Multiobjective Discovery of Human-like Driving Strategies

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ABSTRACT

Human driving models aim at producing human-like driving strategies by mimicking the behavior of drivers. Drivers optimize several objectives when traveling along a route, such as the traveling time and the fuel consumption. However, these objectives are not taken into account when building human driving models. To overcome this shortcoming, we designed a two-level Multiobjective Optimization algorithm for discovering Human-like Driving Strategies (MOHDS) that combines the human driving models with the optimization of the traveling time and the fuel consumption. Consequently, MOHDS enables to simultaneously mimic human driving behavior and optimize relevant driving objectives. MOHDS was tested on a two-lane rural route and compared to the existing approaches for human driving modeling. The results show that, unlike the existing approaches, MOHDS finds the driving strategies with various tradeoffs between the objectives.

CCS CONCEPTS

• **Mathematics of computing** → *Stochastic control and optimization*; • **Applied computing** → *Transportation*;

KEYWORDS

driving strategy, human driving, traveling time, fuel consumption, multiobjective optimization

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1 INTRODUCTION

Autonomous vehicle driving consists of monitoring the vehicle and its surroundings, taking decisions about actions and applying the selected actions. This is a continuous process that optimizes several objectives such as the traveling time and the fuel consumption, while taking into account a set of constraints, e.g., safety constraints, route configuration, traffic rules, etc. Many automotive and other companies, e.g., Toyota [24] and Google [25], have been recently investigating autonomous vehicle driving. As a result, several driver assistance systems are already installed in modern vehicles, such as line assist (see, e.g., Volkswagen [35] and Toyota [32]). In addition, fully autonomous vehicles are starting to drive in urban environments [36].

The main focus of the autonomous driving solutions is on determining the vehicle surroundings, e.g., other vehicles, obstacles, and pedestrians, in order to increase safety and avoid collisions. The obtained driving strategy, however, may miss other objectives that are also important, such as the minimization of traveling time, the minimization of fuel consumption and consequently the reduction of pollution, etc. An important objective is also the acceptability of vehicle behavior by the passengers. More precisely, passengers do not want autonomous vehicle driving to be too unusual, different from their driving or worse than human driving [26]. For example, aggressive behavior of the autonomous vehicle is probably unacceptable for a calm driver. Consequently, the behavior of the autonomous vehicle has to be similar to human driving behavior.

Several objectives are conflicting, e.g., shortening the traveling time will result in increasing the fuel consumption. Therefore, all the objectives need to be considered simultaneously when constructing a driving strategy. Single-objective approaches are able to handle multiple objectives by combining them into a single-objective function. However, when several objectives have to be simultaneously optimized, it is preferable to use the multiobjective approach, since it enables to better explore the multiobjective search space in comparison to the single-objective approach.

The multiobjective approach finds a set of nondominated driving strategies, which enables to select a different strategy without restarting the algorithm when the requirements change or whenever the objectives are a matter of choice [27]. Moreover, Van

Willigen et al. [34] presented the idea of deploying nondominated driving strategies in adaptive cruise control of future intelligent vehicles. In this approach, a user can enter his/her preferences into the vehicle's cruise control at real time. Setting preferences corresponds to real-time selection of the driving strategy with the preferred values of the objectives.

Several methods for discovering driving strategies exist, but they are mostly based on the single-objective approach. In addition, they do not include driver behavior models, but optimize only the traveling time, fuel consumption, etc. Consequently, the obtained driving strategies are not similar to human driving strategies [7]. On the other hand, models that aim to emulate driver behavior are focused only on the replication of this behavior, while other objectives are not taken into account. To overcome the shortcomings of the existing methods, we propose a two-level Multiobjective Optimization algorithm for discovering Human-like Driving Strategies (MOHDS) that mimics human driving behavior and at the same time minimizes the traveling time and the fuel consumption. The lower-level algorithm consists of a set of mathematical models that mimic human driving behavior. The algorithm observes the vehicle state and the state of neighboring vehicles, and selects the best control actions. The upper-level algorithm is a multiobjective optimization algorithm based on DEMO [23, 33] and NSGA-II [6] that searches for the best values of the input parameters for the lower-level algorithm.

The paper is further organized as follows. Related work is presented in Section 2. Section 3 introduces the driving simulation environment. The MOHDS algorithm is described in Section 4. Section 5 presents the numerical experiments. Finally, Section 6 concludes the paper with ideas for future work.

2 RELATED WORK

Driving strategies that mimic human driving behavior can be obtained by using appropriate human driving models. Several models have been developed, where each of them is dedicated to a specific driving operation, such as car following, free driving, lane changing, overtaking, etc.

Car following models, a subset of acceleration models, describe the process of following the preceding vehicle on the same lane. The relation between the preceding vehicle and the following vehicle specifies that each individual vehicle always accelerates or decelerates as a response of its surrounding stimulus. The types of the models vary according to the definitions of the stimulus. Generally speaking, the stimulus may include the velocity and the acceleration of the vehicle, the relative velocity and spacing between the preceding and the following vehicle, etc. [19]. The Gazis-Herman-Rothery (GHR) model, also known as General Motors model, is the best-known stimulus-response model, which was initially proposed by Chandler et al. [5]. The model specifies the stimulus as the relative velocity of vehicles, that is, each vehicle tends to move at the same velocity as its preceding vehicle. Various versions of the model have been developed, which include distance to the preceding vehicle, velocity of the autonomous vehicle, different parameters for acceleration and deceleration phases, etc. [19].

Besides GHR, several other models for the car following operation have been proposed. The linear model relates the acceleration of the following vehicle to the desired following distance, velocity of the following vehicle, relative distance and velocity between the preceding and the following vehicle, and driver's reaction delay [38]. The collision avoidance model, also called the safety distance model, aims to obtain a safe following distance through the manipulation of the basic Newtonian equations of motion [12]. The optimal velocity model assumes that each driver tries to achieve an optimal velocity based on the distance to the preceding vehicle and the velocity difference between the vehicles. In addition, it also assumes that each driver seeks a safe following distance to its preceding vehicle [28]. The psychophysical model, also called the action point model, abstracts the stimuli to the relative motion of the fore-and-aft vehicles, including the velocity difference and the distance. The driver reacts appropriately when these stimuli fall beyond their threshold values [37]. The cellular automaton model describes the traffic as a stochastic discrete automaton model [30]. A fuzzy model uses a set of fuzzy rules to make decisions based on the status messages of the preceding vehicles [17]. Neural networks use the preceding and the following vehicle data to prescribe the car following behavior [21].

Lane changing and overtaking models prescribe the decision-making process of the corresponding driving operations. For example, lane changing on a motorway is usually handled with a lane-changing-desire model, a gap acceptance model and a gap selection model, while overtaking on a rural route is handled with a desire-to-overtake model and a gap acceptance model. Kusuma, Liu, and Francis [18] presented a model for gap acceptance behavior on the motorway. The vehicle driver may choose an available gap based on gap utilities and usually chooses a highest utility. Farah et al. [9, 10] presented overtaking models for two-lane rural routes including the desire-to-pass model and the passing gap acceptance model. The desire-to-pass model returns the utility to the driver from desiring to pass, while the gap acceptance model calculates the minimal acceptable gap. Toledo, Koutsopoulos and Ben-Akiva [31] proposed an integrated lane changing model that combines mandatory and discretionary lane changes into a single utility model. The lane changing process consists of two steps: choice of the target lane and gap acceptance decisions. Ahmed [2] developed a general utility-based framework that captures both mandatory and discretionary lane change situations. The lane changing operation is performed in three steps: a decision to consider the lane change, choice of the target lane and acceptance of gaps in the target lane. Toledo [29] developed a model for motorways, which integrates acceleration, gap acceptance and lane changing, and allows drivers to accelerate in order to facilitate lane changing. Different acceleration models apply depending on the target gap choice. The specification of acceleration models follows the GHR stimulus-response framework.

The human driving models aim at mimicking human driving behavior, however, they miss to achieve other objectives that are relevant when traveling along a route, such as the minimization of the traveling time and the fuel consumption. To optimize such additional objectives, several approaches have been proposed. They aim to minimize either the weighted sum of the fuel consumption and the traveling time, or the fuel consumption only, while considering the traveling time as a constraint. For example, dynamic programming methods that are based on the black-box approach have been developed to minimize the weighted-sum of the objectives [13]. Model-based analytical approaches minimize the weighted-sum of the objectives [15] or fuel consumption only [14]. However, only few researchers in fields such as racing games have focused on multiobjective optimization, without including both traveling time and fuel consumption as objectives. For example, Agapitos et al. [1] studied the driving strategy optimization of racing game competitors based on several objectives, such as avoiding collisions and minimizing steering changes. In our previous work, we have developed a multiobjective algorithm that searches for driving strategies on an empty route and minimizes the traveling time and the fuel consumption [8].

The existing methods for discovering driving strategies focus either on modeling human behavior or on the optimization of the traveling time, the fuel consumption and/or other objectives. This paper presents an algorithm that handles both tasks simultaneously: it ensures human-like driving strategies by implementing human driving models, and optimizes the traveling time and the fuel consumption by tuning the parameters of these models.

3 DRIVING SIMULATION

Vehicle driving is simulated on a two-lane rural route where overtaking is permitted. The simulated scenario includes an autonomous vehicle that is controlled by the autonomous driving algorithm, and a set of traffic vehicles that are controlled by the simulation. The simulation is performed step-wise until the entire route is simulated by the autonomous vehicle or the simulation becomes infeasible. When the simulation concludes, it returns the values of the objectives, i.e., the traveling time t and the fuel consumption c, and the driving feasibility.

3.1 Autonomous Vehicle

The autonomous vehicle has to be controlled with control actions defined as acceleration a, vehicle angle with respect to the route direction α, and the gear. The current gear is determined by considering the previous gear and the engine speed limits ($n_{\mathrm{e,l}}$ and $n_{\mathrm{e,u}}$) as described below. In addition, when selecting the acceleration, the vehicle constraints have to be considered.

The vehicle constraints are determined with a vehicle driving simulator that calculates the wheel friction force, aerodynamic drag force and tangential component of the g-force. These forces are combined in the engine moving force in case of acceleration, or tire braking force in case of deceleration. If the engine moving force exceeds the maximum engine moving force with respect to the current vehicle and route state (i.e., if the engine torque exceeds the maximum engine torque), the actual acceleration is decreased to meet the vehicle constraints. Similarly, if the tire braking force exceeds the maximum tire braking force with respect to the current vehicle and route state, the actual deceleration is decreased to meet the vehicle constraints.

To determine the current vehicle constraints, the current gear has to be selected. The vehicle driving simulator enables to select the current gear by defining the engine speed lower limit $n_{\mathrm{e,l}}$ and the engine speed upper limit $n_{\mathrm{e,u}}$. These limits are used to shift the gears: when the current engine speed exceeds the upper limit, the gear is shifted up; when the current engine speed drops below the lower limit, the gear is shifted down.

The vehicle driving simulator also returns the instantaneous fuel consumption that is obtained by taking into account the engine moving force and the specific fuel-consumption diagram. The vehicle driving simulator is described in detail in [8].

3.2 Traffic Vehicles

The driving simulation also includes a set of traffic vehicles. On the right, i.e., main lane, the traffic vehicles drive in the same direction as the autonomous vehicle, while on the left, i.e., overtaking lane, they drive in the opposite direction. The behavior of traffic vehicles changes with respect to the position of the autonomous vehicle. To that end, a set of action points has to be defined, where each action point determines the absolute position of the autonomous vehicle on the route. When the autonomous vehicle passes the action point, a new target velocity and/or distance between the traffic vehicles are assigned to traffic vehicles. The target velocity is not achieved immediately, but smoothly by applying a predefined acceleration.

3.3 Feasibility Checking

Feasibility checking is performed after each simulation step. The driving is not feasible if the autonomous vehicle stops/drives with a negative velocity, or if there is a collision between the autonomous vehicle and a traffic vehicle. In addition, the route has a velocity limit that must not be exceeded. Besides collision, vehicle stopping and velocity limit detection, the feasibility checking includes also the evaluation of the objective values. More precisely, the upper bounds for all the objectives (that are minimized) are given in advance. Afterwards, the simulation checks whether the current objective values exceed the upper bounds. If an upper bound is exceeded, the driving becomes infeasible. When this happens, the simulation stops. In this case, the distance to the end of the route is also returned in addition to the objective values.

4 ALGORITHM FOR DISCOVERING HUMAN-LIKE DRIVING STRATEGIES

This section presents the two-level Multiobjective optimization algorithm for discovering human-like driving strategies (MOHDS) that simultaneously mimics human driving behavior and minimizes the traveling time and the fuel consumption. The human driving behavior is obtained with the lower-level algorithm, LL-MOHDS, which applies a set of human driving models to determine the vehicle's control actions. The input parameters for the lower-level algorithm are searched by the upper-level algorithm, UL-MOHDS, which is a multiobjective optimization algorithm that minimizes the traveling time and the fuel consumption. The final result is a set of nondominated human-like driving strategies.

4.1 Lower-Level Algorithm for Discovering Human-like Driving Strategies

The lower-level algorithm (LL-MOHDS) consists of mathematical models that mimic human driving behavior and are able to handle the following driving operations: (a) car following, (b) free driving,

(c) emergency deceleration, and (d) overtaking. The car following, free driving and emergency deceleration models determine the vehicle acceleration a, while the overtaking models determine the vehicle angle with respect to the route direction α.

4.1.1 Acceleration Selection. The car following model is based on the GHR models [5, 19] and handles two phases: the acceleration phase and the deceleration phase as shown in Equation (1):

$$a_{\mathrm{cf}}(t)=\begin{cases} k_{\mathrm{cf,a}}v(t)^{k_{\mathrm{cf,a,v}}}\Delta s(t)^{k_{\mathrm{cf,a,s}}}\Delta v(t-\tau)\ ; & \Delta v(t-\tau)>0\\ k_{\mathrm{cf,d}}v(t)^{k_{\mathrm{cf,d,v}}}\Delta s(t)^{k_{\mathrm{cf,d,s}}}\Delta v(t-\tau)\ ; & \Delta v(t-\tau)<0\\ 0\ ; & \Delta v(t-\tau)=0\end{cases} \tag{1}$$

where $\Delta v(t-\tau)=v_{\mathrm{p}}(t-\tau)-v(t-\tau)$, τ is the human reaction time, v_{p} is the velocity of the preceding vehicle, $\Delta s(t)$ is the distance to the preceding vehicle, and $k_{\square}$ are model parameters.

The free driving model handles acceleration when there is no vehicle to follow. In this case an appropriate acceleration is used until the target velocity v_{t} is achieved. This acceleration is calculated as a function of the current velocity [22] as shown in Equation (2):

$$a_{\mathrm{fd}}(t)=\begin{cases} k_{\mathrm{fd,a}}v(t)+n_{\mathrm{fd,a}}\ ; & v(t)<v_{\mathrm{t}}\\ 0\ ; & v=v_{\mathrm{t}}\\ k_{\mathrm{fd,d}}v(t)+n_{\mathrm{fd,d}}\ ; & v(t)>v_{\mathrm{t}}\end{cases} \tag{2}$$

where $k_{\square}$ and $n_{\square}$ are model parameters.

The emergency deceleration model is applied when the autonomous vehicle is too close to the preceding vehicle [22] and is defined as shown in Equation (3):

$$a_{\mathrm{ed}}(t)=\begin{cases} \min\left\{a_{\mathrm{fd,d}}(t),a_{\mathrm{p}}(t)-0.5\frac{\Delta v(t-\tau)^2}{\Delta s(t)}\right\}\ ; & \Delta v(t-\tau)<0\\ \min\left\{a_{\mathrm{fd,d}}(t),a_{\mathrm{p}}(t)+0.25a_{\mathrm{fd,d}}(t)\right\}\ ; & \Delta v(t-\tau)\geq 0\end{cases} \tag{3}$$

where $a_{\mathrm{fd,d}}(t)=k_{\mathrm{fd,d}}v(t)+n_{\mathrm{fd,d}}$ and a_{p} is the acceleration of the preceding vehicle.

At each time step, only one of these models defines the vehicle acceleration, a_{m}. The model selection is based on the time headway h to the preceding vehicle. If the time headway is larger than the upper threshold h_{u}, the vehicle is not constrained by the preceding vehicle and the free driving model is applied. If the time headway is between the upper threshold h_{u} and the lower threshold h_{l}, the car following model is applied. If the time headway is smaller than h_{l}, the vehicle is too close to the preceding vehicle and the emergency deceleration model is applied to extend the headway [22]. This procedure is summarized in Equation (4).

$$a_{\mathrm{m}}(t)=\begin{cases} a_{\mathrm{fd}}(t)\ ; & h>h_{\mathrm{u}}\\ a_{\mathrm{cf}}(t)\ ; & h_{\mathrm{u}}\geq h>h_{\mathrm{l}}\\ a_{\mathrm{ed}}(t)\ ; & h\leq h_{\mathrm{l}}\end{cases} \tag{4}$$

The selection of vehicle acceleration based on car following, free driving and emergency deceleration models has been enhanced in our algorithm to take into account the target velocity v_{t} in all three modes. Although the existing models include the target velocity, this is applied for free driving only [22]. Consequently, when a vehicle follows the preceding vehicle (by applying the car following model), the target velocity is not considered and can be exceeded. LL-MOHDS, on the other hand, applies the target velocity limit during the entire driving. More precisely, at each step the acceleration to achieve the target velocity, $a_{\mathrm{t}}(t)$, is calculated using Equation (2). Finally, the vehicle acceleration $a(t)$ is calculated as shown in Equation (5).

$$a(t)=\min\{a_{\mathrm{m}}(t),a_{\mathrm{t}}(t)\} \tag{5}$$

LL-MOHDS applies two target velocities, one for the main lane, $v_{\mathrm{t,m}}$, and one for the overtaking lane, $v_{\mathrm{t,o}}$.

Although LL-MOHDS selects the vehicle acceleration as shown in Equation (5), the actually applied acceleration can be lower due to the vehicle constraints. For example, the vehicle might not be able to achieve a very high acceleration defined by LL-MOHDS. The vehicle constraints are taken into account as described in Section 3.1.

4.1.2 Handling Overtaking. The overtaking operation is handled with two decision-making models, i.e., the desire-to-pass model and the gap acceptance model. The desire-to-pass model returns the utility to the driver from desiring to pass [10] as shown in Equation (6):

$$u_{\mathrm{dp}}(t)=k_{\mathrm{dp}}+k_{\mathrm{dp,dv}}\Delta v_{\mathrm{t}}(t)+k_{\mathrm{dp,ds}}\Delta s(t) \tag{6}$$

where $\Delta v_{\mathrm{t}}=v_{\mathrm{t}}-v_{\mathrm{p}}$ and $k_{\square}$ are model parameters. If $u_{\mathrm{dp}}(t)\geq 0$, the vehicle desires to pass.

The gap acceptance model calculates the minimal gap that has to be available in order to start the overtaking operation [10]. The minimal acceptable gap is calculated according to Equation (7):

$$g_{\mathrm{ga}}(t)=k_{\mathrm{ga}}+k_{\mathrm{ga,v}}v(t)+k_{\mathrm{ga,vp}}v_{\mathrm{p}}(t)+k_{\mathrm{ga,vo}}v_{\mathrm{o}}(t) \tag{7}$$

where $k_{\square}$ are model parameters and v_{o} is the velocity of the vehicle on the overtaking lane which drives in the opposite direction. If $g_{\mathrm{o}}(t)>g_{\mathrm{ga}}(t)$, the gap is accepted, where $g_{\mathrm{o}}(t)$ is the current gap on the overtaking lane.

In addition, the autonomous vehicle starts to overtake the preceding vehicle only when the gap to the preceding vehicle, $g_{\mathrm{p}}(t)$, is bellow the threshold g_{t}. Equation (8) shows the decision-making process for the overtaking operation.

$$\mathrm{OVERTAKE}(t)=\begin{cases} \text{yes}\ ; & g_{\mathrm{p}}(t)<g_{\mathrm{t}}\ \text{AND}\ u_{\mathrm{dp}}(t)\geq 0\ \text{AND}\\ & g_{\mathrm{o}}(t)>g_{\mathrm{ga}}(t)\\ \text{no}\ ; & \text{otherwise}\end{cases} \tag{8}$$

While overtaking, the autonomous vehicle has to change the lane twice (from the main, i.e., right lane to the overtaking, i.e., left lane and vice versa). While the beginning of overtaking is determined with Equation (8), the overtaking conclusion, i.e., lane changing again to the main lane, is performed only when the autonomous vehicle is in front of the overtaken vehicle and the distance between them is above the rear gap threshold $g_{\mathrm{t,o}}$.

Each lane change is performed by controlling the vehicle angle α. To this end, LL-MOHDS applies quadratic Bézier curves [3] and De Casteljau's algorithm [4]. The vehicle position y_{c} during the lane change is calculated from the starting position y_{s} on the current lane, the target position y_{t} on the target lane (i.e., the middle of the target lane), and the lane change percentage p_{lc} [16] as shown in Equation (9):

$$y_{\mathrm{c}}(t)=\frac{(1-p_{\mathrm{lc}})^2}{p_{\mathrm{lc}}^2+(1-p_{\mathrm{lc}})^2}y_{\mathrm{s}}+\left(1-\frac{(1-p_{\mathrm{lc}})^2}{p_{\mathrm{lc}}^2+(1-p_{\mathrm{lc}})^2}\right)y_{\mathrm{t}} \tag{9}$$

where y is the lateral position of the vehicle with respect to the route direction.

The difference in lateral position during one step is calculated as $\Delta y_c(t) = y_c(t) - y_c(t-1)$, and the traveled route in one step is calculated as $\Delta s(t) = s(t) - s(t-1)$. Finally, the vehicle angle is calculated as shown in Equation (10).

$$\alpha(t) = \arcsin \frac{\Delta y_c(t)}{\Delta s(t)} \tag{10}$$

4.1.3 Gear Shifting. The autonomous vehicle is controlled also by gear shifting, in addition to the vehicle acceleration and driving angle. As described in Section 3.1, gear shifting requires to define the engine speed lower limit $n_{e,l}$ to shift the gear down, and the engine speed upper limit $n_{e,u}$ to shift the gear up. LL-MOHDS defines the engine speed limits for the main lane, i.e., $n_{e,ml}$ and $n_{e,mu}$, and for the overtaking lane, i.e., $n_{e,ol}$ and $n_{e,ou}$.

4.2 Upper-Level Algorithm for Discovering Human-like Driving Strategies

The upper-level algorithm (UL-MOHDS) is a multiobjective optimization algorithm based on DEMO [23, 33] and NSGA-II [6]. It searches for the best values of the input parameters for LL-MOHDS by optimizing two objectives: the traveling time t and the fuel consumption c. While LL-MOHDS assures that the obtained driving strategy is human-like due to the usage of appropriate human driving models, UL-MOHDS enables to find the driving strategies with short traveling time and low fuel consumption.

The set of input-parameter values is stored in an upper-level solution and encoded as a vector of numeric values forming a chromosome. The chromosome encodes the following parameters:

- Target velocities for main and overtaking lanes: $v_{t,m}$ and $v_{t,o}$
- Free driving model parameters for acceleration phase: $k_{fd,a}$ and $n_{fd,a}$
- Free driving model parameters for deceleration phase: $k_{fd,d}$ and $n_{fd,d}$
- Car following model parameters for acceleration phase: $k_{cf,a}$, $k_{cf,a,v}$ and $k_{cf,a,s}$
- Car following model parameters for deceleration phase: $k_{cf,d}$, $k_{cf,d,v}$ and $k_{cf,d,s}$
- Time headway upper and lower thresholds: h_u and h_l
- Gap acceptance model parameters: k_{ga}, $k_{ga,v}$, $k_{ga,vp}$, and $k_{ga,vo}$
- Desire-to-pass model parameters: k_{dp}, $k_{dp,dv}$ and $k_{dp,ds}$
- Rear gap threshold: $g_{t,o}$
- Engine speed upper and lower limits for main lane: $n_{e,mu}$ and $n_{e,ml}$
- Engine speed upper and lower limits for overtaking lane: $n_{e,ou}$ and $n_{e,ol}$

In addition to boundary constraints for the parameters, the following constraints are also to be satisfied:

- $v_{t,m} \leq v_{t,o}$
- $h_l \leq h_u$
- $n_{e,ml} \leq n_{e,mu}$
- $n_{e,ol} \leq n_{e,ou}$
- $k_{fd,a} v_{MAX} + n_{fd,a} \geq 0$
- $k_{fd,d} v_{MAX} + n_{fd,d} \leq 0$

where v_{MAX} is the maximum vehicle velocity.

UL-MOHDS operates with a set of solutions called the population, which is improved through a number of generations. At each generation, for each solution, i.e., parent S_i in the population, a new candidate solution is created using the scheme *DE/rand/1/bin* as follows [23]:

- Randomly select three solutions $S_{i_1}, S_{i_2}, S_{i_3}$ from the population, where i, i_1, i_2 and i_3 are pairwise different.
- Calculate candidate C as $C = S_{i_1} + F(S_{i_2} - S_{i_3})$, where F is a scaling factor.
- Modify the candidate with the parent S_i using the binary crossover with crossover probability p_c.
- Repair the candidate if it falls out of the decision space bounds.

Afterward, the candidate solution is evaluated by applying LL-MOHDS and:

- The candidate solution replaces the parent in the population if it dominates the parent.
- The candidate solution is discarded if the parent dominates it.
- The candidate solution is added to the population if the candidate solution and the parent are incomparable.

After each generation, the best solutions are selected for the population in the next generation in order to maintain a constant population size between the generations. This is carried out using the Fast Nondominated Sort and Crowding Distance mechanisms from the Nondominated Sorting Genetic Algorithm (NSGA-II) [6]. Finally, the UL-MOHDS algorithm returns a set of nondominated driving strategies.

5 EXPERIMENTS AND RESULTS

MOHDS was tested on a two-lane rural route and the results were compared with the results of the existing models for human driving. The route and traffic definition, parameters for the algorithm and the results are presented in the following subsections.

5.1 Experimental Setup

The length of the test two-lane rural route was 46 km. The velocity limit along the entire route was 90 km/h. The velocity of the traffic vehicles and the distance between the traffic vehicles on the same lane varied as described in Section 3.2, where the sets of velocities and distances are shown in Table 1. In particular, all the combinations of velocities and distances were applied on the route. Therefore, for all $s_o \in S_o$, each combination $\{v_m, v_o\}$, $v_m \in V_m$ and $v_o \in V_o$, was applied for a distance of $2s_o$. Consequently, the action points were defined as twice the distances between the vehicles on the overtaking lane. Such a fixed configuration was selected to enable a fair comparison between the driving strategies with respect to the objective values.

Table 1: Traffic Vehicle Parameters

Name	Description	Unit	Values
S_m	Distance between vehicles on the right lane	m	150
S_o	Distance between vehicles on the left lane	m	[300, 800, 1500]
V_m	Velocity on the right lane	km/h	[80, 50, 20]
V_o	Velocity on the left lane	km/h	[90, 60, 30]

Table 2: Decision Space Bounds

Parameter	Unit	Initial population	During evolution
$v_{\mathrm{t,m}}$	km/h	[85, 90]	[40, 90]
$v_{\mathrm{t,o}}$	km/h	[85, 90]	[70, 90]
$k_{\mathrm{fd,a}}$		[-0.5, 0]	
$n_{\mathrm{fd,a}}$		[5, 10]	
$k_{\mathrm{fd,d}}$		[0, 0.5]	
$n_{\mathrm{fd,d}}$		[-10, -5]	[-10, 0]
$k_{\mathrm{cf,a}}$		[-10, 10]	
$k_{\mathrm{cf,a,v}}$		[-0.8, 2.5]	[-10, 10]
$k_{\mathrm{cf,a,s}}$		[-3, 0]	[-10, 10]
$k_{\mathrm{cf,d}}$		[-10, 10]	
$k_{\mathrm{cf,d,v}}$		[0, 2]	[-10, 10]
$k_{\mathrm{cf,d,s}}$		[-3, 0]	[-10, 10]
h_{u}, h_{l}	s	[0.1, 3]	
k_{ga}		[24, 34]	[0, 50]
$k_{\mathrm{ga,v}}$		[-0.5, -0.1]	[-10, 10]
$k_{\mathrm{ga,vp}}$		[0.3, 0.6]	[-10, 10]
$k_{\mathrm{ga,vo}}$		[-0.2, 0]	[-10, 10]
k_{dp}		[-0.7, -0.3]	[-10, 0]
$k_{\mathrm{dp,dv}}$		[0, 0.2]	[0, 10]
$k_{\mathrm{dp,ds}}$		[-0.1, 0]	[-10, 10]
$g_{\mathrm{t,o}}$	m	[5, 50]	
$n_{\mathrm{e,mu}}$, $n_{\mathrm{e,ml}}$, $n_{\mathrm{e,ou}}$, $n_{\mathrm{e,ol}}$	/min	[960, 6000]	

Table 3: UL-MOHDS Parameters

Parameter	Value
Population size	50
Number of generations	50
Crossover probability p_{c}	0.9
Scaling factor F	0.5

The parameters of UL-MOHDS are shown in Tables 2 and 3. Table 2 presents the bounds of the decision space, while Table 3 presents the parameters of the UL-MOHDS algorithm. Note that the bounds for the initial population and during the evolution were different. For the initial population we used the intervals covering the values from related work. For the evolution wider intervals were used in order for the algorithm to consider the traveling time and the fuel consumption in addition to mimicking the human driving behavior.

5.2 Experimental Results

MOHDS is a stochastic algorithm, therefore, it was run 10 times to obtain the driving strategies. The obtained strategies were combined in attainment curves[1] that divide the objective space in attainment surfaces [11] as shown in Figure 1. These results show that the quality of the driving strategies obtained in various algorithm runs was similar. In addition, the objective values of the discovered driving strategies show that the fuel consumption can be reduced by up to 73 % with respect to the maximal fuel consumption of nondominated driving strategies. Similarly, the traveling time can be reduced by up to 55 % with respect to the maximal traveling time of nondominated driving strategies.

[1]The attainment curves were obtained using the PISA library, available online: http://www.tik.ee.ethz.ch/pisa/.

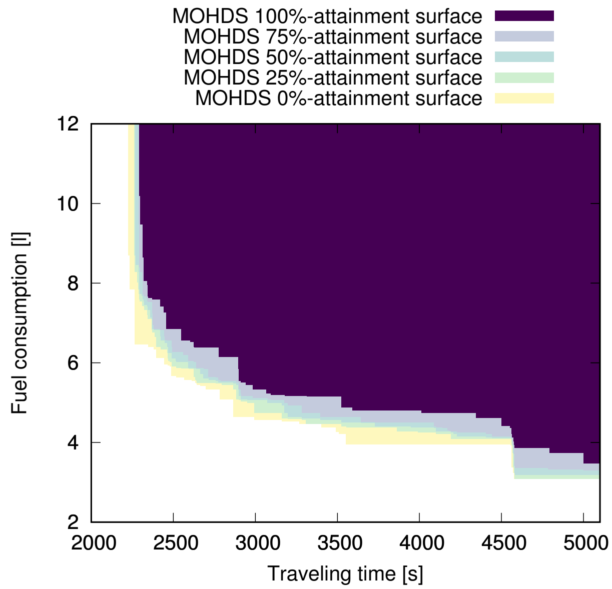

Figure 1: Driving strategies in objective space, obtained in 10 runs of the MOHDS algorithm.

Figure 2 shows the comparison between the MOHDS driving strategies obtained in various simulation runs and the driving strategies with similar parameter values as the existing human driving models that are presented in [2, 10, 19]. These results indicate that the existing driving models can result in either low fuel consumption when only the car following operation is applied (right-hand side of the figure) or short traveling time when overtaking is also applied (left-hand side of the figure). On the other hand, MOHDS enables to find a large set of nondominated driving strategies with various tradeoffs between the traveling time and the fuel consumption. This enables, e.g., to shorten the traveling time without significantly increasing the fuel consumption. The results also show that MOHDS (on average) finds better driving strategies than the existing human driving models.

Examples of the vehicle's behavior obtained when applying a selected number of driving strategies (marked as s_1, s_2 and s_3 in Figure 2) can be seen in Figure 3. Driving strategy s_1 has a short traveling time, s_3 has low fuel consumption, while s_2 has a good balance between both objectives. The first (top) subfigure shows the lateral position, which is 0 in the center of the main lane, and 3.5 in the center of the overtaking lane. The second subfigure shows the vehicle velocity. Note that the velocity limit is 90 km/h along the entire route. The third subfigure shows the distance from the autonomous vehicle to the vehicle in front of it on the same lane. During overtaking, this is the distance to the vehicle on the overtaking lane driving in the opposite direction. The fourth subfigure shows the difference between the velocity of the vehicle in front

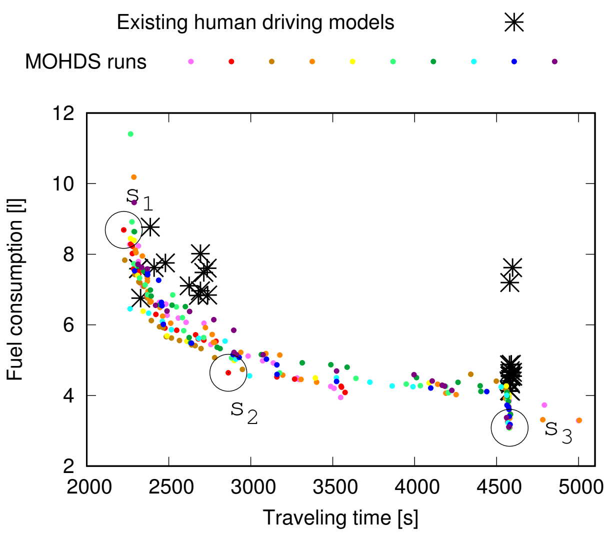

Figure 2: Comparison of driving strategies in objective space between the MOHDS algorithm and existing human driving models.

of the autonomous vehicle and the velocity of the autonomous vehicle. Finally, the fifth (bottom) subfigure shows the cumulative fuel consumption. These results show that s_3 (the driving strategy with low fuel consumption) never overtakes and consequently always follows the preceding vehicle. Its traveling time represents a lower bound for the traveling time of driving strategies that never overtake. As can be seen in Figure 2, other driving strategies with similar traveling time exist, which vary in fuel consumption. As a consequence, MOHDS enables to optimize the fuel consumption even when applying the car following operation only. These results also show that driving strategies s_1 and s_2 combine the overtaking and the car following operations. Among them, s_1 has lower traveling time due to performing the overtaking operation more frequently, and applying higher target velocities on the main and overtaking lanes.

The obtained results can be potentially applied in the fields of autonomous vehicle driving and evaluation of drivers. As suggested by Van Willigen et al. [34], nondominated driving strategies can be deployed in adaptive cruise control of future intelligent vehicles, where a user will be able to select the driving strategy according to his/her preferences. The evaluation of drivers consists of determining the quality of their driving (e.g., near or far from nondominated driving strategies) or classifying the drivers (e.g., fast drivers, low-consumption drivers, far-from-optimal drivers). For example, Lin et al. [20] suggested to use driver models for classifying the drivers into three skill levels: lower, typical, and expert.

6 CONCLUSIONS

We have designed and tested a Multiobjective Optimization algorithm for discovering Human-like Driving Strategies (MOHDS) that applies human driving models and minimizes the traveling time and the fuel consumption. MOHDS is a two-level algorithm, where the lower level consists of a set of human driving models handling

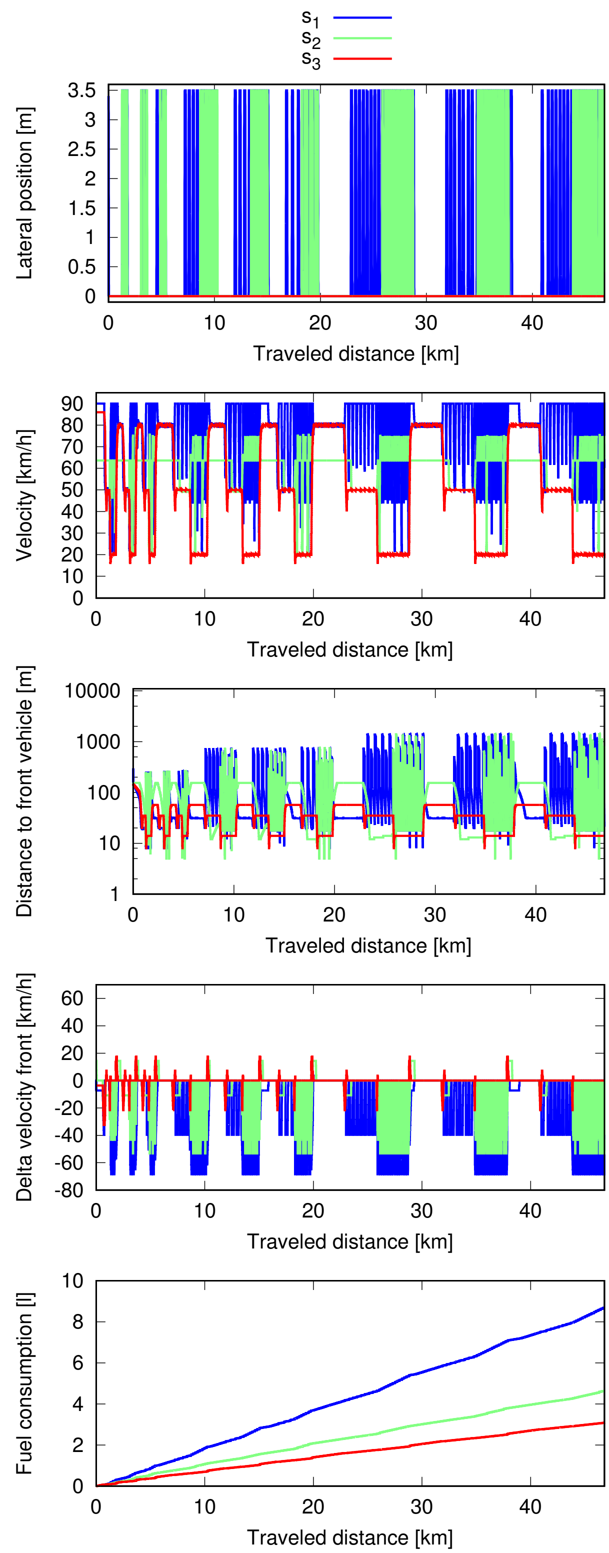

Figure 3: Examples of vehicle behavior obtained by applying the MOHDS driving strategies (s_1, s_2 and s_3 from Figure 2).

various driving operations, and the upper level is a multiobjective evolutionary algorithm that searches for the best input parameters for the lower level algorithm by minimizing the traveling time and the fuel consumption. MOHDS returns a set of nondominated driving strategies with short traveling time and low fuel consumption, which mimic the human driving behavior due to the use of human driving models.

The driving strategies found with MOHDS were compared to driving strategies obtained by applying the existing approaches for obtaining human-like driving strategies. The results show that MOHDS is beneficial since it simultaneously optimizes the human driving models for various driving operations, while, in addition, minimizing the objectives that are relevant when driving along a route. Moreover, MOHDS enables to find the driving strategies with various tradeoffs between the objectives in comparison to the existing approaches.

In our future work we will extend MOHDS to handle additional driving operations, such as lane changing on the motorway which requires dedicated human driving models. In addition, we will test the algorithm on various routes. It would be also interesting to obtain data on human driving and compare them with the driving strategies found with MOHDS. Consequently, a third objective could be added to MOHDS, i.e., similarity with human driving behavior.

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