GECCO 2017 Tutorial: Fitness Landscape Characterisation of Optimisation Problems

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Tutorial overview

- Overview of fitness landscape analysis:
  - Motivation for characterising problems
  - What is a fitness landscape?
  - Features of fitness landscapes
  - Fitness landscape analysis techniques
- Recent contributions with a focus on correlation with algorithm performance, selection and tuning.
  - Vehicle routing problem
  - Failure prediction for PSO
  - Local optima networks
- Interactive demo of metrics in python using Jupyter.

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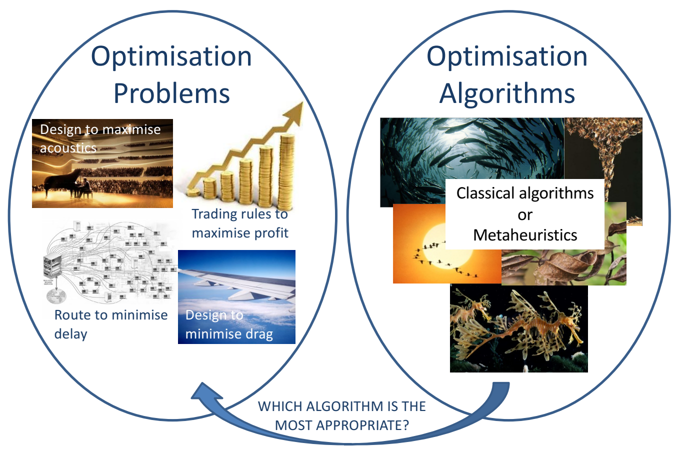

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… unpredictable results

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Too many algorithms

- Too many optimisation algorithms:
  - new algorithms introduced all the time inspired by natural or social phenomenon.
  - Some recent examples: social spider algorithm, water wave optimization algorithms, bat algorithm, election inspired optimization algorithm, football game algorithm, firefly algorithm, honey bee mating algorithm.
  - Are they really “new”?
  - NFL Theorems: a ‘super-algorithm’ cannot exist.
- Not enough understanding of the algorithms:
  - Takes decades of empirical and theoretical research to understand established metaheuristics to a limited extent.
  - Every new approach comes with a blank record of knowledge around algorithm behaviour (? algorithm setup ? parameter choices ? convergence ? suitable / unsuitable problems ?)

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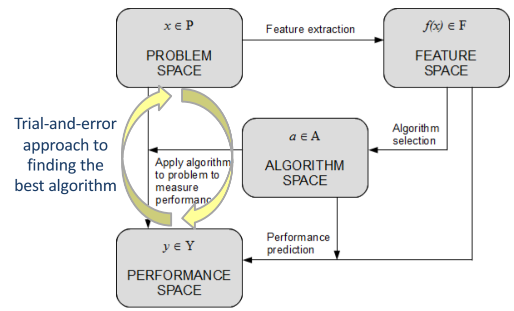
General algorithm selection problem (Rice, 1976)



Research areas:
- Feature extraction (characterising problems)
- Analysis and design of problems
- Understanding algorithm behaviour
- Performance prediction
- Algorithm selection

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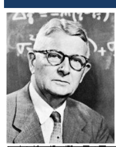
Wright’s fitness landscape



1889 - 1988

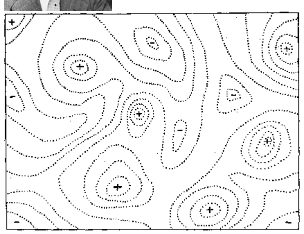


FIGURE 2—Diagrammatic representation of the field of gene combinations in two dimensions instead of many thousands. Dotted lines represent contours with respect to adaptiveness.

- Surface of selective values (Wright, 1932).
- No axes, units or labels.
- Commentary 56 years later:
  - *“useless for mathematical purposes”*
  - Aim: provide an intuitive picture of evolutionary processes taking place in higher dimensional space.

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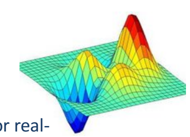
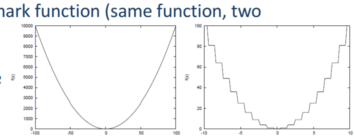
Fitness landscapes today

- We now have (useful) formalised mathematical models.
- Essential elements: search space, fitness function, notion of neighbourhood or accessibility.
- Intuitively, a fitness landscape is a visualisation of the terrain capturing how fitness changes between neighbouring solutions.
- Idea of “valleys”, “peaks”, “ridges”, “plateaus”, etc.
- One fitness function, many fitness landscapes (even for real-valued spaces).
- Example: Step benchmark function (same function, two different landscapes).



$$f(\mathbf{x}) = \sum_{i=1}^{D} (\lfloor x_i + 0.5 \rfloor)^2$$



(a) Larger domain (b) Smaller domain

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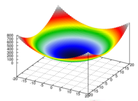
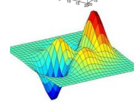
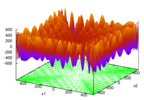
Fitness landscape characterisation

- To understand optimisation problems through analysis of search space in terms of the objective function landscape.
- When problems are simple, classical techniques could be more suitable.
- When are metaheuristics needed?
  - When objective functions do not have the structure required by classical techniques (e.g. uni-modality, differentiability).
  - When problem complexity is too large (classical techniques not feasible).
  - When there is no objective function in mathematical form.
  - Objective function exhibits noise or uncertainty.







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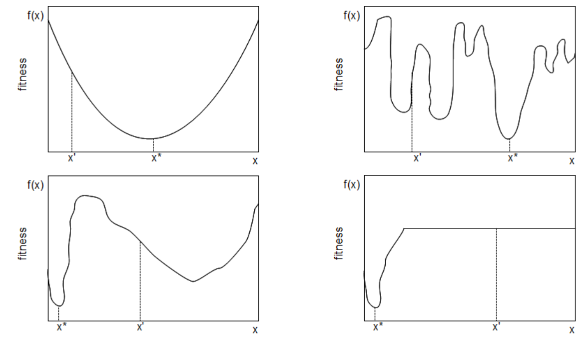
Features of fitness landscapes



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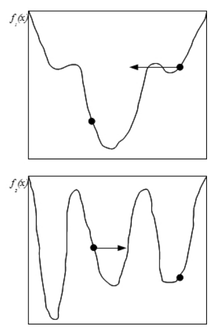
Features of fitness landscapes

- Modality (number of optima) is frequently referred to as affecting difficulty, but too simplistic.
- Example landscapes both with three optima.
- Top landscape: global basin is wider and deeper than local basins.
- Bottom landscape: global basin narrow and local basins deep.
- Consider simple PSO with 2 particles: top landscape not deceptive, bottom landscape is deceptive.
- Distribution & relative sizes of basins of attraction more important than modality.



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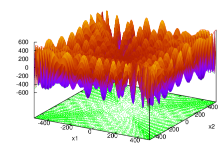
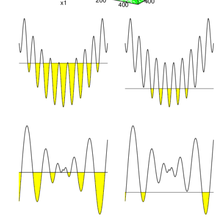
Features of fitness landscapes

- Ruggedness:
  - Quantifies changes in neighbouring fitness values (micro or macro scale).
- Global landscape structure (funnels)
  - Funnel: global basin shape of clustered local optima.
- Gradients:
  - Steepness of gradients measures the magnitude of neighbouring fitness changes.
- Neutrality:
  - Lack of neighbouring information to direct search.





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Some other fitness landscape lingo

- Epistasis
  - Comes from genetics: degree of dependency between genes for expression.
  - More general term: Variable interdependency / non-separability.
- Basins of attraction
  - The set of solutions that lead to the same local optimum via a hill climber / descender.
  - Boundary of a basin of attraction: those solutions in the basin that have at least one neighbour in a different basin.
  - Fitness barrier: minimum fitness value required to reach another optimum.
  - Central massif / Big valley structure (single funnel)
- Evolvability
  - The capacity to produce offspring that are fitter than their parents (`searchability' may be a more general term).
  - Only has meaning with reference to a particular search process.

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An introduction to some fitness landscape analysis techniques

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Autocorrelation

- Oldest and most widely used fitness landscape metric by Weinberger (1990) for measuring ruggedness.
- How it works:
  - Perform a random walk through the fitness landscape to obtain a sequence of fitness values.
  - From this sequence calculate the correlation with the same sequence of fitness values a small distance away.
  - Result 1: Plot of autocorrelation $\rho(s)$ against step size *s*.
  - Result 2: Correlation length (the distance beyond which the majority of points become uncorrelated: a smaller value indicates a more rugged landscape).
- Problems:
  - Assumes that the landscape is statistically isotropic.
  - Length metric assumes that the autocorrelation function is a decaying exponential.

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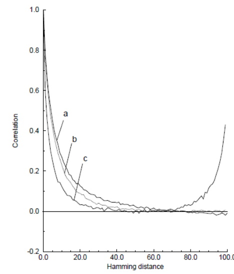
Autocorrelation: example applications

Investigating the landscapes of RNA folding using different alphabets (Fontana et al., 1993).



The role of representation on the multidimensional knapsack problem (Tavares et al., 2006)

TABLE II
SUMMARY RESULTS FOR AUTOCORRELATION ON ALL THE MKP INSTANCES FOR THE FIVE REPRESENTATIONS.

Representation	Instance	n	m	l	ξ
BR	P01	28	10	7.12	0.25
OR	P01	28	10	4.61	0.16
PR	P01	28	10	7.64	0.27
RK	P01	28	10	10.97	0.39
WC	P01	28	10	**14.51**	**0.52**
BR	P02	50	5	16.18	0.32
OR	P02	50	5	6.23	0.12
PR	P02	50	5	11.72	0.23
RK	P02	50	5	17.05	0.34
WC	P02	50	5	**19.43**	**0.39**
BR	CB01	100	5	**35.74**	**0.36**
OR	CB01	100	5	4.09	0.04
PR	CB01	100	5	7.04	0.07
RK	CB01	100	5	8.12	0.08
WC	CB01	100	5	10.85	0.11
BR	CB02	100	5	**36.65**	**0.37**
OR	CB02	100	5	4.78	0.05
PR	CB02	100	5	6.89	0.07
RK	CB02	100	5	9.22	0.09
WC	CB02	100	5	10.78	0.11
BR	GK01	100	15	**27.32**	**0.27**
OR	GK01	100	15	3.95	0.04
PR	GK01	100	15	8.37	0.08
RK	GK01	100	15	8.98	0.09
WC	GK01	100	15	9.33	0.09

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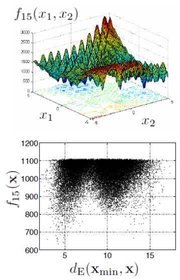
Fitness distance correlation

- Widely used technique introduced by Jones and Forrest (1995).
- How it works:
  - Random sample of points. Measure distance of each point to optimum to generate a set of fitness distance pairs.
  - Result 1: Scatterplot of fitness distance pairs.
  - Result 2: correlation value.
- Problems:
  - Requires knowledge of global optima.
  - Not very useful for algorithm performance prediction.

Characterisation of CEC 2005 benchmark suite (Müller and Sbalzarini, 2011).



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Entropic measures of ruggedness

- Introduced by Vassilev *et al.* (2003).
- How it works:
  - Based on a random walk, a sequence of three-point objects are generated.
  - Ruggedness is estimated using a measure of entropy with respect to the probability distribution of the rugged elements within the sequence.
  - Result: a measure in range [0,1].

Object shape	Classification	Encoding
	neutral	0 0
	rugged	0 1
	rugged	0 $\bar{1}$
	rugged	1 0
	smooth	1 1
	rugged	1 $\bar{1}$
	rugged	$\bar{1}$ 0
	rugged	$\bar{1}$ 1
	smooth	$\bar{1}$ $\bar{1}$

(Malan and Engelbrecht, 2009)

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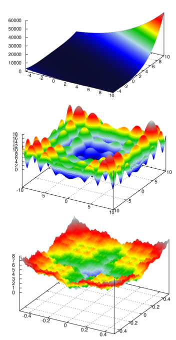
Entropic measures of ruggedness

- Vassilev's (2003) first entropic measure adapted for continuous spaces (Malan and Engelbrecht, 2009).
- First entropic measures of micro and macro ruggedness ($FEM_{0.01}$ and $FEM_{0.1}$).
- Values in range [0,1], where 0 is least rugged and 1 is the most rugged.

Function	Dim	$FEM_{0.01}$	$FEM_{0.1}$
Zakharov	2D	0.291	0.332
	10D	0.284	0.306
	30D	0.237	0.288
Alpine	2D	0.539	0.855
	10D	0.605	0.861
	30D	0.601	0.862
Weierstrass	2D	0.789	0.792
	10D	0.773	0.797
	30D	0.738	0.788



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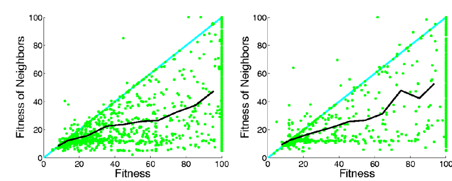
Fitness Cloud

- Fitness cloud (Verel et al. 2003) and negative slope coefficient (Vanneschi et al. 2004): evolvability with reference to a particular search operator.
- How it works:
  - Obtain a sample of solutions from the search space (the parents).
  - Choose a good neighbour of each solution in the sample (the offspring).
  - Fitness cloud: scatter plot of fitness values of parents against offspring.
  - Negative slope coefficient (nsc): Partition the fitness cloud into bins, nsc is the sum of negative slopes of line segments between centroids of adjacent bins.



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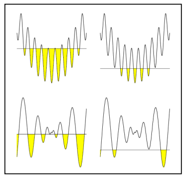
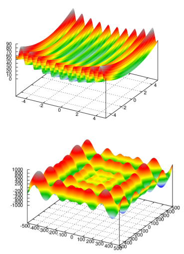
Dispersion metric

- Funnel: global basin shape (local optima form a basin).
- Dispersion metric (Lunacek & Whitley 2006) predicts the presence of funnels.
- How it works:
  - Dispersion of all points in a sample subtracted from the dispersion of a subset of the best points.
  - Adapted to work with normalised distances.
  - A positive DM value is indicative of multiple funnels.





Function	Dim	DM
Rastrigin	2D	-0.224
	10D	-0.245
Schwefel 2.26	2D	0.035
	10D	0.021

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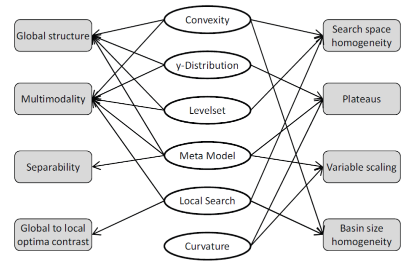
Exploratory landscape analysis (ELA)

- ELA (Mersmann et al. 2011): many simple, low-level features based on a fairly small sample of points from the search space of continuous problems.
- Six low-level feature classes (convexity, y-distribution, etc.) with 50 sub-features.
- Implemented in an R-package called flacco (Kerschke & Trautmann 2016).
- Also see later tutorial on "Exploratory Landscape Analysis".



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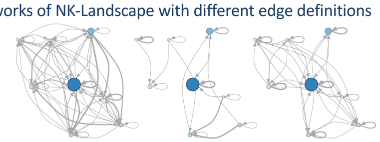
Local optima networks (LON)

- LON (Ochoa et al. 2008): technique for compressing the essential landscape features for combinational optimisation problems in a graph.
- How it works:
  - Run a best-improvement local search to find local optima.
  - Vertices of the LON are local optima and edges between optima indicate that basins are adjacent / chances of escaping the optima (Verel et al. 2012).
  - Statistics are used to characterise the LON.

Local optima networks of NK-Landscape with different edge definitions (Verel et al. 2012)



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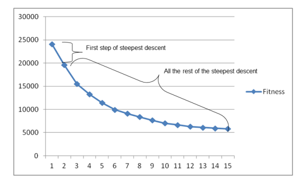
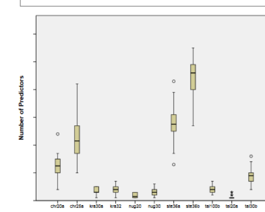
Predictive diagnostic optimisation

- Diagnostic optimisation: combines fitness landscape diagnostics with optimisation.
- Predictive diagnostic optimisation (PDO) for discrete problems (Moser and Gheorghita, 2012).
- How it works:
  - Start with a random solution and perform steepest descent (SD).
  - Calculate ratio of improvement achieved by first step to improvement achieved after the full SD (called a predictor).
  - The number of different predictors is an indicator of the distribution of the basin shapes of the landscape.





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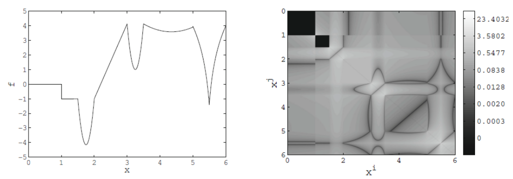
Length scale

- Length scale: characterisation of continuous problems (Morgan and Gallagher, 2012).
- How it works:
  – A random Levy walk is used to sample points in the search space.
  – The length scale between points is calculated as the change in fitness value with respect to distance in decision space.
  – Length scale distribution is defined as the probability density function of the length scale and information entropy is used to summarise the probability density function.



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Recent contributions

- Focus on correlation with
  – algorithm performance
  – algorithm selection
  – algorithm tuning

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FLC for VRP

- Analysed in five studies:
  – Czech 2008
  – Kubiak 2009
  – Runka, Ombuki-Berman and Ventresca 2009
  – Pitzer, Vonolfen, Beham, Affenzeller, Bolshakov, and Merkuryeva 2012
  – Ventresca, Ombuki-Berman and Runka 2013

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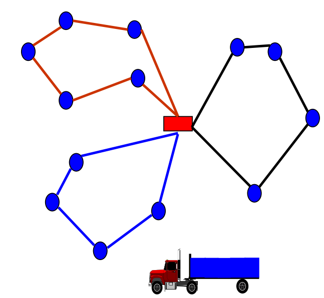
Vehicle routing problem



Time Windows

Repeated Trips

Legal Work Hours

Capacitated

Loading Constraints

Heterogeneous Fleet

Some instances from: w.cba.neu.edu/~msolomon/problems.htm

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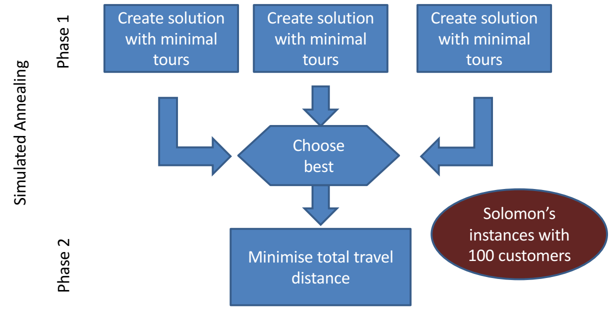
Simulated annealing for VRPTW

- Czech 2008



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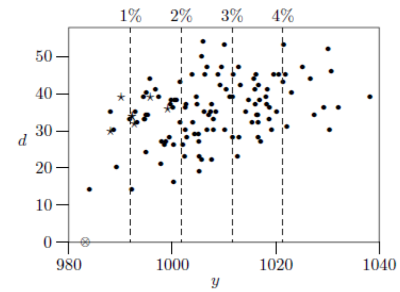
Solutions found by SA

- Czech 2008
- Approach: Probabilities of discovering solutions



Solomon's R112

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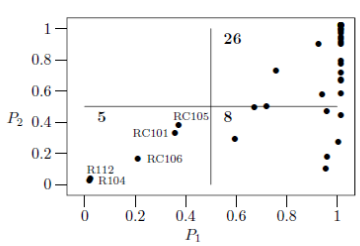
Probabilities of finding quality

- Czech 2008



Instance	Error
R211	3.3
R112	2.4
R110	1.6
R108	1.2
R107	0.9
R111	0.9
R109	0.8
R106	0.6
R103	0.6
R104	0.5
R102	0.4
R105	0.2
R101	0.2

Error $= \frac{mean - min}{min}$

P_1 = prob that 1 solution with min tours is found
P_2 = prob that min tours and 1% of shortest is found
P_3 = prob that best-known is found

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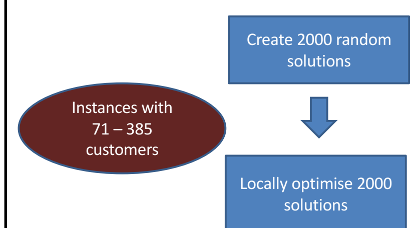
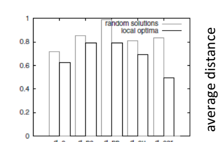
Distance analysis

- Kubiak 2009



Distances:
- edge-based
- tour-based
- tour-edge-based
- tour-edit
- tour-add-remove



Benchmark instances:http://neo.lcc.uma.es/vrp/vrp-instances/capacitated-vrp-instances/

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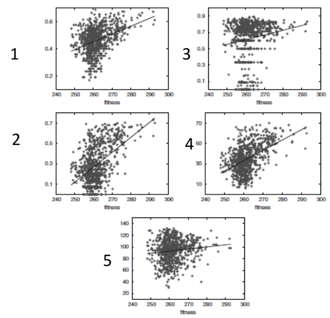
Fitness-distance analysis

- Kubiak 2009



Distances:
1. edge-based
2. tour-based
3. tour-edge-based
4. tour-edit
5. tour-add-remove

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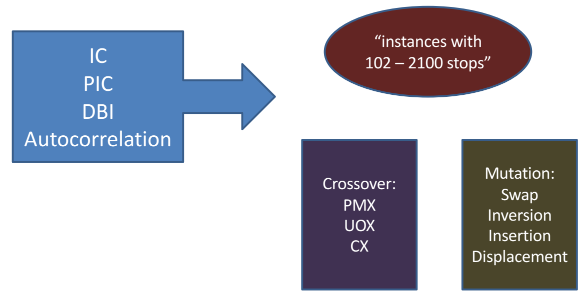
Waste collection VRPTW

- Runka, Ombuki-Berman and Ventresca 2009



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Waste collection VRPTW, results

- Runka, Ombuki-Berman and Ventresca 2009
  - "swap and insertion operators yield smoother landscapes"
    - "does not mean they are superior"
  - "relatively rugged landscapes of the inversion and displacement operators indicate a higher likelihood of skipping over an optimum, but should allow for slower convergence."
  - "Crossovers are destructive"
  - Suggestion to combine or alternate between operators

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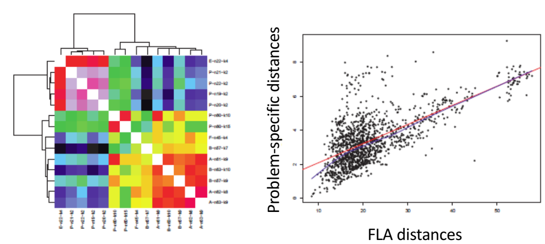
FLA and problem-specific measures

- Pitzer, Vonolfen, Beham, Affenzeller, Bolshakov, and Merkuryeva 2012



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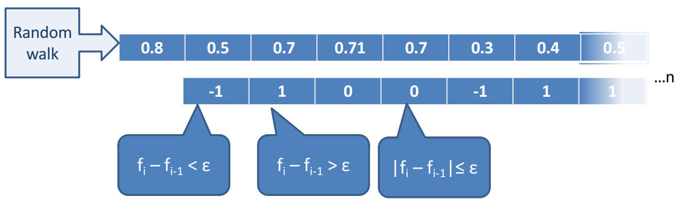
Information content

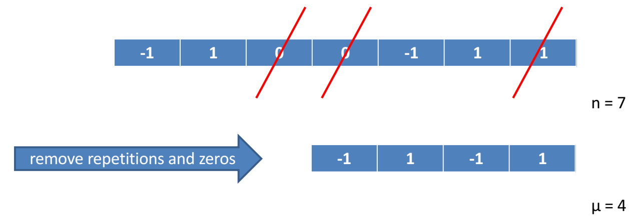


Calculate the probability of two different contiguous items :

$$IC(\varepsilon) = -\sum_{p \neq q} P_{pq} \log_6 P_{pq}$$

$$P_{-1,1} = \frac{|-1,1|}{n}, \quad P_{-1,0} = \frac{|-1,0|}{n} \Rightarrow P_{pq} = \frac{|p,q|}{n}$$

Partial information content



Modality: $M(\varepsilon) = \frac{\mu}{n}$

Density-basin information

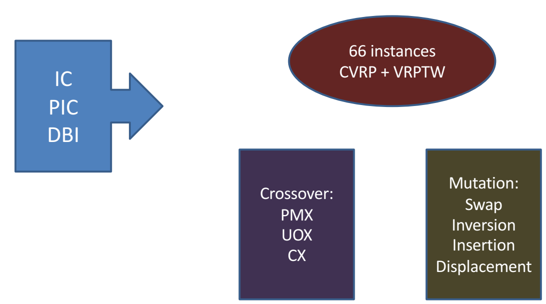
Calculate the probability of two identical contiguous items :

$$\text{DBI}(\varepsilon) = -\sum_{pp} P_{pp} \log_3 P_{pp}$$

$$P_{1,1} = \frac{|1,1|}{n}, \quad P_{0,0} = \frac{|0,0|}{n} \quad\quad P_{pp} = \frac{|p,p|}{n}$$

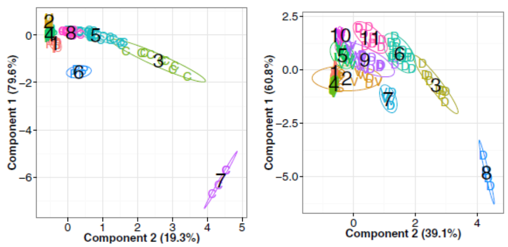
Information content and GA

- Ventresca, Ombuki-Berman and Runka 2013



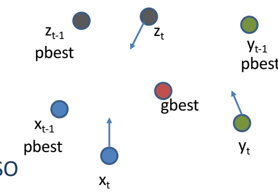
Information content and GA, results

- Ventresca, Ombuki-Berman and Runka 2013
- Results:
  - Clustering by indicators creates almost disjoint clusters of operators
  - There is significant overlap between performance clusters and indicator-based clusters.



Failure prediction for PSO

- Malan and Engelbrecht 2014
  - gbest PSO
  - cognitive PSO
  - social PSO
  - local best PSO
  - asynch global best PSO
  - bare bones PSO
  - modified bare bones PSO

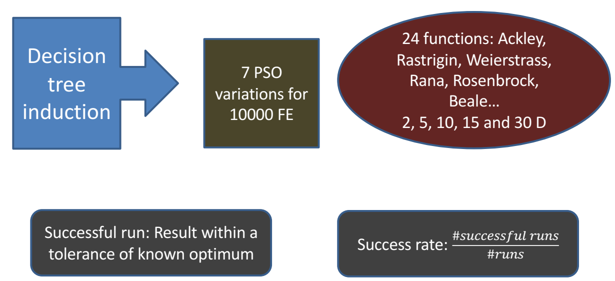


Landscape metrics

- Malan and Engelbrecht 2014
  - Ruggedness (information content)
    - after Vassilev, Fogarty and Miller 2003
  - Dispersion metric for funnel detection
    - by Lunacek and Whitley 2006
  - Gradient measures: average and standard deviation
    - Malan and Engelbrecht 2013
  - Fitness Distance Correlation
    - Jones and Forrest 1995
  - Fitness Cloud Index
    - after Verel, Collard and Clergue 2003

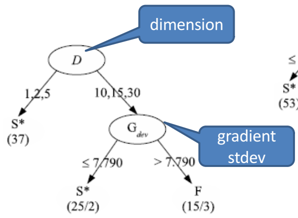
Predicting algorithm failure

- Malan and Engelbrecht 2014

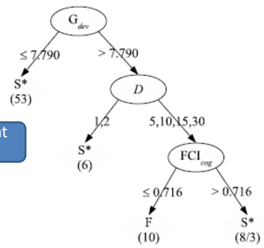


Failure prediction model

- Malan and Engelbrecht 2014



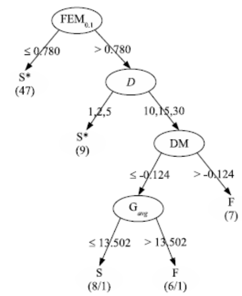
gbest PSO failure prediction



lbest PSO failure prediction

Results

- Malan and Engelbrecht 2014



asynchronous gbest PSO failure prediction

- Easy to identify what makes a problem hard for each variation of PSO
- The most distinguishing metrics tend to be different for the variations.
- All metrics are needed (and possibly more) to identify the causes that stop the algorithm from succeeding.

LON and ILS, NK landscapes

- Daolio, Verel, Ochoa and Tomassini 2012
  - LON-related variables:
    - # local optima
    - average path length to GO
    - average path length between optima
    - NN fitness correlations
    - # self-loops (basin size)
    - clustering coefficient
    - average out-degree of lo
    - average weight disparity of outgoing edges
    - NN degree correlation

Performance:

$$\frac{1-success\ rate}{success\ rate} * \max FE$$

$$+FE(\#successful\ runs)$$

N = 18
K = 2, 4, 6, 8, 10, 12, 14, 16, 17

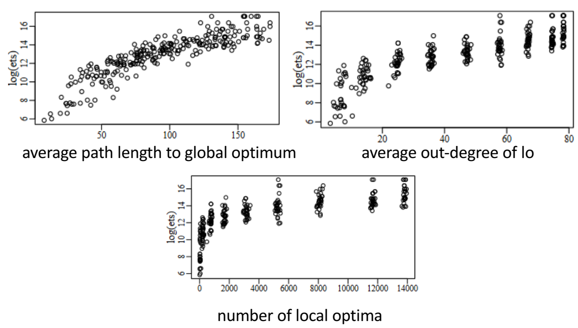
LON metric correlations

- Daolio, Verel, Ochoa and Tomassini 2012

Pearson	Measure
0.5	# local optima
0.52	average path length to global optimum
0.09	average path length between optima
-0.4	NN fitness correlations
-0.4	# self-loops (basin size)
-0.27	clustering coefficient
0.45	average out-degree of lo
-0.3	average weight disparity of outgoing edges
-0.24	NN degree correlation

LON metric correlation plots

- Daolio, Verel, Ochoa and Tomassini 2012



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LON and regression analysis

- Daolio, Verel, Ochoa and Tomassini 2012
  - Linear regression model
    - average path length to GO
    - average out-degree of lo
    - average weight disparity of outgoing edges
    - NN degree correlation

    in combination explain 84% of the variance in the success rate.

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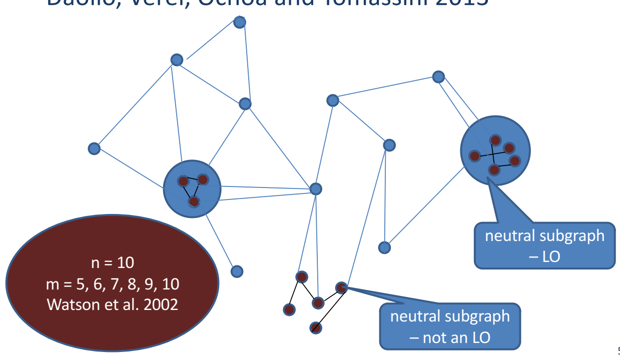
LON and ILS, PFSP

- Daolio, Verel, Ochoa and Tomassini 2013



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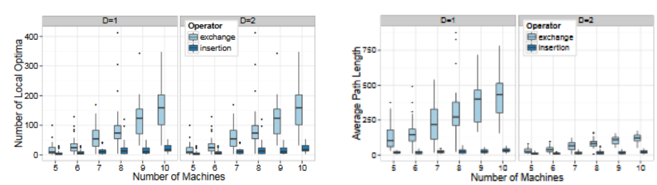
Local optima

- Daolio, Verel, Ochoa and Tomassini 2013



Operators:
Swap (x2)
Insertion (x1)

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LON and ILS, PFSP, results

- Daolio, Verel, Ochoa and Tomassini 2013

1-ins	2-ins	1-exch	2-exch	Measure
0.46	0.54	0.54	0.41	# local optima
0.63	0.69	0.62	0.53	average path length to global optimum
0.40	0.45	0.54	0.35	average path length between optima
0.20	0.32	-0.00	0.22	NN fitness correlations
-0.31	-0.48	-0.24	-0.27	# self-loops (basin size)
-0.22	-0.21	-0.51	-0.26	clustering coefficient
0.48	0.55	0.45	0.41	average out-degree of lo
-0.41	-0.46	-0.47	-0.43	average weight disparity of outgoing edges
0.08	-0.17	0.14	-0.11	NN degree correlation

Performance: $\frac{1-success\ rate}{success\ rate} * \max FE + FE(\#successful\ runs)$

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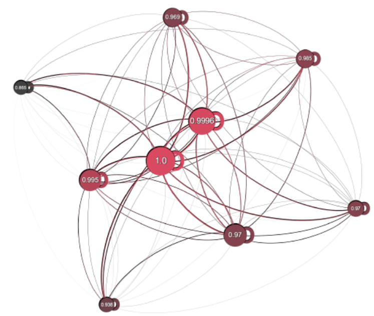
LON and PageRank Centrality

- Herrmann and Rothlauf 2015



$$\pi_j = \sum_i \pi_i \frac{1}{q_i}$$

π_i = rank of page i

q_i = # links on page i

(Franceschet 2011)

also:
NN fitness correlation
FDC to global optimum

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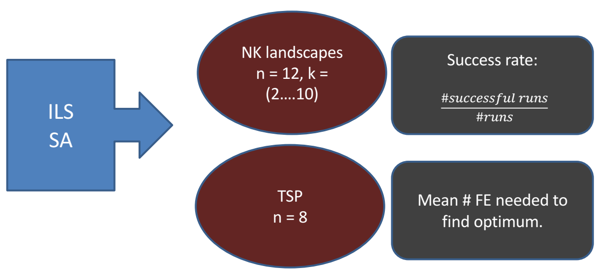
LON and PageRank, experiments

- Herrmann and Rothlauf 2015



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LON and PageRank, results

- Herrmann and Rothlauf 2015

NK	ILS		SA	
	success	# FE	success	# FE
NN fitness correlation	0.48	0.14	0.59	0.37
FDC	0.37	0.10	0.54	0.31
PageRank	0.91	0.31	0.92	0.54

TSP	ILS		SA	
	success	# FE	success	# FE
NN fitness correlation	0.006	0.003	0.001	0.001
FDC	0.11	0.043	0.273	0.001
PageRank	0.757	0.605	0.646	0.338

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References

Z.J. Czech (2008), "Statistical measures of a fitness landscape for the vehicle routing problem", In: IEEE International Symposium on Parallel and Distributed Processing, pp. 1-8.

F. Daolio, S. Verel, G. Ochoa, M. Tomassini (2012), "Local optima networks and the performance of iterated local search", In Proceedings of the 14th annual conference on Genetic and evolutionary computation, pp. 369-376.

F. Daolio, S. Verel, G. Ochoa, M. Tomassini (2013), "Local optima networks of the permutation flow-shop problem", In International Conference on Artificial Evolution (Evolution Artificielle), pp. 41-52.

W. Fontana, P.F. Stadler, E.G. Bornberg-Bauer, T. Griesmacher, I.L. Hofacker, M. Tacker, P. Tarazona, E.D. Weinberger and P. Schuster (1993), "RNA Folding and Combinatory Landscapes", Physical Review E, 47:2083–2099.

M. Franceschet (2011), "PageRank: standing on the shoulders of giants", Communications of the ACM, Vol. 54(6):92.

S. Herrmann, F. Rothlauf (2015), "Predicting heuristic search performance with pageRank centrality in local optima networks", In Proceedings of the 2015 Annual Conference on Genetic and Evolutionary Computation, pp. 401-408.

T. Jones and S. Forrest (1995), "Fitness Distance Correlation as a Measure of Problem Difficulty for Genetic Algorithms", In Proceedings of the Sixth International Conference on Genetic Algorithms, pp. 184–192.

P. Kerschke and H Trautmann (2016), "The R-Package FLACCO for Exploratory Landscape Analysis with Applications to Multi-Objective Optimization Problems", In Proceedings of 2016 IEEE Congress on Evolutionary Computation, pp. 5262-5269.

M. Kubiak (2007), "Distance Measures and Fitness-Distance Analysis for the Capacitated Vehicle Routing Problem", In: Metaheuristics: Progress in Complex Systems Optimization, Operations Research/Computer Science Interfaces, vol. 39, pp. 345-364.

57

References

M. Lunacek and D. Whitley (2006), "The dispersion metric and the CMA evolution strategy", In Proceedings of the 8th Annual Genetic and Evolutionary Computation Conference, pp. 477–484.

K.M. Malan and A.P. Engelbrecht (2009), "Quantifying Ruggedness of Continuous Landscapes using Entropy", In Proceedings of the IEEE Congress on Evolutionary Computation, pp. 1440–1447.

K.M. Malan and A.P. Engelbrecht (2013), "Ruggedness, Funnels and Gradients in Fitness Landscapes and the Effect on PSO Performance", in Proceedings of the IEEE Congress on Evolutionary Computation, Cancun, Mexico, pp. 963-970.

K.M. Malan and A.P. Engelbrecht (2014), "Particle swarm optimisation failure prediction based on fitness landscape characteristics," 2014 IEEE Symposium on Swarm Intelligence, Orlando, FL, pp. 1-9.

O. Mersmann, B. Bischl, H. Trautmann, M. Preuss, C. Weihs and G. Rudolph (2011), "Exploratory landscape analysis", In Proceedings of the 13th annual conference on Genetic and Evolutionary Computation, pp. 829-836.

R. Morgan and M. Gallagher (2012), "Length Scale for Characterising Continuous Optimization Problems", In Parallel Problem Solving from Nature, Lecture Notes in Computer Science, Vol 7491, Springer, pp. 407-416.

I. Moser and M. Gheorghita (2012), "Combining Search Space Diagnostics and Optimisation", in 2012 IEEE Congress on Evolutionary Computation, pp. 1-8.

C.L Müller and I.F. Sbalzarini (2011), "Global characterization of the CEC 2005 fitness landscapes using fitness-distance analysis", In Proceedings of the 2011 International Conference on Applications of Evolutionary Computation – Part I, pp. 294–303.

G. Ochoa, M. Tomassini, S. Verel and C. Darabos (2008), "A Study of NK Landscapes' Basins and Local Optima Networks", In Proceedings of the 10th Annual Conference on Genetic and Evolutionary Computation, pp. 555-562.

58

References

E. Pitzer, M. Affenzeller, A. Beham, S. Wagner (2011), "Comprehensive and automatic fitness landscape analysis using heuristiclab", In Proceedings of the 13th International Conference on Computer Aided Systems Theory, pp. 424-431.

E. Pitzer, S. Vonolfen, A. Beham, M. Affenzeller, V. Bolshakov, and G. Merkuryeva (2012), "Structural Analysis of Vehicle Routing Problems using General Fitness Landscape Analysis and Problem Specific Measures", In Proceedings of 1st Austalian Conference on the Application of Systems Engineering, Sydney, pp. 36-38.

A. Runka, B. Ombuki-Berman, M. Ventresca (2009), "A search space analysis for the waste collection vehicle routing problem with time windows", In Proceedings of the 11th Annual conference on Genetic and evolutionary computation, pp. 1813-1814.

J. Tavares, F.B. Pereira and E. Costa (2006), "The role of representation on the multidimensional knapsack problem by means of fitness landscape analysis", In Proceedings of the 2006 IEEE Congress on Evolutionary Computation, pp. 2307–2314.

L. Vanneschi, M. Clergue, P. Collard, M. Tomassini, and S. Verel (2004). "Fitness Clouds and Problem Hardness in Genetic Programming", In Proceedings of Genetic and Evolutionary Computation Conference, volume 3103 of Lecture Notes in Computer Science, pp. 690–701. Springer Berlin Heidelberg.

V.K. Vassilev, T.C. Fogarty and J.F. Miller (2003), "Smoothness, Ruggedness and Neutrality of Fitness Landscapes: from Theory to Application", In Advances in Evolutionary Computing: Theory and Applications, Springer-Verlag New York, Inc.,pp. 3–44.

M. Ventresca, B. Ombuki-Berman, A. Runka (2013), "Predicting Genetic Algorithm Performance on the Vehicle Routing Problem Using Information Theoretic Landscape Measures", In: Middendorf M., Blum C. (eds) Evolutionary Computation in Combinatorial Optimization, EvoCOP 2013, Lecture Notes in Computer Science, vol 7832.

S. Verel, P. Collard and M. Clergue (2003), "Where are bottlenecks in NK fitness landscapes?", In the 2003 Congress on Evolutionary Computation, pp. 273 - 280.

59

References

S. Verel, F. Daolio, G. Ochoa and M. Tomassini (2012), "Local optima networks with escape edges", In Proceedings of the International Conference on Artificial Evolution, volume 7401 of Lecture Notes in Computer Science, pp. 49-60, Springer.

J. Watson, L. Barbulescu, L. Whitley, A. Howe (2002), "Contrasting structured and random permutation flow-shop scheduling problems: search-space topology and algorithm performance", INFORMS Journal on Computing 14(2), pp. 98-123.

E. Weinberger (1990), "Correlated and Uncorrelated Fitness Landscapes and How to Tell the Difference", Biological Cybernetics, 63(5):325–336.

S. Wright (1932), "The Roles of Mutation, Inbreeding, Crossbreeding, and Selection in evolution", In Proceedings of the Sixth International Congress on Genetics, pp. 356–366.

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