

# Exploratory Landscape Analysis

– Advanced Tutorial at GECCO 2017 –



Pascal Kerschke



Information Systems & Statistics,  
University of Münster, Germany



Mike Preuss

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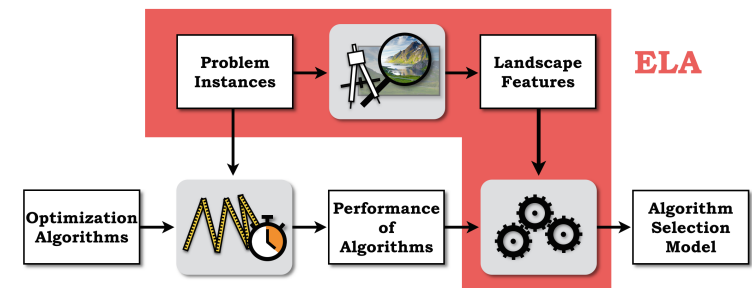
## General Idea of Exploratory Landscape Analysis

## Agenda

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- 5 Closing

## General Idea of ELA

- algorithm selection problem<sup>1</sup>
  - ↪ find the individually best suited algorithm for an unseen optimization problem



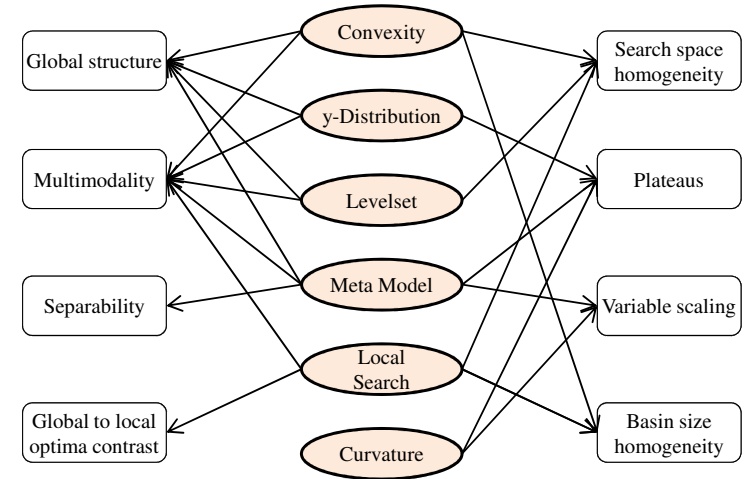
<sup>1</sup>Rice, J. (1976). *The Algorithm Selection Problem*. In *Advances in Computers* (pp. 65-118).

## General Idea of ELA

### Exploratory Landscape Analysis (ELA):

- we aim at finding the right algorithm
- but also at improving problem or algorithm/problem dependency understanding
- basic idea (exploratory!): we start with **very simple features without clear purpose**
- match existing high-level features (expert knowledge) with our ELA features
- currently: mostly continuous (black-box) (global) optimization, but also in other domains (e.g. TSP)

## General Idea of ELA



Mersmann, O., Preuss, M. & Trautmann, H. (2010). *Benchmarking Evolutionary Algorithms: Towards Exploratory Landscape Analysis*. In Proceedings of PPSN XI (pp. 71 - 80).

## General Idea of ELA

- we do not know functional relationships when designing features
- but we can match them to high-level characteristics (multimodality, funnel structure, etc.) of optimization problems
- this enables recognizing important problem properties quickly
- based on initial design of samples  $x_{i1}, \dots, x_{iD}$  and their corresponding fitness value  $y_i$ ,  $i = 1, \dots, n$
- given an evaluated initial design (initial population?), most ELA features are for free
- there are already several different feature sets

## General Idea of ELA

### What is the difference to Fitness Landscape Analysis (FLA)?

- basic idea is similar: extract knowledge on problems in order to select proper algorithm
- however, our viewpoint is always set-based: no single feature has to explain anything on its own, the combination is important
- we heavily rely on Machine Learning for composing good feature sets
- additionally, we strive for (very) small sample sizes, in the range of initial generation samples (100D points and less)

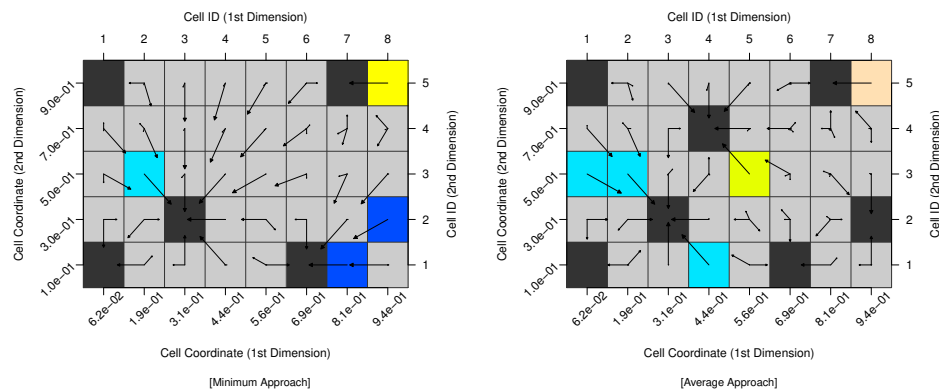
# A Brief Introduction into the FLACCO-GUI

# ELA for Single-Objective Global Optimization Problems

## Block 1 - Part II: Single-Objective Global Opt.

### General Cell Mapping Features

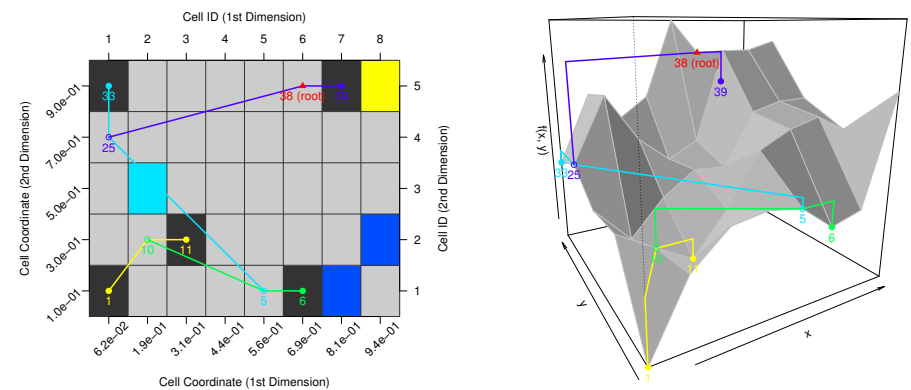
Kerschke, P., Preuss, M., Hernández, C., Schütze, O., Sun, J.-Q., Grimme, C., Rudolph, G., Bischl, B. & Trautmann, H. (2014). *Cell Mapping Techniques for Exploratory Landscape Analysis*. In Proceedings of EVOLVE 2014 (pp. 115 – 131).



## Block 1 - Part II: Single-Objective Global Opt.

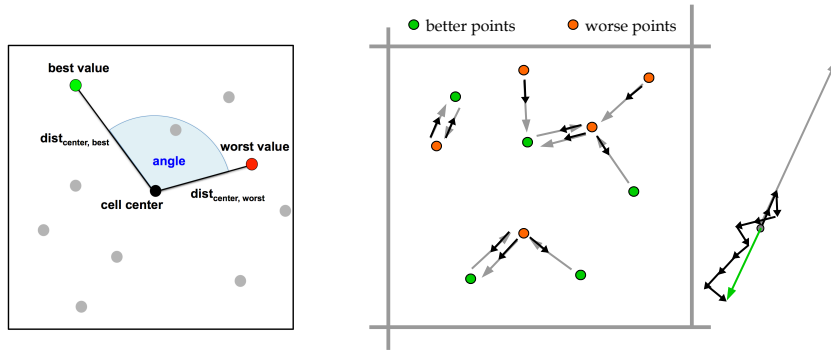
### Barrier Tree Features

Hernández, C., Schütze, O., Emmerich, M. T. M., & Xiong, F. R. (2014). *Barrier Tree for Continuous Landscapes by Means of Generalized Cell Mapping*. In Proceedings of EVOLVE 2014.



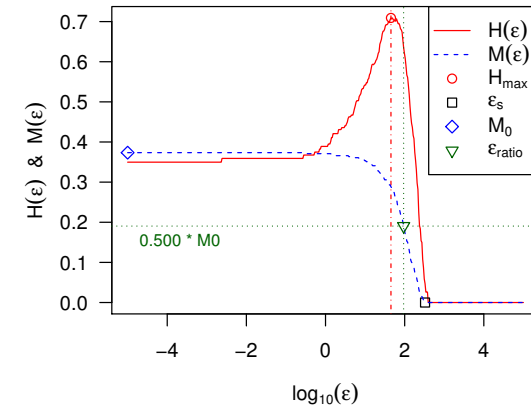
## Cell Mapping Features

Kerschke, P., Preuss, M., Hernández, C., Schütze, O., Sun, J.-Q., Grimme, C., Rudolph, G., Bischl, B. & Trautmann, H. (2014). *Cell Mapping Techniques for Exploratory Landscape Analysis*. In Proceedings of EVOLVE 2014 (pp. 115 - 131).



## Information Content Features

Muñoz, M. A., Kirley, M., Halgamuge, S. K. (2015). *Exploratory Landscape Analysis of Continuous Space Optimization Problems using Information Content*. In IEEE Transactions on Evolutionary Computation (pp. 74 - 87).



## Dispersion Features

Lunacek, M. & Whitley, D. (2006). *The Dispersion Metric and the CMA Evolution Strategy*. In Proceedings of GECCO 2006 (pp. 477 - 484).

## Length Scale Features

Morgan, R. & Gallagher M. (2015). *Analyzing and Characterising Optimization Problems Using Length Scale*. In Soft Computing (pp. 1 - 18).

## Ruggedness Features

Malan, K. M. & Engelbrecht, A. P. (2013). *Ruggedness, Funnels and Gradients in Fitness Landscapes and the Effect on PSO Performance*. In Proceedings of CEC 2013 (pp. 963 - 970).

## Fitness Distance Correlation Features

Jones, T. & Forrest, S. (1995). *Fitness Distance Correlation as a Measure of Problem Difficulty for Genetic Algorithms*. In Proceedings of ICGA 1995 (pp. 184 - 192).

## Violation Landscape Features

Malan, K. M., Oberholzer, J. F. & Engelbrecht, A. P. (1995). *Characterising Constrained Continuous Optimisation Problems*. In CEC 1995 (pp. 1351 - 1358).

## Hill Climbing Features

Abell, T., Malitsky, Y. & Tierney, K. (2013). *Features for Exploiting Black-Box Optimization Problem Structure*. In Proceedings of LION 2013 (pp. 30 - 36).

## Block 1 - Part II: Single-Objective Global Opt.

### Nearest Better Clustering Features

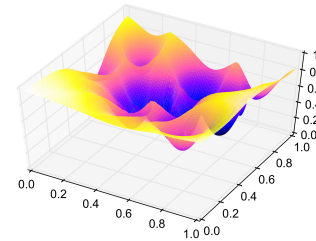
Kerschke, P., Preuss, M., Wessing, S. & Trautmann H. (2015). *Detecting Funnel Structures by Means of Exploratory Landscape Analysis*. In Proceedings of GECCO 2015 (pp. 265 – 272).

Meta-Model & NBC Features  $\rightsquigarrow$  Funnel Detection

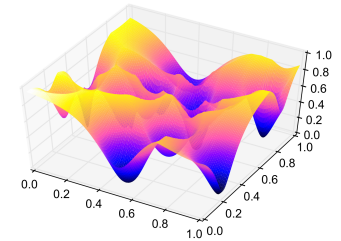
## Block 1 - Part II: Single-Objective Global Opt.

### Funnel Detection

- funnel: local optima are located near to each other and pile up to an “upside-down mountain”
- knowledge about underlying global structure, i.e., funnels, helps selecting the right algorithm



(a) funnel



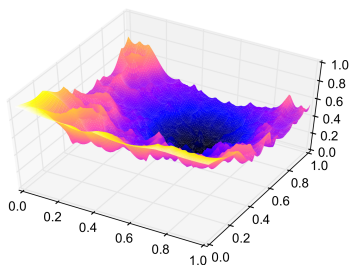
(b) non-funnel (“random”)

Kerschke, P., Preuss, M., Wessing, S. & Trautmann H. (2016). Low-Budget Exploratory Landscape Analysis on Multiple Peaks Models. In Proceedings of GECCO 2016 (pp. 229-236)

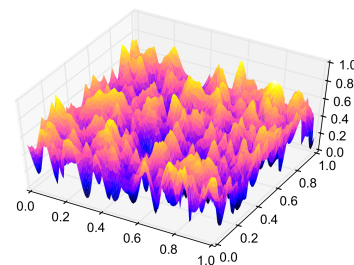
## Block 1 - Part II: Single-Objective Global Opt.

### Funnel Detection

- different algorithm candidates for either category
- wide variety within the classes “funnel” and “non-funnel”



(a) funnel



(b) non-funnel (“random”)

Kerschke, P., Preuss, M., Wessing, S. & Trautmann H. (2016). Low-Budget Exploratory Landscape Analysis on Multiple Peaks Models. In Proceedings of GECCO 2016 (pp. 229-236)

## Block 1 - Part II: Single-Objective Global Opt.

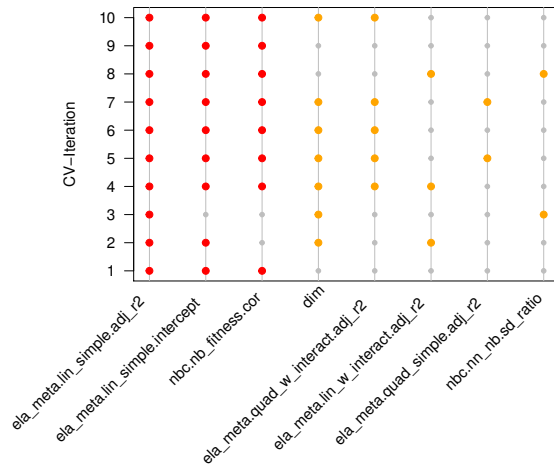
### Funnel Detection

- detailed results in our GECCO 2016 paper
- used MPM2<sup>2</sup> to generate a set of 4,000 training instances
- initial designs of size  $50 \times D$  observations (small!)
- trained four classifiers (random forest, rpart, kkn and ksvm)
- only used a total of 8 Meta-Model and NBC features
- validated results on BBOB and subset of problems from CEC-2013 niching competition

<sup>2</sup>multiple peaks model 2 generator, available in python (optproblems0.9, Wessing, S.) and R (smoof, Bossek, J.)

## Block 1 - Part II: Single-Objective Global Opt.

### Funnel Detection



Kerschke, P., Preuss, M., Wessing, S. & Trautmann H. (2016). Low-Budget Exploratory Landscape Analysis on Multiple Peaks Models. In Proceedings of GECCO 2016 (pp. 229-236)

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## Block 1 - Part III

# Introduction into FLACCO and its GUI

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## Block 1 - Part III: FLACCO and its GUI

- flacco: **F**eature-Based **L**andscape **A**nalysis of **C**ontinuous and **C**onstraint **O**ptimization **P**roblems
- unified interface for multiple (single-objective) sets of configurable features
- stable release on CRAN<sup>3</sup> / developers version on GitHub<sup>4</sup>
- multiple visualisation techniques (partially shown on these slides)
- tracks # of function evaluations and run time - per feature set

<sup>3</sup>Stable Release: <https://cran.r-project.org/package=flacco>

<sup>4</sup>Developers Version: <https://github.com/kerschke/flacco>

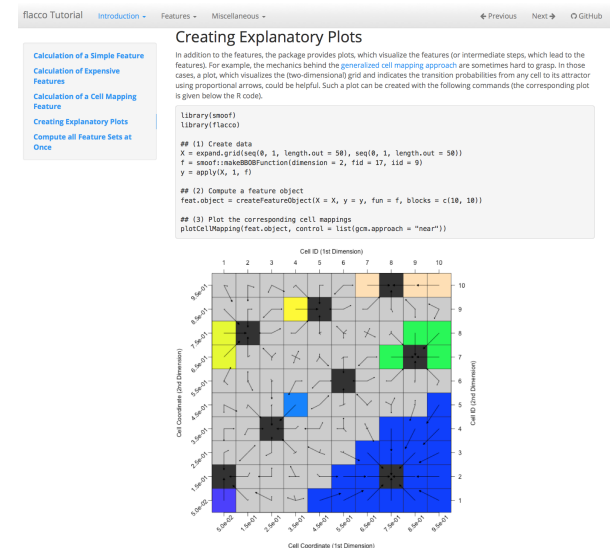
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## Block 1 - Part III: FLACCO and its GUI



Tutorial: <http://kerschke.github.io/flacco-tutorial/site/>

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## Block 1 - Part III: FLACCO and its GUI

- flacco also comes with a GUI, which provides many functionalities of the package itself
- the GUI can be started (within R) using the commands below:

```
> # first, install "flacco" from CRAN
> install.packages("flacco", dependencies = TRUE)
>
> # then, load the package and start the app
> library(flacco)
> runFlaccoGUI()
```

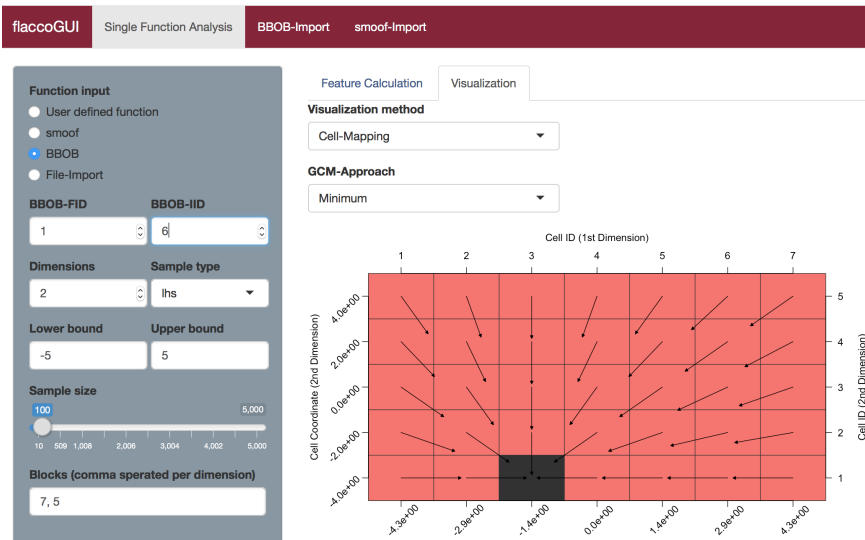
- alternatively, one can completely avoid the usage of R and use the online version of the GUI, which can be found here:  
<https://flaccogui.shinyapps.io/flaccogui>

## Block 1 - Part III: FLACCO and its GUI

Feature	Value
ela_meta.lin_simple.adj_r2	0.98
ela_meta.lin_simple.intercept	178.65
ela_meta.lin_simple.coef.min	34.87
ela_meta.lin_simple.coef.max	43.22
ela_meta.lin_simple.coef.max_by_min	1.24
ela_meta.lin_w_interact.adj_r2	0.98
ela_meta.quad_simple.adj_r2	0.99
ela_meta.quad_simple.cond	2.95
ela_meta.quad_w_interact.adj_r2	1.00
ela_meta.costs_fun_evals	0.00
ela_meta.costs_runtime	0.01

<https://flaccogui.shinyapps.io/flaccogui/>

## Block 1 - Part III: FLACCO and its GUI



<https://flaccogui.shinyapps.io/flaccogui/>

## Block 1 - Part III: FLACCO and its GUI

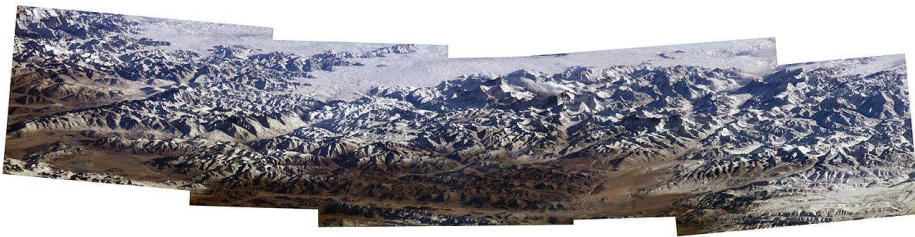
Further information on flacco and/or its GUI can be found here:

- GECCO 2017 workshop paper:  
Hanster, C. & Kerschke, P. (2017). *flaccogui: Exploratory Landscape Analysis for Everyone*. In Proceedings of GECCO 2017.
- presentation at the EvoSoft-Workshop at GECCO 2017:  
<http://dev.heuristiclab.com/trac.fcgi/wiki/EvoSoft>
- CEC 2016 paper:  
Kerschke, P. & Trautmann, H. (2016). *The R-Package FLACCO for Exploratory Landscape Analysis with Applications to Multi-Objective Optimization Problems*. In Proceedings of CEC 2016.

## Live-Session Using FLACCO and its GUI

## ELA for Single-Objective Multimodal Optimization Problems

### Block 2: Single-Objective Multimodal Optimization



### Block 2: Single-Objective Multimodal Optimization

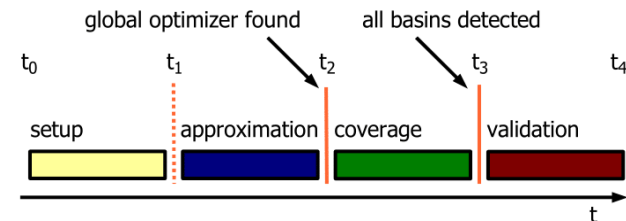
- core difference: we are looking for solution sets, not for one optimal solution
- sample definition:  
“In a multimodal optimization task, the main purpose is to find multiple optimal solutions (global and local), so that the user can have a better knowledge about different optimal solutions in the search space and as and when needed, the current solution may be switched to another suitable optimum solution.”  
(from Deb, Saha: Multimodal Optimization Using a Bi-Objective Evolutionary Algorithm, ECJ, 2012)
- many things are fuzzy here

## Block 2: Single-Objective Multimodal Optimization

- different aims possible
- currently most important (competitions): multiglobal  
= find all search space points that are globally optimal
- two main algorithmic approaches:
  - parallel, large populations
  - sequential, coordinated restarts
- several components that may be used: archives, clustering methods, methods for obtaining well distributed samples
- ELA could be helpful for selecting components/methods

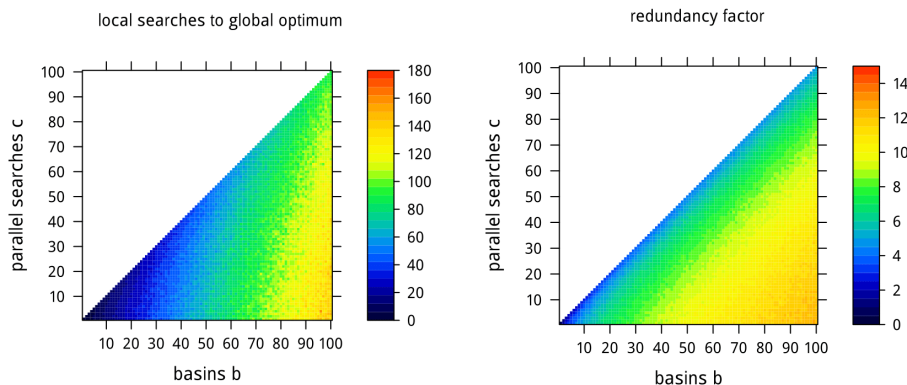
## Block 2: Single-Objective Multimodal Optimization

- funnel detection is important, because many methods need to partition space into basins
- for two-stage methods, we know that **restart organization** does not make much sense for global optimization
- but it does for multimodal optimization (because we have to look “everywhere”, Preuss: Multimodal Optimization by Means of Evolutionary Algorithms, Springer 2015)



## Block 2: Single-Objective Multimodal Optimization

- how do  $t_2$  and  $t_3$  depend on organizing restarts well?



**Figure:** left:  $t_2$  (time to global optimum), right:  $t_3$  (time until all basins have been visited) for unequal basin sizes (1:10) and moderately well working ( $p = 0.5$ ) basin identification

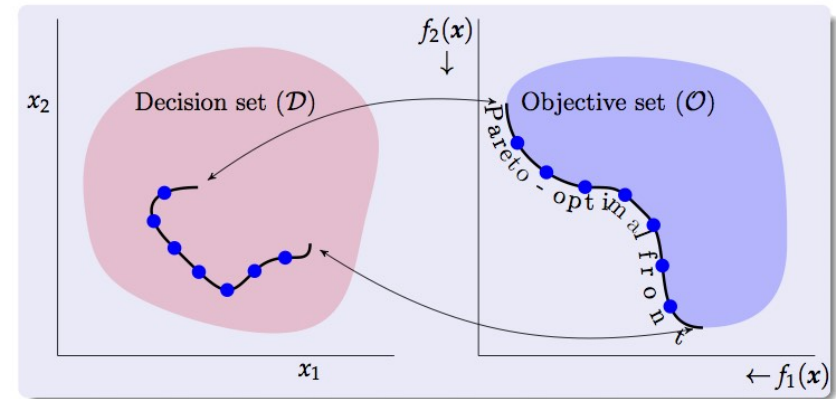
## Block 2: Single-Objective Multimodal Optimization

### So what do we need ELA to do for us?

- having an idea of how many basins are there would be great
- knowing how homogenous basin sizes are is important
- but even if we do not understand everything, it would be good to properly match algorithms/components to problems
- there is little activity in this direction: get active!

# ELA for Multi-Objective “Global” Optimization Problems

$$\text{minimize } F(x) = \langle f_1(x), f_2(x) \rangle, \\ \text{with } x \in \mathcal{D} \subseteq \mathbb{R}^2.$$



source: lmarti.github.io

## Block 3 - Part I: Multi-Objective “Global” Opt.

- in single-objective optimization, ELA has shown to be useful for describing the problem landscape based on a small initial design
- currently, there exist almost no landscape features for continuous multi-objective optimization problems
- first approaches<sup>5,6</sup> towards ELA in the multi-objective setting

<sup>5</sup>Kerschke, P. & Trautmann, H. (2016). *The R-Package FLACCO for Exploratory Landscape Analysis with Applications to Multi-Objective Optimization Problems*. In Proceedings of CEC 2016.

<sup>6</sup>Kerschke, P., Wang, H., Preuss, M., Grimme, C., Deutz, A., Trautmann, H., & Emmerich, M. (2016). *Towards Analyzing Multimodality of Multiobjective Landscapes*. In Proceedings of PPSN 2016 (pp. 962-972)

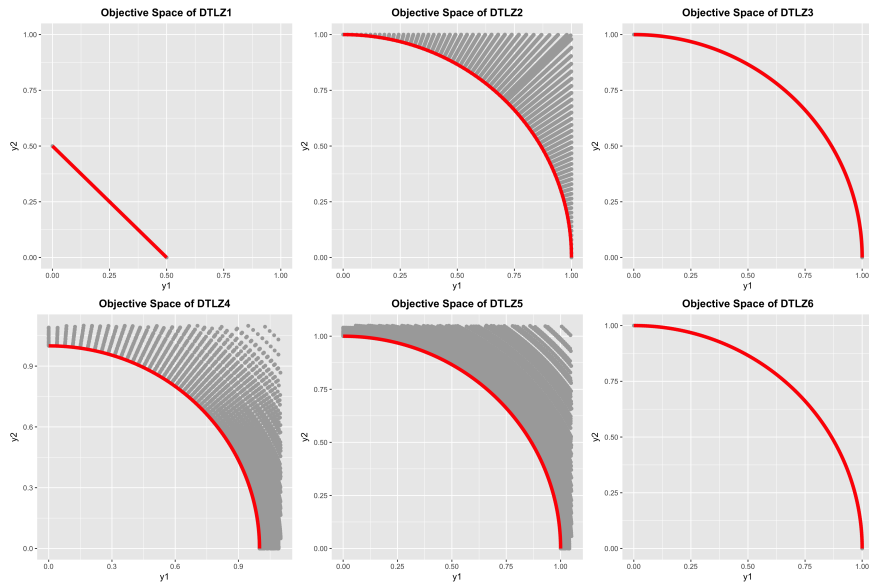
## Block 3 - Part I: Multi-Objective “Global” Opt.

- DTLZ1 to DTLZ7<sup>7</sup> and ZDT1 to ZDT6<sup>8</sup> (without ZDT5)  
~> 120 instances (12 functions with 10 replicates each)
- initial designs:  $100 \times D$  samples with  $D = 3$
- considered 131 artificially designed “interaction-features”:
  - all 15 feature sets except for GCM and Barrier Trees
  - aggregated objectives (objective 1 / objective 2) per feature
  - discarded runtimes, as well as all features that contained infinite or non-defined values

<sup>7</sup>Deb, K., Thiele, L., Laumanns, M. & Zitzler, E. (2001). Scalable Multi-Objective Optimization Test Problems. In Proceedings of CEC 2002 (pp. 825 - 830)

<sup>8</sup>Zitzler, E., Deb, K. & Thiele (2000). Comparison of Multiobjective Evolutionary Algorithms: Empirical Results. In Journal of Evolutionary Computation (pp. 173 - 195)

## Block 3 - Part I: Multi-Objective “Global” Opt.



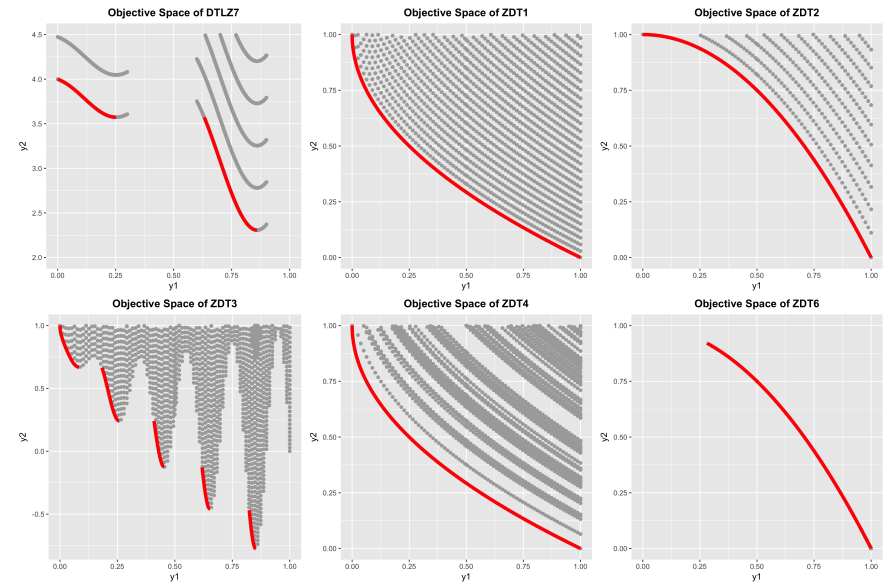
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## Block 3 - Part I: Multi-Objective “Global” Opt.



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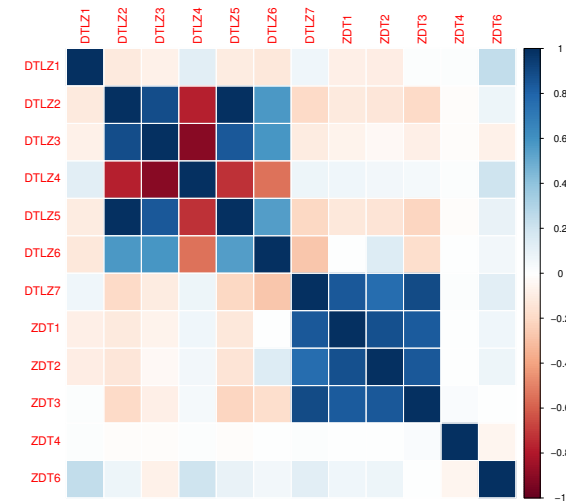
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## Block 3 - Part I: Multi-Objective “Global” Opt.

- the objective spaces of the 12 MOPs show some similarities across the problems, e.g.
  - $DTLZ2 \approx DTLZ4$
  - $ZDT4 \approx DTLZ5 \approx ZDT1$
- the objective space of DTLZ7 looks very different to all the others

Do the features meet our expectations and group the MOPs accordingly?

## Block 3 - Part I: Multi-Objective “Global” Opt.



Kerschke, P. & Trautmann, H. (2016). *The R-Package FLACCO for Exploratory Landscape Analysis with Applications to Multi-Objective Optimization Problems*. In: Proceedings of CEC 2016.

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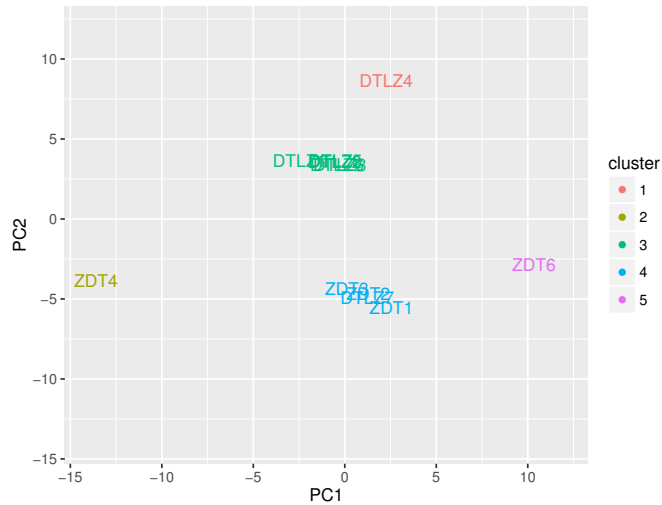
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Kerschke, P. & Trautmann, H. (2016). *The R-Package FLACCO for Exploratory Landscape Analysis with Applications to Multi-Objective Optimization Problems*. In: Proceedings of CEC 2016.

### Why do the features group the MOPs differently?

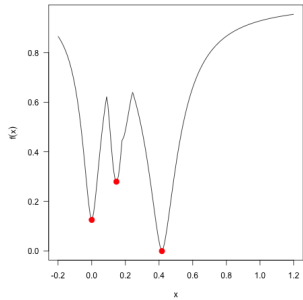
- many features are based on the decision space
- remember: many algorithms also act in the decision space (e.g., mutation / recombination within EAs)
- did not use any sophisticated features (just the feature-wise ratios between the objectives)

**We need something to characterize and/or distinguish the multi-objective landscapes!**

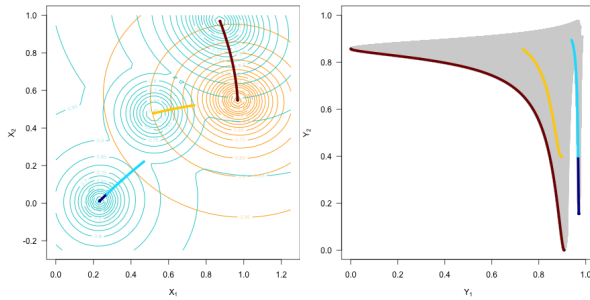
- 1 start with a “white-box”-approach and “measure” some (rather obvious) characteristics
- 2 once we know which of the characteristics might be useful, we can (and should!) develop landscape features that measure the information of these characteristics

# ELA for Multi-Objective Multimodal Optimization Problems

## Single-Objective Optimization



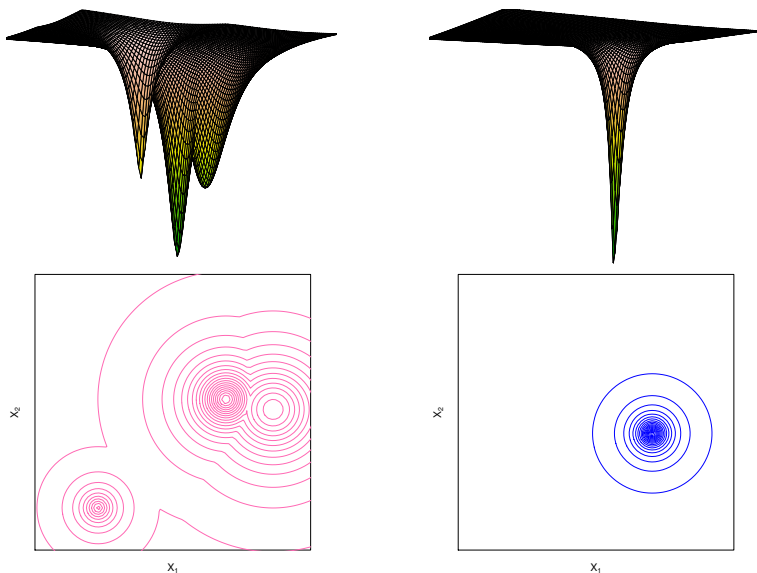
## Multi-Objective Optimization



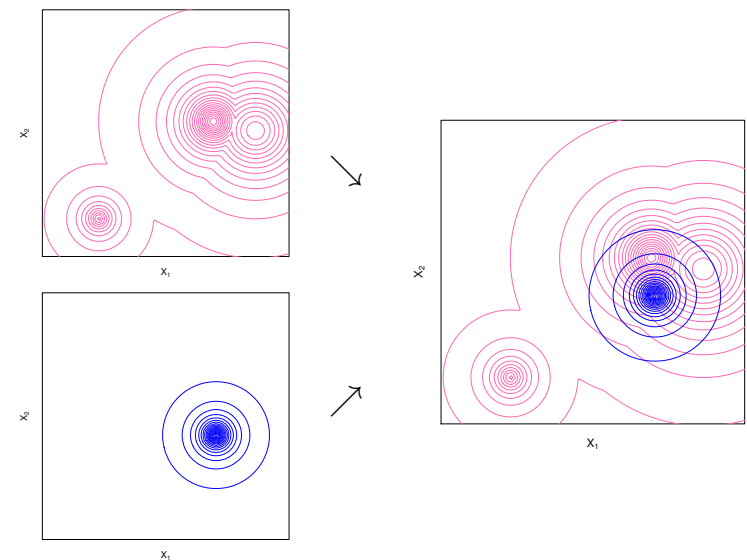
- definition of multimodality for multi-objective problems within our PPSN paper from 2016:  
Kerschke, P., Wang, H., Preuss, M., Grimme, C., Deutz, A., Trautmann, H., & Emmerich, M. (2016). *Towards Analyzing Multimodality of Multiobjective Landscapes*. In: Proceedings of PPSN XIV, Edinburgh, Scotland, pp. 962–972 (Best Paper Award).
- visualized multimodality on a set of simple, but configurable problems  $\rightsquigarrow$  bi-objective mixed-sphere problems (using an adaptation of the MPM2-generator<sup>9</sup>)

<sup>9</sup> multiple peaks model 2 generator, available in python (optproblems0.9, Wessing, S.) and R (smoof, Bossek, J.)

## Block 3 - Part II: Multi-Objective Multimodal Opt. Mixed-Sphere Problems

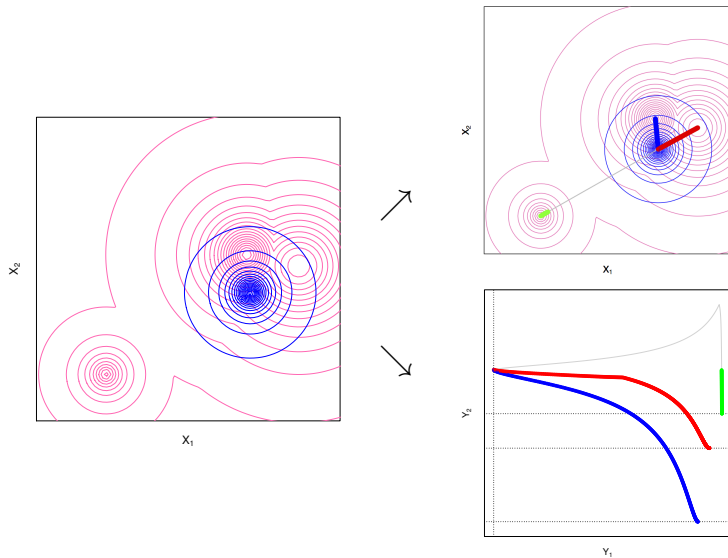


## Block 3 - Part II: Multi-Objective Multimodal Opt. Mixed-Sphere Problems



## Block 3 - Part II: Multi-Objective Multimodal Opt.

### Mixed-Sphere Problems



## Block 3 - Part II: Multi-Objective Multimodal Opt.

### Characteristics $\neq$ Features

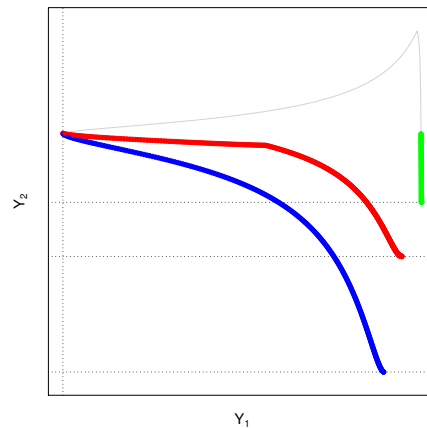
- characteristics use knowledge of the entire landscape (white-box)
- features are based on a small (!) sample of points from the problem

## Block 3 - Part II: Multi-Objective Multimodal Opt.

### Measuring the Multimodality

possible characteristics:

- 1 percentage of counts of global to local Pareto fronts
- 2 percentage of lengths of global to local Pareto fronts
- 3 (1) for connected fronts
- 4 (2) for connected fronts
- 5 ...



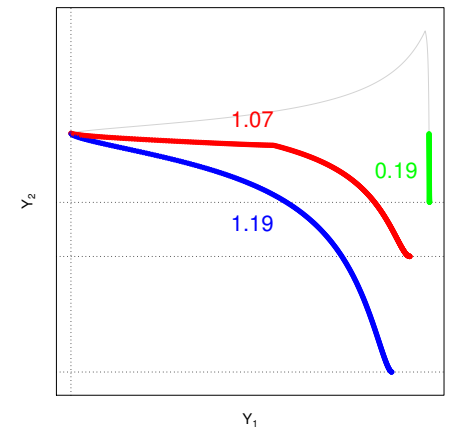
$$\frac{1}{1 + 1 + 1} = \frac{1}{3} \approx 0.33$$

## Block 3 - Part II: Multi-Objective Multimodal Opt.

### Measuring the Multimodality

possible characteristics:

- 1 percentage of counts of global to local Pareto fronts
- 2 percentage of lengths of global to local Pareto fronts
- 3 (1) for connected fronts
- 4 (2) for connected fronts
- 5 ...



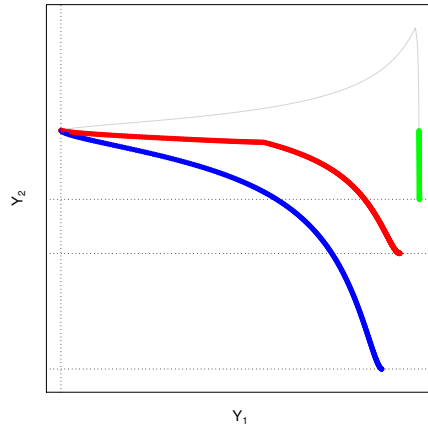
$$\frac{1.19}{1.19 + 1.07 + 0.19} = \frac{1.19}{2.45} \approx 0.49$$

## Block 3 - Part II: Multi-Objective Multimodal Opt.

### Measuring the Multimodality

possible characteristics:

- ❶ percentage of counts of global to local Pareto fronts
- ❷ percentage of lengths of global to local Pareto fronts
- ❸ (1) for connected fronts
- ❹ (2) for connected fronts
- ❺ ...



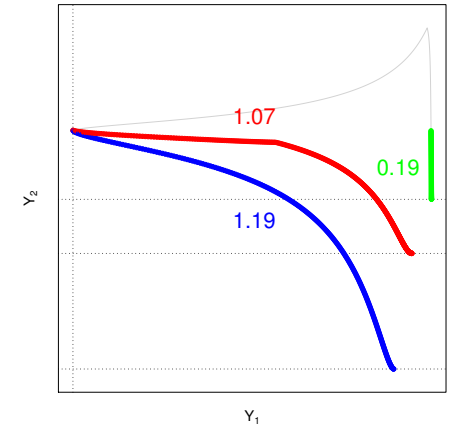
$$\frac{1 + 1}{1 + 1 + 1} = \frac{2}{3} \approx 0.67$$

## Block 3 - Part II: Multi-Objective Multimodal Opt.

### Measuring the Multimodality

possible characteristics:

- ❶ percentage of counts of global to local Pareto fronts
- ❷ percentage of lengths of global to local Pareto fronts
- ❸ (1) for connected fronts
- ❹ (2) for connected fronts
- ❺ ...

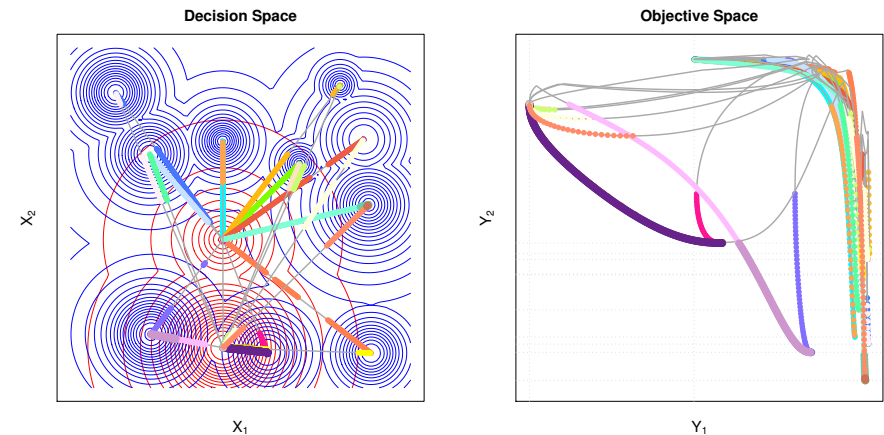


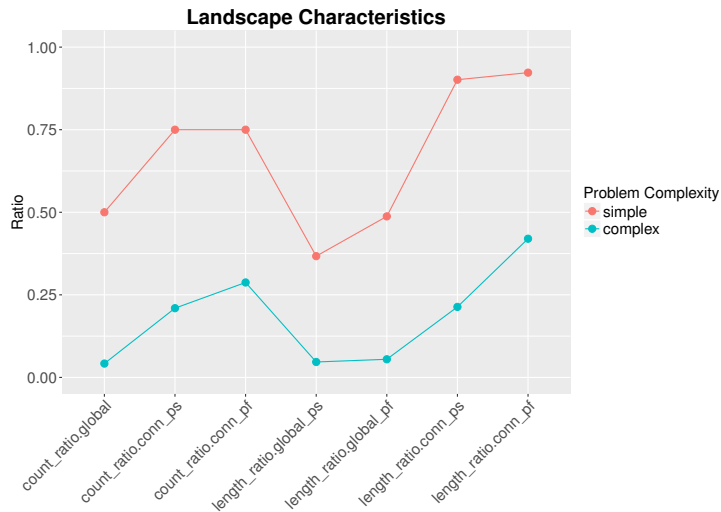
$$\frac{1.19 + 1.07}{1.19 + 1.07 + 0.19} = \frac{2.26}{2.45} \approx 0.92$$

## Block 3 - Part II: Multi-Objective Multimodal Opt.

Quite simple for small problems. But what happens if the problems become (just a little bit) more multimodal?

## Block 3 - Part II: Multi-Objective Multimodal Opt.



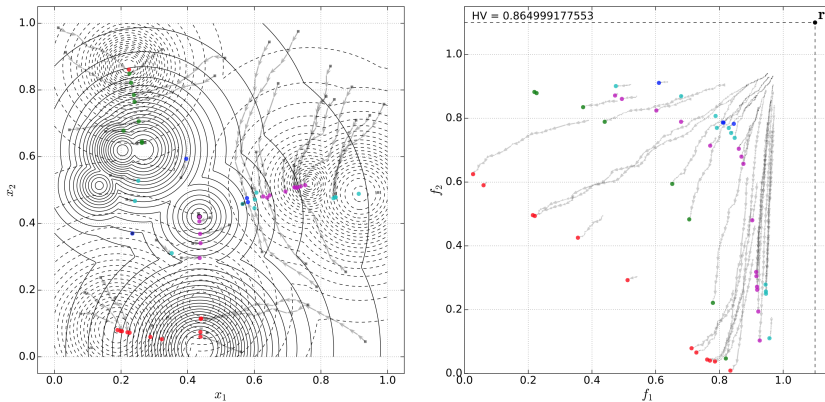


Kerschke, P., Wang, H., Preuss, M., Grimme, C., Deutz, A., Trautmann, H., & Emmerich, M. (2016). *Towards Analyzing Multimodality of Multiobjective Landscapes*. In Proceedings of PPSN 2016 (pp. 962-972)

## Why is it necessary / useful to know the multimodality?

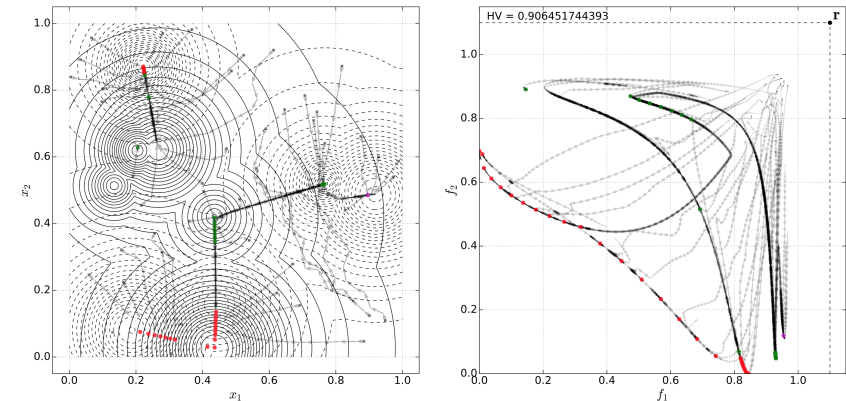
- optimizers behave differently:
  - a general optimizer rather detects the local fronts
  - a global optimizer tries to find the global Pareto front(s)

## Naive Stochastic Local Search (SLS) Algorithm:



Kerschke, P., Wang, H., Preuss, M., Grimme, C., Deutz, A., Trautmann, H., & Emmerich, M. (2016). *Towards Analyzing Multimodality of Multiobjective Landscapes*. In Proceedings of PPSN 2016 (pp. 962-972)

## Hypervolume Indicator Gradient Ascent Multi-Objective Optimization (HIGA-MO) Algorithm:

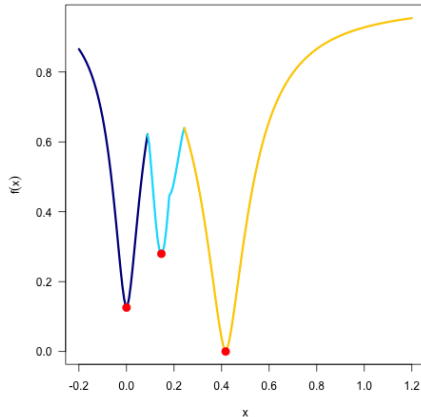


Kerschke, P., Wang, H., Preuss, M., Grimme, C., Deutz, A., Trautmann, H., & Emmerich, M. (2016). *Towards Analyzing Multimodality of Multiobjective Landscapes*. In Proceedings of PPSN 2016 (pp. 962-972)

## Block 3 - Part II: Multi-Objective Multimodal Opt.

### Basin of Attraction

#### Single-Objective Optimization



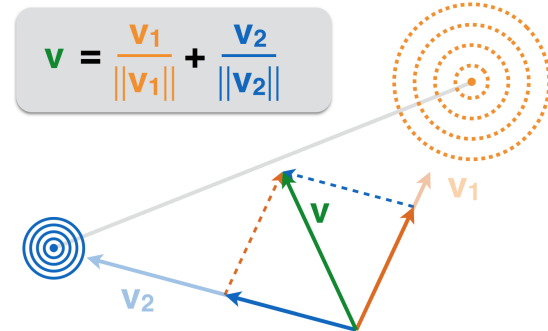
#### Multi-Objective Optimization



## Block 3 - Part II: Multi-Objective Multimodal Opt.

### Visualizing the Basins of Attraction

#### 1. Combined Gradient



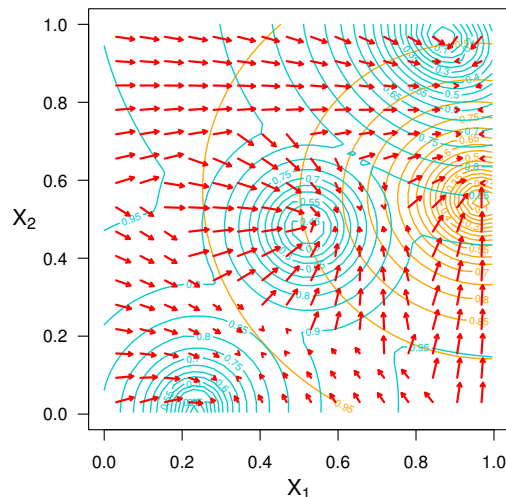
- length of combined gradient  $\longleftrightarrow$  cone of dominance
- move to next cell  $\iff ||\mathbf{v}|| > \epsilon$

Kerschke, P. & Grimme, C. (2017). *An Expedition to Multimodal Multi-Objective Optimization Landscapes*. In Proceedings of EMO 2017 (pp. 329-343)

## Block 3 - Part II: Multi-Objective Multimodal Opt.

### Visualizing the Basins of Attraction

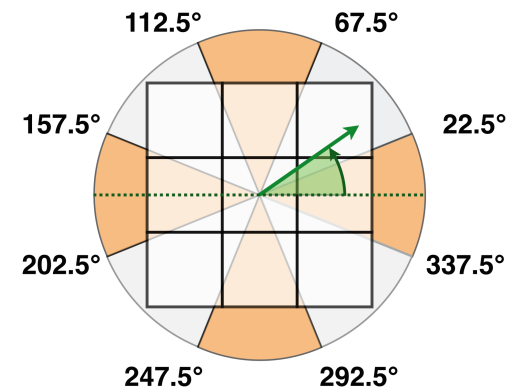
#### 2. Gradient Field



## Block 3 - Part II: Multi-Objective Multimodal Opt.

### Visualizing the Basins of Attraction

#### 3. Direction for Next Step

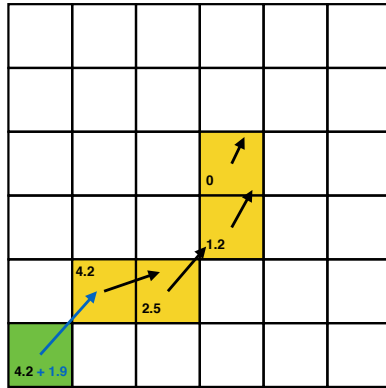


Kerschke, P. & Grimme, C. (2017). *An Expedition to Multimodal Multi-Objective Optimization Landscapes*. In Proceedings of EMO 2017 (pp. 329-343)

## Block 3 - Part II: Multi-Objective Multimodal Opt.

Visualizing the Basins of Attraction

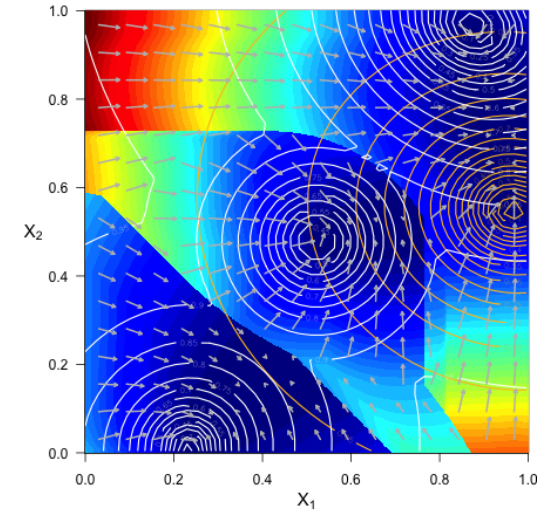
### 4. Cumulated Gradient Paths



## Block 3 - Part II: Multi-Objective Multimodal Opt.

Visualizing the Basins of Attraction

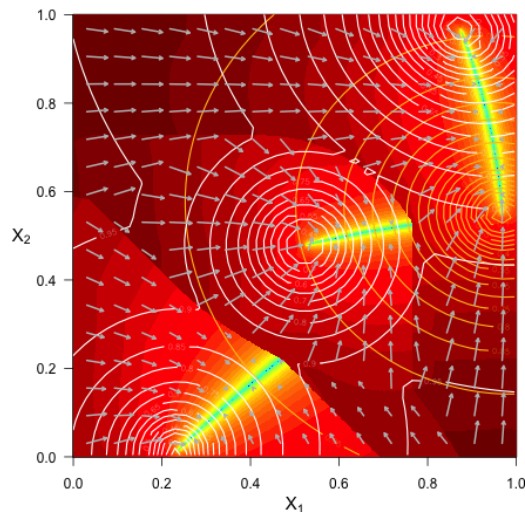
### 5. Heatmap of Cumulated Gradient Paths



## Block 3 - Part II: Multi-Objective Multimodal Opt.

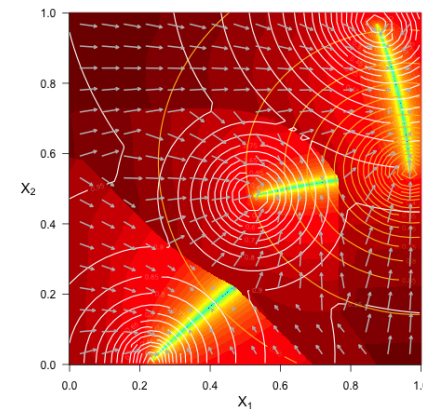
Visualizing the Basins of Attraction

### 5. Heatmap of Cumulated Gradient Paths



## Block 3 - Part II: Multi-Objective Multimodal Opt.

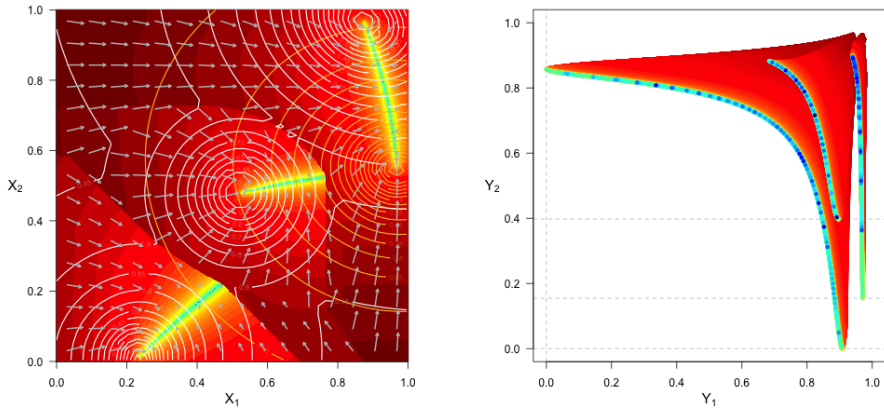
Visualizing the Basins of Attraction



- basin of attraction
- (joint vs. disconnected) local efficient sets
- multi-objective ball
- discontinuities & ridges

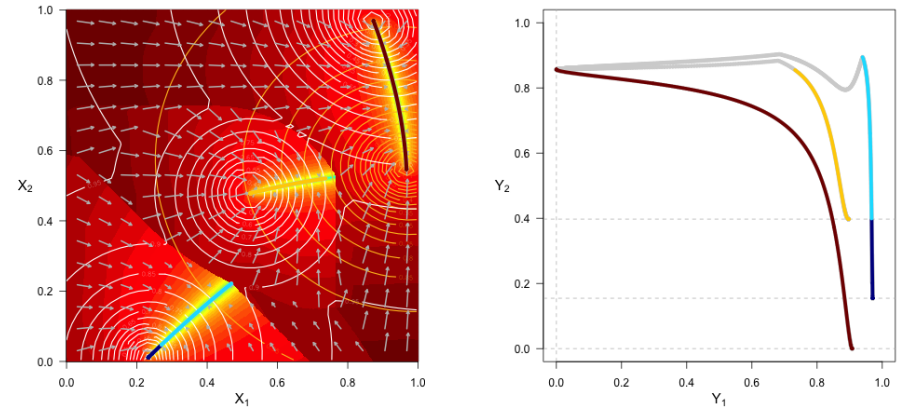
## Block 3 - Part II: Multi-Objective Multimodal Opt.

Visualizing the Basins of Attraction



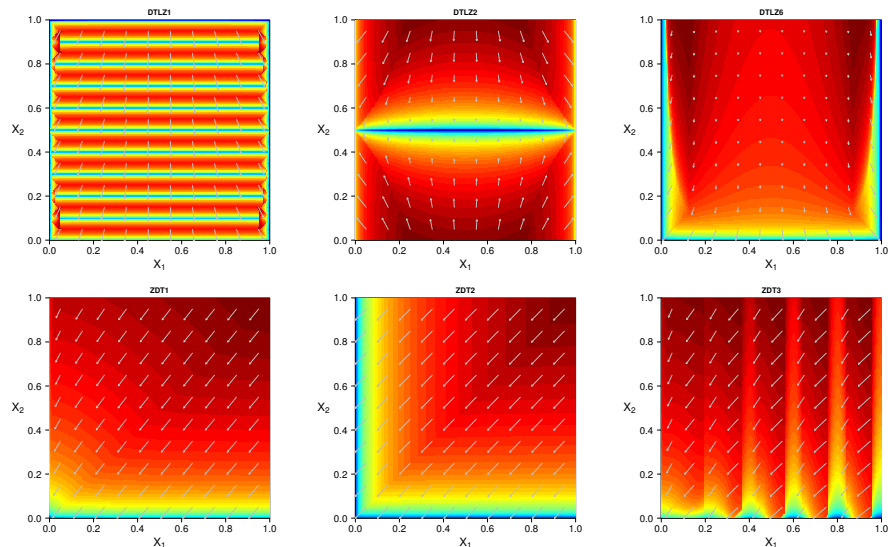
## Block 3 - Part II: Multi-Objective Multimodal Opt.

Visualizing the Basins of Attraction



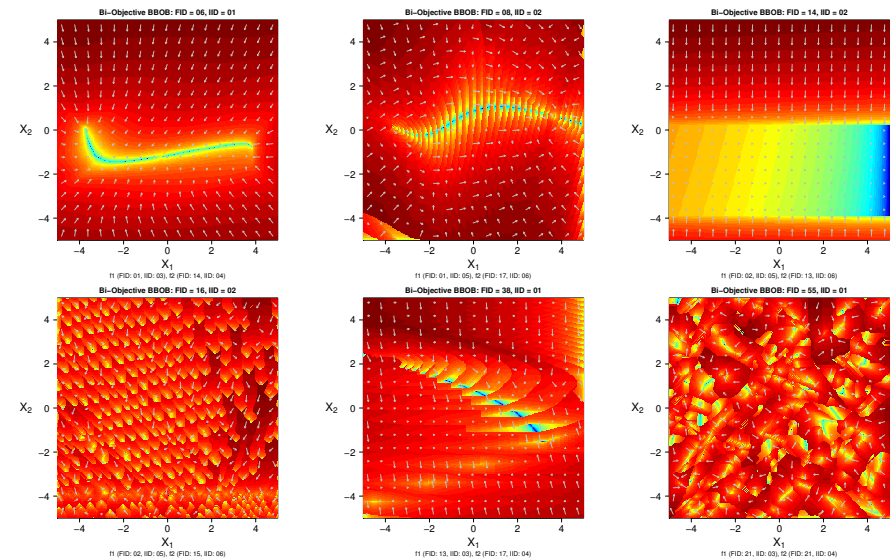
## Block 3 - Part II: Multi-Objective Multimodal Opt.

Visualizing the Basins of Attraction – DTLZ & ZDT



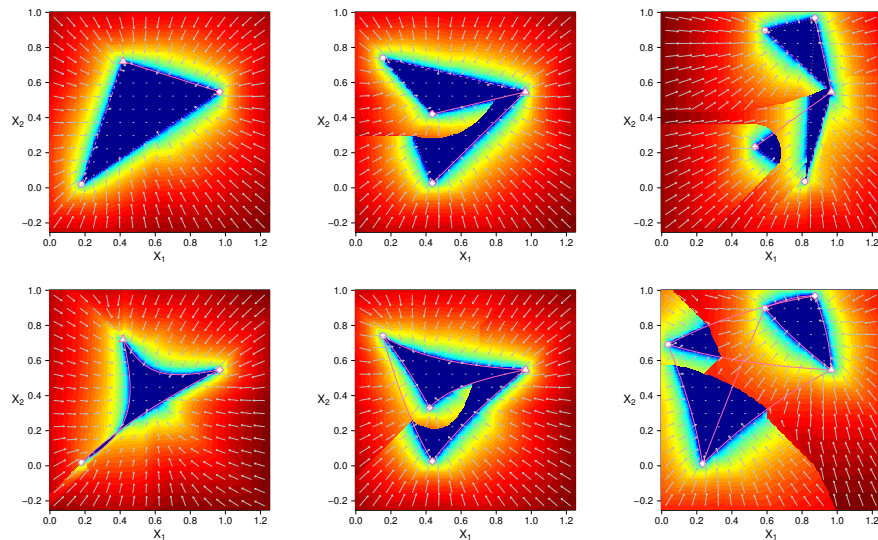
## Block 3 - Part II: Multi-Objective Multimodal Opt.

Visualizing the Basins of Attraction – Bi-Objective BBOB



## Block 3 - Part II: Multi-Objective Multimodal Opt.

Visualizing the Basins of Attraction – 3 Objectives



# Closing

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## Closing

- enhance flacco with more ELA features
- how can we find the smallest most informative feature set?
- by how much can we still reduce the size of the initial designs without losing (too much) information?!
- where can we find representative real-world problems / appropriate benchmarks?
- can we transfer landscape features from / to different domains?
- use ELA features for improved algorithm selection and/or configuration on different benchmarks (e.g., BBOB)

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