A Fast Hybrid Evolutionary Algorithm with Inexact Fitness Evaluation for Solving Two-Stage Stochastic Scheduling Problems

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ABSTRACT

Scheduling in the real-world has to be performed under significant uncertainty. This uncertainty can be introduced into mathematical optimization by using two-stage stochastic optimization, where the uncertainty is modeled by a discrete set of scenarios, and recourse decisions represent the degrees of freedom to react to the actual evolution of the uncertainties.

Most scheduling problems can be formulated as MILP, and due to the progress in problem formulations and solvers, problems of realistic size can be solved nowadays. However, in a monolithic formulation of two-stage programs, the size of the problem increases linearly with the number of scenarios and the solution of the resulting MILP becomes computationally very challenging.

In this contribution we present a modification of a hybrid evolutionary algorithm [5, 6] based upon stage-decomposition by incorporating ideas from Ordinal Optimization. We replace the time-consuming computation of optimal solutions for all second stage scenario problems during the fitness evaluation by fast but inexact methods. The proposed algorithm is evaluated by numerical experiments using a real-world case-study from the polymer industry. Two inexact evaluation methods are tested and compared to the original approach: a LP-relaxation and an evaluation method which uses the expected value problem in combination with a test for feasibility.

CCS CONCEPTS

• **Theory of computation** → **Evolutionary algorithms**; **Integer programming**; • **Software and its engineering** → *Scheduling*;

KEYWORDS

Scheduling, Hybrid Evolutionary Algorithm, Ordinal Optimization

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1 INTRODUCTION

Two-stage stochastic mixed-integer linear problems can be used to introduce uncertainty into mathematical optimization. The uncertainty is modelled by a discrete set of scenarios Ω. A general formulation of such a problem can be written as follows:

$$\min c^T x + \sum_{\omega=1}^{\Omega} \pi_\omega q_\omega^T y_\omega \tag{1}$$

$$s.t. \quad Ax \leq b \tag{2}$$

$$T_\omega x + W_\omega y_\omega \leq h_\omega \tag{3}$$

$$x \in X, y_\omega \in Y, \omega = 1, \ldots, \Omega. \tag{4}$$

This problem formulation distinguishes between first-stage (here-and-now) decisions x which comprise the decisions that have to be made at the current point in time and the second-stage (wait-and-see) decisions y_ω which represent future decisions that can be different for each scenario $\omega \in \Omega$. The first-stage decisions are optimized such that the expected value over the scenarios of the total cost is minimized and the solution is feasible for all scenarios.

Scheduling problems can be modelled as *mixed-integer linear problems (MILP) formulations* which can be extended to two-stage stochastic mixed-integer problems (2S-MILP) [2]. However with an increasing number of uncertain parameters these problems become very hard to solve in a monolithic fashion. Especially in a rolling horizon approach, where the schedule has to be renewed multiple times due to new events, a fast solution method is needed.

In this contribution a new method EA+OO is presented which incorporates ideas of *Ordinal Optimization (OO)* [3] into the *hybrid evolutionary algorithm (HEA)* for 2S-MILP from [5, 6] and is based on the ideas of [4].

2 NEW APPROACH: EA+OO

In the HEA [5], the problem is decomposed into a master problem (MASTER) and into $|\Omega|$ scenario problems (SUB_ω). An Evolutionary Algorithm is used to search for good first-stage solutions x for (MASTER) while an exact solver for MILP (e. g. CPLEX) is utilized to solve the subproblems $(\text{SUB})_\omega$ during the fitness evaluation of an individual. During the evaluation of an individual all variables of the first-stage x are fixed to the values induced by the individual and afterwards solved separately. Also a penalty function is used in the fitness function which measures the violation of constraints to tackle the constrained problem.

The ideas that OO is based upon are: "*Order* is easier than *Value*" and "Nothing but the best is very costly". OO's first principle expresses that it is easier to show that the performance of a solution

θ_1 is better than the performance of a solution θ_2, than it is to calculate the exact performance of both designs. The second principle states that searching for multiple *good enough* solutions instead of searching for a single optimal one is much easier. The combination of both principles leads to a reduction of the computational time for finding adequate solutions for practical problems. An OO-based method searches for a set of solutions which are obtained by using a heuristic or an approximate solution. While the ranking in this set might be erroneous, the set as a whole can be robust against perturbations and contains one of the best solutions with a high probability [3].

Combining the ideas of OO with the idea of the HEA leads to the proposed method EA+OO. This method replaces the exact fitness function F_{ex} which is computed by determining the optimal solution for each scenario problem $(\text{SUB})_\omega$ in the HEA by inexact fitness evaluations F_{inex} which use a simplification of the problem. The value F_{inex} is used to evaluate an individual and provides a performance indicator for the first-stage solution that makes it possible to compare the performance of two solutions. The first-stage solution is the solution which will be be implemented in a real-world application. Beside using en inexact evaluation the EA+OO method searches for a set of s solutions, where s is a problem-specific value. It is assumed that at least one of these s solutions is of high quality, when evaluated exactly. This assumption is based upon the idea that if the goal of the optimization is relaxed to find at least one solution of a set of good enough solutions G (e. g. the best 1%) and at the same time a set S of candidate solutions is considered with $|S| = s$, there is high level of confidence that there is a reasonable degree of matching between both sets, even in the presence of large noise [3].

Using inexact evaluations might cause erroneous feasibility evaluations. Therefore all feasible individuals are stored and ranked according to the inexact fitness function F_{inex}. Afterwards these individuals are re-evaluated using F_{ex}. During this re-evaluation the individuals are re-evaluated in the same order as they were ranked in the first stage of the evaluation until s feasible solutions have been found.

3 EXPERIMENTAL EVALUATION

To evaluate the performance of the EA+OO for scheduling problems in comparison to the HEA we used a real-world batch scheduling problem from the polymer industry with different parameter sets. It concerns a plant for the production of a expandable polystyrene (EPS) and was also used to test the HEA [5, 6]. We tested four parameter sets with 256 scenarios and four instances with 512 scenarios, using 100 generations as the termination criterion and also a time-limit of 7.5 minutes for the first four and of 15 minutes for the second four instances. For each dataset the optimization was repeated 20 times.

The basic configuration of the HEA and EA+OO is the same that was used in [6]. The optimization is performed by a (μ, κ, λ)-*integer evolution strategy*. For EA+OO two inexact evaluation methods $F_{\text{inex}}^{\text{LP}}$ and $F_{\text{inex}}^{\text{EV}}$ were used. In $F_{\text{inex}}^{\text{LP}}$ the LP-relaxation of the second-stage is used as an approximation to estimate the performance of a first-stage solution. Due to the relaxation it cannot be implied that when a feasible solution for the relaxed second-stage was found a feasible solution for each not relaxed scenario problem exists. $F_{\text{inex}}^{\text{EV}}$ uses the *Expected Value Problem (EVP)* [2] as a fitness indicator for an individual. Instead of solving up to $|\Omega|$ scenario problems only one artificial scenario representing the EVP is solved using a standard MILP-solver. To improve the reliability of this evaluation method this fitness function incorporates a test for feasibility using the *feasibility pump (FP)* heuristic [1]. This heuristic is used to quickly test whether a MILP has a feasible solution. If one exists the objective value of the EVP is used as the fitness value otherwise a penalty term is calculated.

It was observed that the EA+OO approach using the inexact fitness functions $F_{\text{inex}}^{\text{LP}}$ and $F_{\text{inex}}^{\text{EV}}$ finds solutions of even significantly better quality (after the re-evaluation with F_{ex}) than the original algorithm which uses the exact fitness function F_{ex}. For all datasets the median value of all runs is better when using $F_{\text{inex}}^{\text{EV}}$ instead of F_{ex}. This is also the case when $F_{\text{inex}}^{\text{LP}}$ is used as a fitness function with the exception of one test-case which can be seen as an outlier. It can also be observed that the usage of the inexact fitness values reduces the spread of the results.

A reason for the improved performance might be a side-effect of the usage of the inexact fitness functions. For all individuals which induce a solution which violates only constraints of the second-stage, the original exact fitness function F_{ex} only assigns the same fitness value while both inexact fitness functions provide more differentiated fitness values for more individuals than F_{ex} which seems to improve the optimization process.

For $F_{\text{inex}}^{\text{LP}}$ in over 90% of all cases less than 200 re-evaluations were necessary to find the best individual of the run. Hence, the same amount of exact evaluations had to be performed as during only four generations of the HEA. For $F_{\text{inex}}^{\text{EV}}$ in over 95% of the runs it was sufficient to re-evaluate ten or less candidate solutions to find the best individual of a run. Hence, a value of $s = 10$ is sufficient to find the best result in each run with a high probability when using the EA+OO-method.

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