# ED-LS - A Heuristic Local Search for the Firefighter Problem

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## ABSTRACT

This abstract summarizes the results reported in the paper [4]. In this paper a new method of performing the local search for the multiobjective Firefighter Problem (FFP) is proposed. The proposed method reduces the size of the neighbourhood in which the local search looks for improved solutions by using heuristics to decide which solutions in the neighbourhood should be visited.

In the paper the proposed local search method is used for improving solutions produced by two commonly used evolutionary algorithms: the MOEA/D and the NSGA-II. In the experiments the proposed method outperformed both the evolutionary algorithms without any local search (the 'None' method) as well as the algorithms combined with the local search visiting all solutions in the neighbourhood (the 'Full' method). Comparison to the local search selecting the same number of solutions as the ED-LS, but at random (the 'Random' method) shows that the proposed heuristics are indeed selecting good solutions to visit.

Presented research can be extended in further studies. One possibility is to study in-depth how to balance computational resources between the evolutionary and local search algorithms. Second possibility is to employ more advanced models instead of the heuristics used in the paper.

## CCS CONCEPTS

• **Mathematics of computing → Evolutionary algorithms**; • **Applied computing → Multi-criterion optimization and decision-making**;

## KEYWORDS

Graph-based Optimization, Multiobjective optimization, Combinatorial problems

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## 1 INTRODUCTION

The Firefighter Problem (FFP) [2] is a combinatorial optimization problem that concerns the prevention of the spreading of a threat in a graph. It can be used as an abstraction of real-life problems, such as epidemics prevention, fire and flood containment, systemic risk minimization (e.g. in the context of cascading bankruptcies or infrastructure failures). In the FFP the spreading of fire is modelled on an undirected graph $G = \langle V, E \rangle$ with $N_v$ vertices. The vertices of the graph can be in one of the states 'B' - burning, 'D' - defended, or 'U' - untouched. The state of the graph changes in discrete time steps $t = 0, 1, \ldots$ with fire propagating from 'B' vertices to adjacent 'U' vertices. The state of the vertices at time $t$ is represented by an $N_v$-element vector $S_t \in \{ \text{'B'}, \text{'D'}, \text{'U'} \}^{N_v}$ in which an element $S_t[v]$ represents the state of vertex $v$ at time $t$. Initially (at $t = 0$) some of the graph vertices are burning and in each of the subsequent time steps two actions changing the state of the graph take place. First, $N_f$ vertices become defended - they change their state to 'D'. Then, fire spreads from burning vertices to the untouched ones. Solutions in the FFP are permutations which indicate in what order to defend the vertices in the graph. An evaluation of a solution $\pi$ is performed by simulating the spreading of fire starting from the initial state $S_0$. At each subsequent time step the first $N_f$ elements from $\pi$ are selected for which the corresponding vertices are in the state 'U'. The vertices corresponding to these elements of $\pi$ become defended. After the simulation stops, the evaluation of the solution $\pi$ is calculated as the number of non-burning nodes (in the single-objective FFP) or by summing, for the non-burning vertices, $m$ weights assigned to each of them (in the $m$-objective FFP).

In this paper a local search method is studied which improves solutions of the FFP found by evolutionary algorithms. Because in the FFP solutions are permutations, the neighbourhood of a given solution $\pi$ can be explored by applying transpositions which swap pairs of elements of $\pi$. The method presented in the paper uses heuristics to assess which transpositions can be expected to produce good solutions. Exploring neighbourhoods of smaller size improves the performance in comparison to visiting neighbours generated by all possible transpositions of the elements of $\pi$.

## 2 THE ED-LS ALGORITHM

The ED-LS is a local search method that is applied to an initial solution $\pi$, which in this paper is produced by an evolutionary algorithm. This method uses two heuristics to decide which transpositions can be expected to produce good results when applied to $\pi$. These heuristics prioritize vertices close to fire and with a high degree, because results in previous works on the FFP [1, 5] suggest that these characteristics of graph vertices make them good locations to defend.

Both heuristics used in the ED-LS are calculated for a given graph state $S_t$ for a vertex $v$. $E(S_t, v)$ is the number of edges that separate $v$ from fire in the graph state $S_t$. The minimum number of time steps that are left before fire reaches vertex $v$ is $E(S_t, v)$, because fire can only travel one graph edge per a time step. Defending vertices with lower values of $E(S_t, v)$ is preferable, because doing

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so may help cutting off fire from other vertices and preventing its spread. $D(S_t, v)$ is the "untouched degree" of $v$, that is, the number of edges adjacent to $v$ leading to vertices that are in the state 'U' at the time step $t$. Defending vertices with higher values of $D(S_t, v)$ is preferable, because if fire gets to them it can spread to many undefended vertices. Vertex $w$ is preferred to $v$ if it is closer to fire or if both vertices are equally close to fire, but $w$ has a higher "untouched degree":

$$\left[ \begin{array}{l} E(S_t, w) < E(S_t, v) \quad \text{or} \\ \Big(E(S_t, w) = E(S_t, v) \text{ and } D(S_t, w) > D(S_t, v)\Big) \end{array} \right] \tag{1}$$

To determine which elements of $\pi$ to transpose, generating new solutions in the neighbourhood of $\pi$, the ED-LS performs the following steps.

(1) An $N_v \times N_v$ Boolean array $T$ is created with all elements initially set to *false*. This array will store information which transpositions to check.
(2) The spreading of fire is simulated starting from the initial state $S_0$. The solution $\pi$ is used to determine which vertices of the graph $G$ to protect as described in the introduction.
(3) At each time step $t$ of the simulation the elements $T[v, w]$ and $T[w, v]$ are set to *true* if:
   - $v$ became defended at the time step $t$, i.e. $S_{t-1}[v]$ = 'U' and $S_t[v]$ = 'D'
   - $w$ is still untouched at the time step $t$, i.e. $S_t[w]$ = 'U'
   - condition (1) holds for $v$ and $w$

When performing the local search around $\pi$, the ED-LS methods checks only the neighbours of $\pi$ generated by swapping these elements $i$ and $j$ for which $T[\pi[i], \pi[j]] = true$. Formally, the neighbourhood $N(\pi)$ explored by the ED-LS method can be defined as:

$$N(\pi) = \left\{ \pi' \in \Pi_{N_v} : \exists_{\substack{1 \le i < j \le N_v \\ T[\pi[i],\pi[j]]=true}} \Big( (\pi'_i = \pi_j) \wedge (\pi'_j = \pi_i) \wedge (\forall_{\substack{k \ne i \\ k \ne j}} \pi'_k = \pi_k) \Big) \right\}, \tag{2}$$

In the experiments the size of the neighbourhood in the ED-LS was reduced to a few percent of that used by the 'Full' method.

## 3 EXPERIMENTS AND RESULTS

In the experiments the proposed method was tested with two evolutionary algorithms: NSGA-II and MOEA/D solving 2-objective FFP instances with $N_v$ = 50, 75, 100, 125, 150, 175, 200, 225 and 250 vertices and 3-objective FFP instances with $N_v$ = 50, 75, 100, 125 vertices. Also, the local search was performed using three different solution acceptance criteria: Pareto-based (PLS), decomposition-based (DLS) and directional (DirLS) [3]. The ED-LS was compared to evolutionary algorithms without a local search (denoted 'None'), EAs with a local search searching the entire neighbourhood ('Full') and a local search which selected the same number of solutions as ED-LS, but randomly without using any heuristics ('Random'). Results were compared using a median of the hypervolume attained in 30 runs with respect to the number of solution evaluations $E_{max}$ and also with respect to the maximum running time $T_{max}$. Table 1 presents a brief summary of the results presented in the paper.

**Table 1: Summary of the results.**

| | $NSGA-II$ | | $MOEA/D$ | |
|---|---|---|---|---|
| | $E_{max}$ | $T_{max}$ | $E_{max}$ | $T_{max}$ |
| Number of better median hypervolumes in 27 tests (2 objectives) | | | | |
| ED-LS > Full | 27 | 27 | 27 | 26 |
| ED-LS > None | 27 | 27 | 26 | 27 |
| Full > None | 23 | 23 | 17 | 18 |
| ED-LS > Random | 27 | 27 | 27 | 27 |
| Number of better median hypervolumes in 12 tests (3 objectives) | | | | |
| ED-LS > Full | 12 | 12 | 12 | 12 |
| ED-LS > None | 12 | 12 | 10 | 11 |
| Full > None | 0 | 6 | 2 | 4 |

The comparison between the 'Full' and 'None' methods shows that performing a local search using transposition-based neighbourhoods is more effective than not performing the local search at all. On the other hand the ED-LS clearly outperforms both 'None' and 'Full' local searches. Comparison with the 'Random' local search shows that the proposed heuristics indeed point to good solutions.

## 4 CONCLUSION

In the paper a local search method was presented that uses heuristics for deciding which solutions to visit in the neighbourhood of an initial solution $\pi$. Results presented in the paper show that the proposed method effectively solves the Firefighter Problem with 2 and 3 objectives. This paper suggests two directions of further research. One is to develop methods for balancing the use of computational resources between the global and local search, because preliminary results presented in the paper shown that it can be beneficial not to perform the local search in initial generations of the evolutionary algorithm. The other possibility is to employ more advanced models instead of the heuristics to decide which neighbours to visit.

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