A Multi-Objective Decomposition-based Evolutionary Algorithm with Enhanced Variable Space Diversity Control

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ABSTRACT

Most Multi-objective Evolutionary Algorithms (MOEAs) operate without explicitly promoting the diversity of the variable space. Nevertheless, in the single-objective domain it has been shown that properly managing this kind of diversity might lead to higher-quality solutions. In this paper the diversity of the variable space is analyzed for several state-of-the-art MOEAs with well-known benchmarks, showing that in the long term, the diversity is lost in a subset of variables. This loss implies an important degradation of the performance. In order to show that increasing the diversity can solve these issues, MOEA/D with Enhanced Variable-Space Diversity (MOEA/D-EVSD) is proposed. This variant induces a gradual loss of diversity by altering the mating selection process. In addition, a final phase to properly intensify is included. The experimental validation was carried out with the Walking Fish Group (WFG) benchmark and several state-of-the-art MOEAs showing the benefits of the proposal. Particularly, WFG1 and WFG8, which are not properly solved by most state-of-the-art approaches, are readily solved by our proposal.

CCS CONCEPTS

•**Mathematics of computing** → *Continuous optimization; Stochastic control and optimization; Bio-inspired optimization;*

KEYWORDS

Multi-Objective, Decomposition, Diversity, Optimization

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1 INTRODUCTION

Multi-objective optimization is an area of multiple criteria decision making that involves the simultaneous optimization of several, usually conflicting, objective functions [5]. A continuous minimization multi-objective problem can be defined as follows:

$$\begin{aligned} &minimize \quad F(x) = (f_1(x), f_2(x), ..., f_m(x)) \\ &subject \quad to \quad x \in \Omega \end{aligned} \tag{1}$$

where $x = (x_1, ..., x_n) \in R^n$ is a decision variable vector, n is the dimensionality of the variable space, Ω is the feasible space, $F : \Omega \rightarrow R^m$ consist of m real-valued objective functions and R^m is called the *objective space*. In a minimization Multi-objective Optimization Problem (MOP) with m objective functions, and given two solutions $x, y \in \Omega$, x dominates y, denoted by $x \prec y$, if $f_i(x) \leq f_i(y)$ for all objectives $i \in \{1, ..., m\}$, and $F(x) \neq F(y)$. This means that the solution x is not worse than y in any of the objectives and x is strictly better than y in at least one objective. The Pareto dominance definition states that the best solutions of a multi-objective optimization problem are those whose objective vectors are not dominated by any other feasible vector. A solution $x^* \in \Omega$ is known as Pareto optimal solution if no other solution $x \in \Omega$ dominates x^*. The Pareto set is the set of all the Pareto optimal solutions and the Pareto front are the images of the Pareto set. The goal of multi-objective optimization approaches is to obtain a proper approximation of the Pareto front. Particularly, a set of solutions that are diverse and close to the Pareto front are desired.

Multi-objective Evolutionary Algorithms (MOEAs) are one of the most typical approaches to address MOPs. In spite of the popularity of Evolutionary Algorithms (EAs) both in single-objective and multi-objective domains, there are several issues that can affect their performance. In the case of single-objective problems, premature convergence has been recognized as one of the most typical failures modes of EAs [20]. Premature convergence arises when most of the population members are placed in a small region of the search space and the components selected do not allow escaping from this region. This issue is related with the loss of diversity in the variable space, so several methods have been proposed to deal with the proper management of this kind of diversity [8, 12]. Some recent successful methods are based on taking into account the stopping criterion set by the user with the aim of provoking a gradual loss of diversity [18]. In this work the stopping criterion was set to a maximum number of generations[1]. Thus, initial phases of the optimization induce a larger degree of exploration, whereas final phases are dedicated to intensification. In the case of MOEAs, given that diversity is promoted in the objective space, some degree of diversity is usually maintained inherently in the variable space, so a complete convergence in the variable space does not appear. However, if this amount of diversity is not large enough, a similar situation to premature converge might arise because the genetic operators might not produce better trade-offs when applied to the current members of the population.

The aim of this paper is to show that by inducing a gradual loss of diversity in the variable space, the state-of-the-art of MOEAs can be improved further. Our proposal (MOEA/D-EVSD) is an extension of MOEA/D (MOEA based on Decomposition) [21] that includes an enhanced variable-space diversity control. The main novelty of MOEA/D-EVSD is that it promotes the preservation of the diversity of the decision space whereas the majority of the state-of-the-art MOEAs focus only in the preservation of the diversity in the objective space. The stopping criterion set by the user bias the decisions taken internally by MOEA/D-EVSD with the aim of attaining a gradual progress between exploration and intensification. Additionally, a final phase to further promote intensification is included. The experimental validation is performed with the well-known Walking Fish Group (WFG) tests [10]. Results show the promising performance of MOEA/D-EVSD, which is specially clear in some of the most complex problems. The experimental validation has considered the hypervolume and the attainment surfaces and proper statistical tests have been applied to confirm the benefits attained by MOEA/D-EVSD. In the cases where MOEA/D-EVSD provokes a degradation of the performance, the reasons have been studied.

The rest of this paper is organized as follows. Section II provides a brief review of the state of the art of MOEAs. Section III discusses a way in which MOEAs can fail. Section IV describes a mechanism to increase the diversity in the variable space through a special mating selection scheme. The experimental validation of the proposal is shown in Section V. Finally, conclusions and some lines of future work are given in Section VI.

2 LITERATURE REVIEW

In the last decades, the field of MOEAs has gained popularity. As a result, there is a large number of MOEAs available. In order to better classify the different schemes, several taxonomies have been proposed [19]. Particularly, MOEAs can be based on Pareto dominance, indicators and/or decomposition [4]. In this paper, our validation has been carried out by including the Non-Dominated Sorting Genetic Algorithm (NSGA-II) [6], the MOEA Based on Decomposition (MOEA/D) [21], and the S-metric Selection Evolutionary Multi-objective Optimization Algorithm (SMS-EMOA) [2]. They are representative methods of the domination-based, decomposition-based and indicator-based paradigms, respectively. Additionally, since our proposal implements differential evolution (DE) operators, the Generalized Differential Evolution (GDE3) [14] has also been taken into account. It is important to note that none of the selected methods introduce special mechanisms to promote diversity in the variable space. In fact, contrary to the case of single-objective domain [18], we could not find any paper proposing a MOEA that relates the degree of exploration to the stopping criterion set by the user with the aim of adapting the search capabilities to the different optimization stages. Thus, current state-of-the-art methods do not provide a way to simultaneously promote diversity in the objective and variable space.

[1]In other implementations a period of time is considered.

2.1 Domination-Based

This kind of MOEAs are based on the application of the Pareto dominance relation. Since the dominance relation does not inherently promote the preservation of diversity in the objective space, auxiliary techniques such as niching, crowding and/or clustering are usually integrated with the aim of obtaining a proper spread and diversity of the objective space. One popular algorithm of this group is the NSGA-II. This algorithm [6] implements a special parent selection operator and incorporates the use of elitism in the replacement phase. The selection operator is based on two mechanisms: fast-non-dominated-sort and crowding. The first one provides a bias based on the Pareto dominance relation, whereas the second one promotes the preservation of diversity in the objective space. This method has been extended in a large number of ways [13]. NSGA-II is now a mature algorithm that has been incorporated in many popular frameworks. Thus, it is almost mandatory to compare the results of new algorithms against this method, so NSGA-II has been one of the methods selected of this group.

Differential Evolution (DE) is a simple population-based metaheuristic, closely related to EAs. One of its main features is the relation between the mutation operator and the content of the population. DE has reported promising results in single-objective optimization [17], and several multi-objective variants of this metaheuristic have been devised [1]. One popular multi-objective variant is the third evolution step of Generalized Differential Evolution (GDE3). GDE3 is a generalized variant of DE that can deal with single-objective, multi-objective and constrained problems. The creation of new individuals is based on the DE/rand/1/bin strategy, and the replacement takes into account the dominance concept and crowding. Since our proposal incorporates the operators of DE, GDE3 is also used in our experimental validation.

2.2 Indicator Based

In order to compare the performance of MOEAs, several quality indicators that map Pareto set approximations to real numbers have been devised. Since these indicators measure the quality of the approximations attained by MOEAs, a paradigm based on the application of these indicators was proposed. In these cases, instead of the Pareto dominance, the indicators are used in the MOEAs to guide the optimization process. Particularly, the parent selection and replacement phase are usually modified by incorporating the use of an indicator. Among the different indicators, hypervolume is a widely accepted Pareto-compliance quality indicator. One of the advantages of indicator-based schemes is that the indicators usually take into account both the quality of the candidate solutions and their diversity. Among the different indicator-based MOEAs, the SMS-EMOA [2] has been used extensively, probably due to its simplicity and superiority over many other approaches [9].

2.3 Decomposition Based

Decomposition-based MOEAs [3] are based on transforming the MOP into a set of single-objective optimization problems that are tackled simultaneously. This transformation can be performed in several ways, e.g. with a linear weighted sum or with a weighted Tchebycheff function. Given a set of weights to establish different single-objective functions, the MOEA searches for a single high-quality solution for each of them. The weight vectors should be selected with the aim of obtaining a well-spread set of solutions. However, this is a difficulty of these kinds of approaches because the selection of proper weights might depend on the form of the Pareto front [5].

MOEA/D [21] is a designed decomposition-based MOEA. Its main principles include problem decomposition, weighted aggregation of objectives and mating restrictions through the use of neighborhoods. Different ways of aggregating the objectives have been tested with MOEA/D. Among them, the use of the Tchebycheff approach is quite popular.

One special feature of the MOEA/D is the definition of neighborhoods. Each subproblem is associated with a set of close subproblems in terms of the distances of their weights. These subproblems are said to belong to its neighborhood. Then, in each mating operation a subproblem is selected and two individuals belonging to the corresponding neighborhood are used as input of the genetic operators. Note that the best individual for each weighted function is always preserved, so MOEA/D is an elitist algorithm. The principle that governed this design is that, usually, close subproblems are properly solved by close solutions; thus, MOEA/D promotes the mating of close solutions, resulting in further intensification. Given that some forms of the Pareto front might provoke difficulties when applying uniformly distributed weights, some methods to automatically adapt the weights with the aim of improving the diversity in the objective space have been devised [16]. Additionally, some authors have noted that close subproblems do not necessarily induce close solutions in the search space. Thus, in MOEA/D-AMS [3], the neighborhood is defined in terms of the space of the variables. In any case, the principle is the same because this restriction favors the mating of similar individuals.

3 A FAILURE MODE OF MOEAS

In the last decades, a large amount of MOEAs have been devised. Two aims are usually considered: the solutions should be close to the Pareto front, and the diversity of the approximation — measured in the objective space — should be maximized [23]. In spite of the large amount of methods, some well-known benchmarks such as the WFG tests are not solved to optimality.

In the case of single-objective optimization, it has been found experimentally that integrating rules to promote the diversity in the variable space can bring benefits in terms of the objective function. The main reason behind this finding is that EAs have a tendency to quickly lose diversity, so premature convergence might appear, meaning that many resources might be wasted. Recently, a new principle of design has been successfully used to obtain new best-known solutions in several well-known single-objective problems [18]. Note that in MOEAs, complete convergence does not appear because some degree of diversity is explicitly maintained in the multi-objective space. However, our hypothesis is that depending on the problem, the total amount of diversity maintained in the variable space might be too low. In such cases, a situation similar to premature convergence might appear with MOPs, i.e. the diversity might not be large enough to improve further the results.

Several benchmarks to study the performance of MOEAs have been proposed [11]. Among them, the WFG benchmark [10] is one of the most widely accepted ones. As a result, it has been selected to perform our studies. The WFG problems divide the decision variables in two kinds of parameters: the distance parameters and the position parameters. A parameter x_i is a distance parameter when for all parameter vectors **a**, modifying x_i in **a** results in a parameter vector that dominates **a**, is equivalent to **a**, or is dominated by **a**. However, if x_i is a position parameter, modifying x_i in **a** always results in a vector that is incomparable or equivalent to **a** [10].

In this section we show that state-of-the-art MOEAs do not always maintain a high enough diversity. Particularly, a problem is used to show that premature convergence appears in the set of distance parameters. As a result, the crossover loses its exploratory strength. In order to illustrate the previously mentioned drawback, the WFG1 test has been selected. We selected WFG1 because it has a simple definition, but most current MOEAs face difficulties with it. In fact, WFG1 is an uni-modal and separable problem. The distance parameters values associated to Pareto optimal solutions for WFG1 [10] has exactly the same values in the distance parameters. This value is shown in (2).

$$X_{i=k+1:n} = 2i \times 0.35 \qquad (2)$$

The jMetalcpp [15] framework was used to perform our executions. Taking into account the stochastic behavior of MOEAs, 35 independent executions were run. In all of them, the stopping criterion was set to $50,000$ generations and the size of the population was fixed to 250. In order to analyze the diversity, the average Euclidean distance among individuals (ADI) is calculated, i.e. the mean value of all pairwise distances among individuals in the population is reported. Fig. 1 shows the evolution of the diversity for GDE3, SMS-EMOA, MOEA/D and NSGA-II (the case of MOEA/D-EVSD is commented later). The parameterization is quite standard and is detailed in Section 5. The top part of the figure shows the evolution of the ADI by taking into account the whole set of parameters, whereas in the bottom part, only the distance parameters are taken into account. Note that a logarithmic scale is used; in the case of the distance parameters, the regions where no information is plotted correspond to generations where the distance is zero. As we can appreciate, all the methods maintain some degree of diversity when all the parameters are considered. Otherwise, the diversity in the objective space would be lost. However, when the distance parameters are the only ones considered, all the methods reach a zero distance relatively fast. In fact, after about $5,000$ generations all of them have reached a generation where the distance is equal to zero. This means that in relatively few generations, all the methods have converged in the distance parameters. We can see that in some generations, some degree of diversity is recovered; this is because of the action

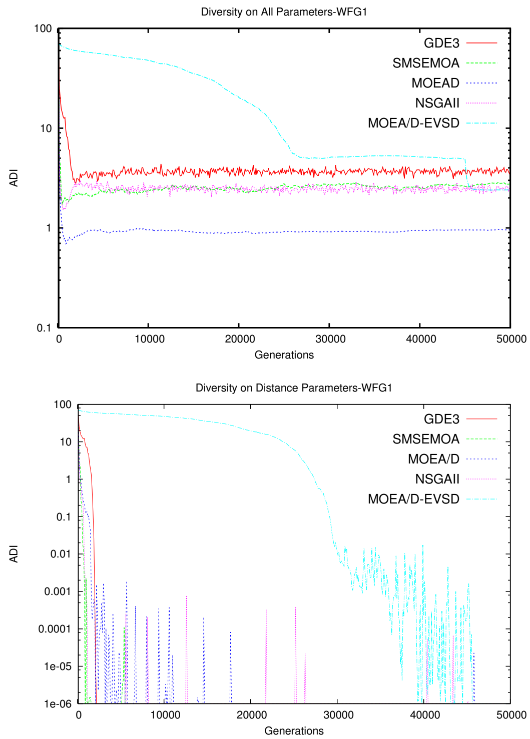

Figure 1: Evolution of the diversity for instance WFG1

Algorithm 1 MOEA/D-EVSD (First phase)

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Initialize the weight vectors λ^1, λ^2, ..., λ^N and neighborhoods B(i) using the traditional MOEA/D approach.
Generate an initial population x^1, ..., x^N randomly.
Initialize z = (z_1, ..., z_m)^T to a high value.
while (not stopping criterion) do
    for i=1,...,N do
        Mating Pool: Randomly fill a pool P with α individuals, selecting each individual with replacement from neighborhood B(i) with probability δ or from the entire population with probability (1 − δ).
        Reproduction: Select the most distant individuals from P and apply genetic operators to them to generate a new offspring (y).
        Update the reference z: For each j = 1,..,m, if z_j > f_j(y), then set z_j = f_j(y)
        Update of Neighboring solutions: For each index j ∈ B(i), if g(y|λ^j, z) < g(x_j|λ^j, z), then set x^j = y
    end for
    Update the value of δ
end while
```

of the mutation. Thus, one of the reasons of the poor performance of some state-of-the-art MOEAs in this case is the apperance of premature convergence in the distance parameters. In fact, after this loss of diversity, the MOEAs are basically modifying the position parameters, so the majority of the time is invested in improving further the diversity in the objective space instead of the quality.

4 PROPOSAL

Our proposal (MOEA/D-EVSD) extends the traditional variant of MOEA/D. Particularly, MOEA/D-EVSD is a decomposition-based MOEA that applies the Tchebycheff approach to generate the single-objective subproblems. MOEA/D-EVSD is divided in two phases: the first phase starts with a large degree of exploration, and gradually alters this degree towards intensification, whereas the second phase is dedicated to intensification. The main distinguishing feature of the first phase is the inclusion of a special mating selection approach (see Algorithm 1). Similarly to MOEA/D, each subproblem has a neighborhood whose size is denoted by T_r. The aim of altering the mating selection is to have a better control of the diversity induced in the variable space. The mating selection operates as follows. Similarly to MOEA/D, for each subproblem P_i, a new individual is created. It is known that in most crossover operators, such as the SBX, the exploratory power increases when distant individuals are taken into account. Thus, a heuristic approach to try to induce a larger diversity lies in promoting the mating of dissimilar individuals. Thus, the mating selection process of MOEA/D is modified in our proposal. Specifically, instead of selecting two individuals of the neighborhood of P_i for proceeding with the mating process, the next steps are performed. First, a pool P with size α of candidate parents is filled. Each candidate parent is randomly selected from the neighborhood of problem P_i with probability δ, whereas it is randomly selected from the whole population with probability $1 - \delta$. Then, the two selected individuals whose distance is the largest one are selected for the mating process. Note that the previous process requires the setting of the δ parameter to fill the mating pool. Since we aim to alter dynamically the degree of exploration, this parameter is set as follows: $\delta = \frac{t_i}{Total \quad Generations}$, where t_i denotes the current generation. This way, at the beginning of the first phase, every individual is selected from the whole population but the proportion of globally selected individuals is linearly decreased during the execution. Thus, a gradual change between exploration and exploitation can be induced.

In the second phase of our proposal, the traditional mating selection of MOEA/D[21] is used with the aim of further intensifying. Additionally, since the DE operators are quite useful to intensify, the Rand/1/bin scheme is used to create new trials. Thus, our second phase is just a traditional MOEA/D with DE operators. Given the different exploratory power of the DE operators, a different T_r value might be appropriate for this second phase. The T_r of the first and second phase are identified as $T_{r,1}$ and $T_{r,2}$, respectively.

5 EXPERIMENTAL VALIDATION

This section is devoted to validate our proposal and to show that controlling the diversity in the variable space is a way to improve further some of the results obtained by state-of-the-art MOEAs. The nine WFG tests proposed in [10] have been used for our purpose. Our experimental validation includes the MOEA/D-EVDS, as well as four well-known state-of-the-art algorithms. Given that all of them are stochastic algorithms, each execution was repeated 35 times with different seeds. The common configuration in all of them was the following: the stopping criterion was set to 50,000 generations, the population size was fixed to 250, and the WFG were configured with two objectives and 24 parameters, where 20 of them are distance parameters and 4 are position parameters. In general (except for GDE3), the crossover and mutation operators are SBX and polynomial respectively, with a crossover probabilty of 0.9 and mutation probability of 1/24, also the crossover and mutation

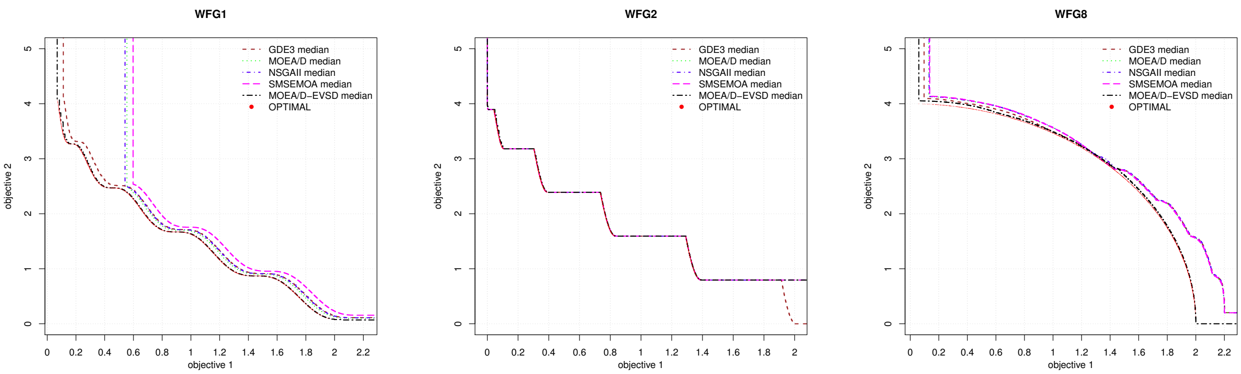

Figure 2: 50% Attainment Surfaces achieved for the WFG instances.

distribution indexes were assigned to 20 and 50 respectively. The extra parameterization[2] of each algorithm is as follows:

- **GDE3**: CR = 0.5 and F = 0.5.
- **SMS-EMOA**: offset = 250.
- **MOEA/D**: size of neighborhood = 20, max updates by sub-problem (nr) = 2 and $\delta = 0.9$.
- **MOEA/D-EVSD**: first phase uses 80% of total generations, second phase use 20% of total generations, in second phase F = 0.5, α = 20 individuals, $T_{r,1} = 2$, $T_{r,2} = 25$.

Our experimental analysis has been performed in base of attainment surfaces and hypervolume. In order to statistically compare the hypervolume results, a similar guideline than the one proposed in [7] was used.

In four of the problems (WFG 3, 4, 5 and 7), all the methods reported quite similar results. In fact the differences among the mean of the hypervolume attained by the methods was lower than 0.1. Thus, our study focuses on the remaining problems. The 50% attainment surfaces for WFG1, WFG2 and WFG8 problems are shown in Fig. 2. In addition, the Pareto front is plotted. The analysis of this figure shows that the modifications introduced in the MOEA/D provokes significant changes in the obtained results. However, there are cases where MOEA/D-EVSD performs significantly better, whereas in other problems there is a degradation in the performance. The WFG1 and WFG8 are the two problems where the benefits of MOEA/D-EVSD are clearer. In such cases, no one of the state-of-the-art MOEAs were able to attain high-quality results. However, the MOEA/D-EVSD could obtain a really good approximation of the Pareto front.

In order to better understand the internal behavior of MOEA/D, the diversity obtained in the variable space was analyzed for WFG1. Fig. 1 shows, in the top part, the evolution of the average distance when considering all the variables, whereas in the bottom part only the distance parameters are taken into account. When taking into account all the parameters, all the methods preserve some degree of diversity. However, it is clear that in MOEA/D-EVSD the loss of diversity is more gradual. Differences are clearer when attending to the distance parameters. In this case, the only method that is able to preserve diversity in the long-term is MOEA/D-EVSD, meaning that it is the only one that does not converge prematurely in this subset of variables. The increase of diversity explains the improvements in WFG1 and WFG8.

MOEA/D-EVSD performs worse than some of the other state-of-the-art MOEAs in WFG2, WFG6 and WFG9. By inspecting the content of the population in such cases, we could find out that, very soon in the optimization process, in MOEA/D-EVSD some of the position variables were set to quite large or small values in every individual. The reason is that, due to the way in which selection is done, there is a bias to select more frequently individuals that present very large or very small values in their variables. This bias arises because the most distant individuals of the mating pool are selected to proceed with the mating process. Thus, when there are individuals placed near the corners of the variable space that present relatively high-quality values, they are selected very frequently, so a premature convergence to these regions might appear. While our proposal has been useful to show that increasing the diversity of the variable space is a way to improve further the results in problems that are not solved by current state-of-the-art MOEAs, a bias towards some zones have been introduced. Therefore, other ways of increasing the diversity that do not include such biases should be analyzed.

Finally, in order to fully validate our previous conclusions, analyses of the hypervolume (see Table 1 and Table 2) were also performed. Table 2 shows the minimum, maximum, mean and standard deviation of the hypervolume attained by the different tested optimizers. The reference point was established to $(3.0, 5.0)$ [22]. In addition, pair-wise statistical tests were performed (Table 1). For each instance, the column "↑" reports the number of comparisons where the statistical tests confirmed the superiority of the MOEA listed in the corresponding row, whereas the column "↓" reports the number of cases where it was inferior. The attained hypervolume values and the corresponding statistical tests confirm the superiority of MOEA/D-EVSD in WFG1 and WFG8. In fact, in both cases the statistical tests confirm that MOEA/D-EVSD is superior to all the remaining MOEAs. However, as expected, the tests also confirm the inferiority of MOEA/D-EVSD in the remaining cases.

[2]The proposed tuned parameters for the MOEA/D-EVSD are: $\alpha = 0.1xN$, $T_{r,1} = 0.01xN$ and $T_{r,2} = 0.1xN$, where N is the population size.

Table 2: Statistics HV

	MOEA/D-EVSD				GDE3				MOEA/D				NSGAII				SMS-EMOA			
	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD
WFG1	11.53	11.54	**11.54**	2.02E-03	10.90	11.40	11.12	1.50E-01	9.63	10.68	10.36	2.63E-01	10.11	10.65	10.38	2.21E-01	9.51	10.09	9.89	2.42E-01
WFG2	10.62	11.46	10.89	3.88E-01	11.47	11.47	**11.47**	4.75E-05	10.63	10.63	10.63	2.53E-04	10.63	10.63	10.63	2.73E-04	10.63	10.63	10.63	5.75E-04
WFG6	7.99	8.11	8.05	3.01E-02	8.60	8.65	**8.61**	2.06E-02	7.81	8.50	8.35	1.30E-01	8.31	8.44	8.37	3.60E-02	8.28	8.48	8.39	4.23E-02
WFG8	7.96	8.60	**8.44**	2.39E-01	7.93	7.94	7.93	3.97E-03	7.83	7.89	7.87	1.85E-02	7.82	7.86	7.84	1.01E-02	7.82	7.89	7.86	1.73E-02
WFG9	7.72	8.21	7.73	8.21E-02	7.72	7.79	7.75	2.27E-02	7.72	8.57	**8.30**	2.43E-01	7.72	8.58	7.82	2.71E-01	7.72	8.58	8.21	3.59E-01

Table 1: Statistical Test HV

	WFG1		WFG2		WFG6		WFG8		WFG9	
	↑	↓	↑	↓	↑	↓	↑	↓	↑	↓
MOEA/D-EVSD	4	0	3	1	0	4	4	0	0	4
GDE3	3	1	4	0	4	0	3	1	1	3
MOEA/D	1	2	0	4	1	1	2	2	3	0
NSGAII	1	2	1	2	1	1	0	4	2	2
SMS-EMOA	0	4	1	2	1	1	1	3	3	0

6 CONCLUSIONS AND FUTURE WORK

MOEAs are one of the most popular approaches to deal with complex MOPs. In the case of MOEAs, a complete convergence in the variable space does not usually appears, because most current approaches explicitly maintain diversity in the objective space, thus maintaining some diversity in the variable space as a side effect. However, we show that in some of the problematic benchmarks, the reason of the poor performance of state-of-the-art MOEAs is that they do not maintain a large enough diversity in the variable space. In order to better illustrate these findings, MOEA/D-EVSD, which is an extension of MOEA/D, is proposed. MOEA/D-EVSD is based on the principles of inducing a large degree of exploration in the initial stages of the optimization and on moving towards intensification in a gradual way. In order to attain this gradual behavior, the stopping criterion set by the user is used to alter the mating process. Particularly, MOEA/D-EVSD tends to recombine more distant individuals in the initial stages than in subsequent stages. The experimental validation is carried out with long-term executions and the well-known WFG tests. This validation shows that the novel proposal is able to properly solve the WFG1 and WFG8 instances, which are quite problematic for the remaining tested MOEAs. The advantages of MOEA/D-EVSD are shown both in terms of the attainment surfaces and hypervolume. Moreover, statistical tests to confirm the superiority are carried out. However, we also show that the way in which diversity is promoted is problematic in some cases. First, we show that the search is biased towards solutions that are close to the corners of the search space. Second, since the maintenance of diversity is not explicit, this bias is problematic because when relatively high-quality solutions are found in such regions, premature convergence is not avoided, so high-quality results are not obtained. We would like to devise some explicit strategies to control diversity, which might attain better results in the long-term and avoid the current bias that is included in MOEA/D-EVSD.

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Code

The code is available in C++ language in:
https://github.com/joelchaconcastillo/GECCO17_MOEA_D_MATING.git.

REFERENCES

[1] Slim Bechikh, Maha Elarbi, and Lamjed Ben Said. 2017. Many-objective Optimization Using Evolutionary Algorithms: A Survey. In *Recent Advances in Evolutionary Multi-objective Optimization*. 105–137.

[2] Nicola Beume, Boris Naujoks, and Michael Emmerich. 2007. SMS-EMOA: Multiobjective selection based on dominated hypervolume . *European Journal of Operational Research* 181, 3 (2007), 1653 – 1669.

[3] T. C. Chiang and Y. P. Lai. 2011. MOEA/D-AMS: Improving MOEA/D by an adaptive mating selection mechanism. In *2011 IEEE Congress of Evolutionary Computation (CEC)*. 1473–1480.

[4] S. Das and P. N. Suganthan. 2011. Differential Evolution: A Survey of the State-of-the-Art. *IEEE Transactions on Evolutionary Computation* 15, 1 (Feb 2011), 4–31.

[5] Kalyanmoy Deb and Deb Kalyanmoy. 2001. *Multi-Objective Optimization Using Evolutionary Algorithms*. John Wiley & Sons, Inc., New York, NY, USA.

[6] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan. 2002. A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation* 6, 2 (Apr 2002), 182–197.

[7] J. J. Durillo, A. J. Nebro, C. A. C. Coello, J. Garcia-Nieto, F. Luna, and E. Alba. 2010. A Study of Multiobjective Metaheuristics When Solving Parameter Scalable Problems. *IEEE Transactions on Evolutionary Computation* 14, 4 (Aug 2010), 618–635.

[8] Larry J. Eshelman. 1991. The CHC Adaptive Search Algorithm: How to Have Safe Search When Engaging in Nontraditional Genetic Recombination. In *Foundations of Genetic Algorithms*, Gregory J.E. Rawlins (Ed.), Vol. 1. Morgan Kaufmann Publishers, 265 – 283.

[9] Raquel Hernández Gómez, Carlos A. Coello Coello, and Enrique Alba. 2016. *A Parallel Version of SMS-EMOA for Many-Objective Optimization Problems*. Springer International Publishing, Cham, 568–577.

[10] Simon Huband, Luigi Barone, Lyndon While, and Phil Hingston. 2005. *A Scalable Multi-objective Test Problem Toolkit*. Springer Berlin Heidelberg, Berlin, Heidelberg, 280–295.

[11] S. Huband, P. Hingston, L. Barone, and L. While. 2006. A review of multiobjective test problems and a scalable test problem toolkit. *IEEE Transactions on Evolutionary Computation* 10, 5 (Oct 2006), 477–506.

[12] V.K. Koumousis and C.P. Katsaras. 2006. A saw-tooth genetic algorithm combining the effects of variable population size and reinitialization to enhance performance. 10, 1 (Feb 2006), 19–28.

[13] S. Kukkonen and K. Deb. 2006. Improved Pruning of Non-Dominated Solutions Based on Crowding Distance for Bi-Objective Optimization Problems. In *2006 IEEE International Conference on Evolutionary Computation*. 1179–1186.

[14] S. Kukkonen and J. Lampinen. 2005. GDE3: the third evolution step of generalized differential evolution. In *2005 IEEE Congress on Evolutionary Computation*, Vol. 1. 443–450 Vol.1.

[15] Antonio J. Nebro and Esteban Lpez-Camacho. 2016. jMetal. http://jmetalcpp.sourceforge.net/. (2016). [Online; accessed 24-March-2017].

[16] Yutao Qi, Xiaoliang Ma, Fang Liu, Licheng Jiao, Jianyong Sun, and Jianshe Wu. 2014. MOEA/D with Adaptive Weight Adjustment. *Evol. Comput.* 22, 2 (June 2014), 231–264.

[17] Carlos Segura, Carlos A. Coello Coello, and Alfredo G. Hernndez-Daz. 2015. Improving the vector generation strategy of Differential Evolution for large-scale optimization . *Information Sciences* 323 (2015), 106 – 129.

[18] C. Segura, C. A. Coello Coello, E. Segredo, and A. H. Aguirre. 2016. A Novel Diversity-Based Replacement Strategy for Evolutionary Algorithms. *IEEE Transactions on Cybernetics* 46, 12 (Dec 2016), 3233–3246.

[19] Abhishek Gupta Slim Bechikh, Rituparna Datta. *Recent Advances in Evolutionary Multi-objective Optimization*. springer.

[20] Matej Črepinšek, Shih-Hsi Liu, and Marjan Mernik. 2013. Exploration and Exploitation in Evolutionary Algorithms: A Survey. *Comput. Surveys* 45, 3 (July 2013), 35:1–35:33.

[21] Q. Zhang and H. Li. 2007. MOEA/D: A Multiobjective Evolutionary Algorithm Based on Decomposition. *IEEE Transactions on Evolutionary Computation* 11, 6 (Dec 2007), 712–731.

[22] Qingling Zhu, Qiuzhen Lin, Zhihua Du, Zhengping Liang, Wenjun Wang, Zexuan Zhu, Jianyong Chen, Peizhi Huang, and Zhong Ming. 2016. A novel adaptive hybrid crossover operator for multiobjective evolutionary algorithm . *Information Sciences* 345 (2016), 177 – 198.

[23] Eckart Zitzler, Marco Laumanns, and Stefan Bleuler. 2004. *A Tutorial on Evolutionary Multiobjective Optimization*. Springer Berlin Heidelberg, Berlin, Heidelberg, 3–37.