GA and Entropy Objective Function for Solving Sudoku Puzzle

Katya Rodríguez-Vázquez
IIMAS, UNAM
Circuito Escolar, CU, 04510
México
katya.rodriguez@iimas.unam.mx

ABSTRACT

In this paper, a genetic algorithm for solving Sudoku puzzles is presented. Objective function has been defined as maximization of an entropy function in order to get a solution of Sudoku by generating rows, columns and 3x3 sub-matrices containing each integer [1, 2, 3, 4, 5, 6, 7, 8 ,9] without duplication, for the case of 9x9 grid puzzle. A permutation and row-crossover operators are designed to this problem. The proposed algorithm is tested on different instances of Sudoku: easy and multimodal Sudoku. Simulation results show a competitive performance for these two instances of Sudoku.

CCS CONCEPTS

• **Computing Methodologies → Artificial Intelligence**; *Search Methodologies*

KEYWORDS

Sudoku puzzle, Genetic Algorithms, Entropy Fitness Function, Combinatorial Problems, Benchmarking

1 INTRODUCTION

Sudoku puzzles can be considered a combinatorial optimisation problem for benchmarking. Typically, Sudoku puzzle is composed of a 9x9 matrix (81 positions) which is divided into nine 3x3 sub-matrices. The idea is to fill matrix with each integer [1, 2, 3, 4, 5, 6, 7, 8, 9] without repeating integer by row, column and 3x3 sub matrix starting from some givens clues (givens values for some grids). There are also Sudoku puzzles of higher dimension (e.g. 16x16 or 25x25 Sudoku matrices) but few algorithms to solve such Sudoku dimensions have been reported. One of these approaches is proposed by (Horn, 2017). But, even for the 3x3 dimension, there are different instances (easy, medium, difficult, expert and multimodal that makes Sudoku puzzle a challenging problem for benchmarking.

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Also, many evolutionary and bioinspired approaches have been reported in the literature in order to solve this problem. Examples of them are the work presented by Deng and Li (2013) introducing a hybrid approach based on genetic algorithm and particle swarm optimization and an adaptive probability of mutation operator. Sato and Inoue (2010) proposed a genetic algorithm that preserve building blocks when applying genetic operators and uses a local search strategy. On the other hand, an important issue in any applications bioinspired algorithm and even more any metaheuristics, is the definition of objective function. In this paper, the optimization of an entropy fitness function is proposed. The advantage of this function is that constraints about no duplication of integer in each row, column or sub-matrix are implicit in objective function. Examples of similar proposal are presented in Gunther and Moon (2012), Pang et al (2010) and Zhai and Zhang (2013).

2 ENTROPY FITNESS FUNCTION

In general term, entropy can be defined as a measure of the disorder or uncertainty. Using this idea, the measure of information entropy associated with each possible value in each grid of Sudoku matrix is the negative logarithm of the probability function for the value. A variation of the entropy shows a variation of the order and, an increment of entropy reflects an increment of diversity of values in each row, column or sub-matrix of Sudoku solution. This concept of information entropy was introduced in 1948 by Claude Shannon. Then, the objective is to maximise the entropy and this occurs when all values in each row, in each column and in each sub-matrix are different. Thus, the information entropy can be defined as,

$$h(X) = -\sum_{i=1}^{n} p(x_i) log\left(p(x_i)\right) \tag{1}$$

where $p(x_i)$ is the probability of presence of symbol x_i. In terms of Sudoku puzzle, the objective function is defined as follows.

$$fr_i = -\sum_{j=1}^{9} p(x_i) log\left(p(x_j)\right) \tag{2}$$

where r_i corresponds to row i of Sudoku, j represents each of the nine integers and x_j is the frequency of present of integer j. Thus, considering the row (fr), column (fc) and submatrix (fs) contributions, the entropy objective function is given by,

$$f = \sum_{i=1}^{9} (fr_i + fc_i + fs_i) \tag{3}$$

3 GENETIC ALGORITHM SUDOKU ENCODING AND GENETIC OPERATORS

3.1 Coding Sudoku

Starting by a given Sudoku clues (see Figure 1), an individual is created by performing a permutation of complement values for each row. This means that maximum entropy is reach since the generation of individuals for each row. Column and submatrix diversity have then to be solved. Thus, for each of nine rows, a permutation is performed of size given by *row_r* vector taken values of this permutation as the indexes of each row of *complement* matrix and putting them in available spaces of *given* matrix. Based on this criterion, all rows accomplish with the restriction of having different values. But, it is not the case for column and sub-matrix.

3.2 Crossover operator for Sudoku puzzle

The crossover operator is performed by rows, with a equal probability of 1) exchanging only one randomly selected row of each parent, 2) randomly selecting a row crossover point and exchanging submatrices, or 3) applying uniform crossover of rows generating a mask of size nine.

3.3 Mutation operator for Sudoku puzzle

For the case of mutation operator, a mutation is performed by row with a certain probability. If mutation takes place, two positions (excluding given clues) are selected and values are exchanged.

8		2			3	5	1	
	6			9	1			3
7		1				8	9	4
6		8			4		2	1
			2	5	8		6	
9	2		3	1		4		
			4		2	7	8	
		5		8	9			
2					7	1		

given=[8 0 2 0 0 3 5 1 0	Comp=[4 6 7 9 0 0 0 0 0
0 6 0 0 9 1 0 0 3	2 4 5 7 8 0 0 0 0
7 0 1 0 0 0 8 9 4	2 3 5 6 0 0 0 0 0
6 0 8 0 0 4 0 2 1	3 5 7 9 0 0 0 0 0
0 0 0 2 5 8 0 6 0	1 3 4 7 9 0 0 0 0
9 2 0 3 1 0 4 0 0	5 6 7 8 0 0 0 0 0
0 0 0 4 0 2 7 8 0	1 3 5 6 9 0 0 0 0
0 0 5 0 8 9 0 0 0	1 2 3 4 6 7 0 0 0
2 0 0 0 0 7 1 0 0]	3 4 5 6 8 9 0 0 0]

row_r = [4 5 4 4 5 4 5 6 6];

Figure 1: Sudoku Puzzle encoding for Genetic Algorithms.

4 RESULTS

For easy Sudoku puzzle, example of Figure 1 is taken as a test where 38 clues are provided and 43 values have to be estimated. This example is taken from (Deng and Li, 2013). A GA based on Entropy objective function was used to solve this instance with a population of 200 individuals, 50% of mutation and 50% of row crossover. Also, a reinitialisation process was considered consisting of 50% of news individuals every 20 generations. Ten runs were performed and in all runs the optimal solution was found. In average, 37400 evaluations of objective function are needed. However, in most of the cases, less than 100 generations are only required, that means, less than 20000 evaluations of objective function of a search space of 6.0415e+52, but because of the permutation by row, the search space is reduced to 2.9720e+17. An interesting instance of Sudoku puzzle was taken from paper by Zhai and Zhang (2013). This is an example of multiple solutions of Sudoku with 26 clues. This example is presented in Figure 3, and in Figure 4 different solutions produced by the Entropy fitness function based GA are shown.

4 CONCLUSIONS

In this paper, a genetic algorithm based on an entropy fitness function evaluation was presented. A row crossover consisting of three types of crossover was also introduced and a reinitialisation process. The algorithm was tested in two different instances of Sudoku puzzle: an easy Sudoku with 38 clues and a Sudoku with 26 clues and multiple solutions. Results showed a competitive performance of the proposed algorithm.

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1	3	5	4	6	9	8	7	**2**
9	**2**	**7**	3	1	8	**5**	**6**	4
4	**6**	8	7	**5**	2	3	**9**	1
6	9	1	**8**	**7**	**4**	2	3	5
2	7	**3**	5	**9**	1	**4**	8	6
5	8	4	**2**	**3**	**6**	9	1	7
7	**4**	9	6	**2**	3	1	**5**	8
8	**1**	**6**	9	4	5	**7**	**2**	3
3	5	2	1	8	7	6	4	9

1	3	5	9	6	7	8	4	**2**
9	**2**	**7**	4	1	8	**5**	**6**	3
4	**6**	8	3	**5**	2	1	**9**	7
6	9	1	**8**	**7**	**4**	2	3	5
2	7	**3**	1	**9**	5	**4**	8	6
5	8	4	**2**	**3**	**6**	9	7	1
7	**4**	9	6	**2**	1	3	**5**	8
8	**1**	**6**	5	4	3	**7**	**2**	9
3	5	2	7	8	9	6	1	4

Figure 2: Two solutions of multiple solutions Sudoku puzzle.