tanh(\dot{q}_d(t)-\dot{q}(t))-\dot{q}(t))+\dot{q}_d(t))-tanh(tanh(\dot{q}_d(t)-\dot{q}(t)))+tanh(\dot{q}_d(t)-2*\dot{q}(t))-tanh(2*\dot{q}_d(t)-\dot{q}(t))$	0.92
u_{GP_3}	$(min(sech(sech(\ vmax))-2*\dot{q}_d(t)+3*\dot{q}(t), asin(\epsilon)))^2-\dot{q}(t)+\dot{q}_d(t)+tanh(min(tanh(\dot{q}_d(t)-\dot{q}(t)), \dot{q}(t)))-min(tanh(\dot{q}(t)-\dot{q}_d(t)), asin(\epsilon))-2*(\dot{q}(t)+\dot{q}_d(t))+cosh((min(\dot{q}(t)-asin(\epsilon), asin(min(tanh(\dot{q}(t)-\dot{q}_d(t)), \dot{q}_{max}))-sech(\dot{q}(t)-\dot{q}_d(t))))^2)-tanh(\dot{q}_{max}-\dot{q}_d(t)+\dot{q}(t))$	0.89
$u_{\tanh}$	$(\tanh((\dot{q}(t)+\dot{q}_{max})/\Delta))/2-(\tanh((\dot{q}(t)-\dot{q}_{max})/\Delta))/2$	2989.68
u_{pcf}	$\begin{cases} 1, & \dot{q}(t)<\dot{q}_{max} \\ (1-\cos((\pi/\Delta)*(\dot{q}(t)-\dot{q}_{max})))/2, & -\dot{q}_{max}\le\dot{q}(t)<-\dot{q}_{max}+\Delta \\ 0, & -\dot{q}_{max}+\Delta\le\dot{q}(t)\le\dot{q}_{max}-\Delta \\ (-1+\cos((\pi/\Delta)*(\dot{q}(t)+\dot{q}_{max})))/2, & \dot{q}_{max}-\Delta<\dot{q}(t)\le\dot{q}_{max} \\ -1, & \dot{q}(t)>\dot{q}_{max} \end{cases}$	0.96

2 SYNTHESIS OF TRACKING CONTROLLERS

The proposed control input for the double-integrator system is a combination of two controllers given by

$$u(t) = u_{CT} + u_{GP}, \tag{4}$$

where u_{CT} is a traditional PD-like controller used to achieve asymptotic tracking of the reference and u_{GP} is used to keep the velocity within the desired limits. The design of u_{GP} is not evident and is a real challenge since there is no established procedure to propose such analytic controller. An alternative approach is then proposed for designing u_{GP} by using Genetic Programming, in which the system stability, the tracking performance and the maximal allowed velocity can be considered simultaneously. Thus, the fitness function is defined in (5), including the Root Mean Square Error (RMSE) between the position error and the velocity error, the parameter N_{oc} which takes into account the number of milliseconds where boundary limits are exceeded by the controlled system, and the initial conditions $q(0)$ and $\dot{q}(0)$. Thus, the fitness function is defined as the average performance of a set of M initial conditions

$$f = \frac{\sum_{i=1}^{M}\left(\sqrt{\frac{\sum_{t=0}^{n}(q_d(t)-q(t))^2}{n}}+\sqrt{\frac{\sum_{t=0}^{n}(\dot{q}_d(t)-\dot{q}(t))^2}{n}}+N_{oc}\right)_i}{M} \tag{5}$$

3 SIMULATION RESULTS

Some discovered fittest solutions u_{GP_i}, $i = 1,2,3$ as well as state-of-the-art proposals u_{tanh} and u_{pcf}, are shown in Table 1. A performance comparison against our fittest solution using the same parameters and initial conditions is shown in Figure 1.

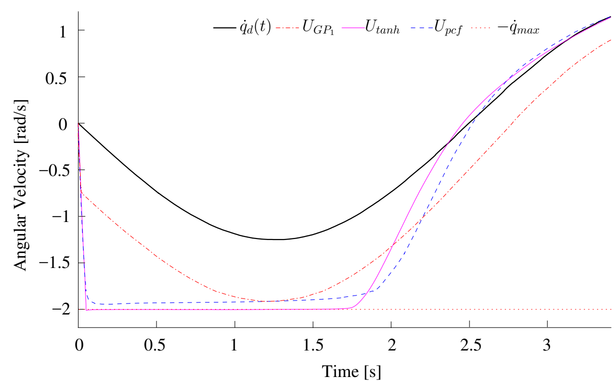

Figure 1: Detailed comparison of exhibited velocity with the different approaches during the settling time.

4 CONCLUSIONS

A new methodology towards the development of controllers merging CT and GP approaches was proposed. It was applied to solve the motion control problem for the double-integrator system subject to velocity constraints. The proposal allows the system to achieve asymptotic tracking of a given reference while bounding the velocity to a desired value. The family of controllers found by the evolutionary process are fittest, compared to solely Control-Theory-based solutions, under the same settings. Future work will extend the proposed framework to improve performance of the controlled system applying a Multi-Objective Genetic-Programming-based approach considering physical constraints of other variables (e.g. torques and/or displacements) to increase feasibility of real-world implementations and to facilitate the stability analysis of the closed-loop system.

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REFERENCES

[1] S. Ding and W.X. Zheng. 2015. Robust control of multiple integrators subject to input saturation and disturbance. *Internat. J. Control* 88, 4 (2015), 844–856. DOI: http://dx.doi.org/10.1080/00207179.2014.982710

[2] D. Gu and H. Hu. 2008. Using Fuzzy Logic to Design Separation Function in Flocking Algorithms. *IEEE Transactions on Fuzzy Systems* 16, 4 (Aug 2008), 826–838. DOI: http://dx.doi.org/10.1109/TFUZZ.2008.917289

[3] C. Knoll and K. Röbenack. 2011. Control of an underactuated manipulator using similarities to the double integrator. *IFAC Proceedings Volumes* 44, 1 (2011), 11501 – 11507. DOI: http://dx.doi.org/10.3182/20110828-6-IT-1002.02812

[4] H.B. Oza, Y.V. Orlov, and S.K. Spurgeon. 2014. Finite time stabilization of a perturbed double integrator with unilateral constraints. *Mathematics and Computers in Simulation* 95 (2014), 200 – 212. DOI: http://dx.doi.org/10.1016/j.matcom.2012.02.011 Discontinuous Differential Systems : Theory and Numerical Methods.

[5] E. Serpelloni, M. Maggiore, and C. Damaren. 2016. Bang-bang hybrid stabilization of perturbed double-integrators. *Automatica* 69 (2016), 315–323. DOI: http://dx.doi.org/10.1016/j.automatica.2016.02.028