Studying the effect of techniques to generate reference vectors in many-objective optimization

Miriam Pescador-Rojas*
ESCOM, Instituto Politécnico Nacional,
Computer Science Department
CINVESTAV-IPN
pescador@computacion.cs.cinvestav.mx

Carlos A. Coello Coello†
Computer Science Department
CINVESTAV-IPN
Mexico City, Mexico
ccoello@cs.cinvestav.mx

ABSTRACT

A large number of Multi-Objective Evolutionary Algorithms employ reference directions in order to establish relative preferences for each objective function. Uniform Design (UD), Simplex Lattice Design (SLD) and their variants are techniques commonly used to generate a set of uniformly distributed reference directions with the aim of capturing the whole Pareto optimal front. In this paper, we present a comparative study of UD and SLD methods when solving Many-objective Optimization problems and we design a new strategy that combines SLD with multiple layers and UD techniques. Our preliminary results indicate that our proposed approach is able to outperform state-of-the-art methods in many-objective optimization problems.

CCS CONCEPTS

• **Computing methodologies → Continuous space search**;

KEYWORDS

Reference directions, many-objective optimization, uniform design, simplex-lattice design.

ACM Reference Format:
Miriam Pescador-Rojas and Carlos A. Coello Coello. 2018. Studying the effect of techniques to generate reference vectors in many-objective optimization. In *GECCO '18 Companion: Genetic and Evolutionary Computation Conference Companion, July 15–19, 2018, Kyoto, Japan.* ACM, New York, NY, USA, 3 pages. https://doi.org/10.1145/3205651.3205684

1 INTRODUCTION

We are interested in solving Many-objective Optimization Problems (MaOPs), which are multi-objective problems having more than three objective functions. Several Multi-objective Evolutionary Algorithms (MOEAs) that tackle MaOPs use a set of reference vectors also known as direction vectors, reference lines or weight vectors in order to establish the relative importance for each objective function via a set of N convex weight vectors $\Lambda = \{\boldsymbol{\lambda}^1, \dots, \boldsymbol{\lambda}^N\}$ where each $\boldsymbol{\lambda}^j = (\lambda_1^j, \dots, \lambda_m^j)^T$ must satisfy $\sum_{i=1}^m \lambda_i^j = 1$ and $\lambda_i^j \geq 0$ for all $i \in \{1, \dots, m\}$ objectives. MOEAs can capture the whole Pareto optimal front if these reference vectors are evenly distributed in the whole objective space. Reference vectors are adopted mainly by decomposition-based MOEAs and by approaches based on the $R2$ indicator. Here, we present a review of techniques commonly adopted to generate a set of uniformly distributed weight vectors such a Simplex Lattice Design, Uniform Design and their variants. We analyze their effect in solving MaOPs and we propose a new combination of techniques to improve the distribution of the reference vectors. We validate our proposal using the MOEA/D framework and several MaOPs.

*The first author acknowledges support from CONACyT and IPN to pursue graduate studies in computer science at CINVESTAV-IPN
†The second author acknowledges support from CONACyT project no. 221551.

GECCO '18 Companion, July 15–19, 2018, Kyoto, Japan

ACM ISBN 978-1-4503-5764-7/18/07…$15.00
https://doi.org/10.1145/3205651.3205684

2 METHODS TO GENERATE UNIFORMLY DISTRIBUTED REFERENCE VECTORS

The Simplex Lattice Design (SLD) method [5] has been widely used in decomposition-based MOEAs. SLD generates N points arranged in a uniform way on a $\{m-1\}$ unity simplex-lattice (hyperplane). The number of weight vectors is $N = C_{H+m-1}^{m-1}$, where H is a positive integer that represents the number of subdivisions for each objective. Unfortunately, N increases exponentially with respect to the number of objectives. To deal with this issue, the authors of NSGA-III [1] proposed the use of the SLD with two layers (boundary and inside layers), each one with a different value of H. Recently, Jiang and Yang [2] proposed a strategy that uses multiple layers (the so-called MSLD) This method computes a central reference point and partions the unit simplex into m sub-simplexes to generate k subdivisions along each objective coordinate. The number of reference directions generated by MSLD is fixed at $N = C_m^k$. In this case, the increase of N is lower than when using the SLD method. The Uniform Design (UD) method [4] is a space-filling approach that generates uniform scattering points. Let $(N; h_1, \dots, h_s)$ be a vector with integral components satisfying $1 \leq h_i < N$, $h_i \neq h_j (i \neq j)$ and $m < N$. UD uses prime numbers to compute a sequence of component values h_i. An alternative is proposed in [6] replacing prime numbers by trascendental numbers to generate different distributions of points. Ma et al. [3] employed a combination of SLD and UD techniques via different UD measurements.

3 OUR PROPOSED APPROACH

We propose to combine the MSLD and UD methods as follows. Given a number of reference vectors (N), we apply the MSLD and

Table 1: Mean hypervolume values (the standard deviations are shown in parentheses)

m	MOP	SLD	MSLD	UD (prime)	UD (transc.)	SLD+UD	MSLD+UD
03D	DTLZ1	6.3845e+01(9.6895e-03)	6.3860e+01(2.2429e-02)	6.3839e+01(2.4008e-02)	6.3874e+01(1.7111e-02)	6.3853e+01(1.7896e-02)	6.3902e+01(2.0695e-02)
	DTLZ3	6.3339e+01(2.5841e-02)	6.3305e+01(4.3467e-02)	6.3236e+01(5.8583e-02)	6.3253e+01(3.3543e-02)	6.3328e+01(3.0968e-02)	6.3294e+01(3.6717e-02)
	DTLZ5	5.9425e+01(1.6462e-03)	5.9433e+01(3.7705e-03)	5.9582e+01(7.4090e-03)	5.9654e+01(4.4190e-03)	5.9411e+01(1.7725e-03)	5.9628e+01(3.5558e-03)
	DTLZ7	5.5595e+01(9.8809e+00)	6.0509e+01(4.0647e+00)	5.8305e+01(6.5102e+00)	5.8482e+01(7.8085e+00)	5.8332e+01(6.5119e+00)	5.8606e+01(6.5202e+00)
	WFG1	5.4901e+01(1.6490e+00)	5.4801e+01(1.8503e+00)	5.5168e+01(1.3374e+00)	5.4222e+01(1.4025e+00)	5.5024e+01(1.5132e+00)	5.4974e+01(1.6104e+00)
	WFG2	9.9509e+01(9.5344e-01)	9.9415e+01(8.5808e-01)	9.8790e+01(1.2416e+00)	9.9095e+01(8.5094e-01)	9.9173e+01(9.8708e-01)	9.8926e+01(9.5866e-01)
	WFG3	7.4427e+01(6.5659e-01)	7.4400e+01(5.2923e-01)	7.4895e+01(4.2265e-01)	7.4742e+01(5.6676e-01)	7.4559e+01(5.3659e-01)	7.4368e+01(3.6941e-01)
05D	DTLZ1	1.0236e+03(1.2674e-01)	1.0232e+03(2.8754e-01)	1.0235e+03(1.4836e-01)	1.0236e+03(1.1320e-01)	1.0238e+03(5.9120e-02)	1.0238e+03(7.5356e-02)
	DTLZ3	1.0233e+03(1.7916e-01)	1.0230e+03(2.3260e-01)	1.0226e+03(2.1389e-01)	1.0227e+03(2.0909e-01)	1.0232e+03(3.0501e-01)	1.0233e+03(8.1759e-02)
	DTLZ5	9.4339e+02(7.9288e-01)	9.0762e+02(1.3880e+01)	9.5436e+02(9.0703e-01)	9.5448e+02(2.1474e+00)	9.5416e+02(1.6589e+00)	9.5700e+02(4.4003e-01)
	DTLZ7	1.3725e+03(2.3886e+02)	1.1950e+03(1.4316e+02)	1.4230e+03(1.8324e+02)	1.4552e+03(1.6845e+02)	1.4633e+03(2.5601e+02)	1.4557e+03(2.1674e+02)
	WFG1	4.9716e+03(9.5905e+01)	4.9167e+03(1.3784e+02)	4.8739e+03(1.3850e+02)	4.9054e+03(1.5283e+02)	5.0158e+03(6.4513e+01)	4.9141e+03(1.4957e+02)
	WFG2	9.9726e+03(1.2748e+02)	9.7444e+03(2.0201e+02)	9.8531e+03(1.7826e+02)	9.8306e+03(1.5083e+02)	1.0020e+04(6.1733e+01)	9.9750e+03(1.2811e+02)
	WFG3	6.5071e+03(1.2130e+02)	5.9370e+03(1.6655e+02)	7.1785e+03(6.1070e+01)	7.1127e+03(6.9684e+01)	6.9655e+03(1.0966e+02)	7.0600e+03(8.4196e+01)
07D	DTLZ1	1.6379e+04(2.9019e+00)	1.6369e+04(5.0149e+00)	1.6379e+04(1.0603e+00)	1.6379e+04(1.0720e+00)	1.6381e+04(1.1547e+00)	1.6382e+04(7.0198e-01)
	DTLZ3	1.6370e+04(1.8298e+01)	1.6375e+04(3.9086e+00)	1.6368e+04(4.7229e+00)	1.6364e+04(8.2971e+00)	1.6375e+04(4.0959e+00)	1.6376e+04(5.2650e+00)
	DTLZ5	1.4616e+04(3.2112e+01)	1.4755e+04(5.3847e+01)	1.5135e+04(7.1776e+01)	1.5190e+04(4.1269e+01)	1.5157e+04(3.7834e+01)	1.5191e+04(9.7254e+01)
	DTLZ7	2.4953e+04(4.3683e+03)	2.1583e+04(2.2366e+03)	2.7710e+04(4.5638e+03)	2.8721e+04(4.0265e+03)	2.9517e+04(3.2320e+03)	3.0371e+04(3.9915e+03)
	WFG1	8.4562e+05(2.4569e+04)	6.4826e+05(5.3342e+04)	8.2521e+05(2.1132e+04)	8.3102e+05(1.1061e+04)	8.7547e+05(1.6101e+04)	8.4805e+05(1.9906e+04)
	WFG2	1.9332e+06(3.2438e+04)	1.8771e+06(5.7370e+04)	1.9133e+06(3.5399e+04)	1.9242e+06(2.3325e+04)	1.9461e+06(1.3024e+04)	1.9216e+06(2.1272e+04)
	WFG3	1.0356e+06(5.2868e+04)	1.0811e+06(1.9087e+04)	1.3469e+06(1.7406e+04)	1.2630e+06(2.5358e+04)	1.2750e+06(2.3843e+04)	1.2864e+06(2.1210e+04)
10D	DTLZ1	1.0476e+06(4.5505e+02)	1.0463e+06(9.7115e+02)	1.0482e+06(9.1661e+01)	1.0483e+06(6.5512e+01)	1.0483e+06(7.1079e+01)	1.0483e+06(7.9886e+01)
	DTLZ3	1.0474e+06(8.3095e+02)	1.0451e+06(2.6238e+03)	1.0475e+06(3.0090e+02)	1.0473e+06(4.0236e+02)	1.0476e+06(5.7818e+02)	1.0476e+06(4.8984e+02)
	DTLZ5	9.1297e+05(5.6492e+03)	9.1498e+05(1.6392e+04)	9.7259e+05(1.0131e+03)	9.7158e+05(2.1098e+03)	9.7054e+05(1.1569e+03)	9.7216e+05(2.9843e+03)
	DTLZ7	1.5757e+06(7.1807e+04)	1.3130e+06(1.5193e+05)	2.1619e+06(2.7942e+05)	2.1867e+06(2.7655e+05)	2.2023e+06(2.2328e+05)	2.1876e+06(3.0165e+05)
	WFG1	4.1765e+09(2.8532e+08)	3.5970e+09(2.0376e+08)	5.1407e+09(9.3265e+07)	5.0935e+09(5.5796e+07)	5.1238e+09(8.6872e+07)	5.1497e+09(5.9976e+07)
	WFG2	1.1997e+10(9.7337e+08)	1.1630e+10(1.1788e+09)	1.3067e+10(2.3850e+08)	1.3008e+10(2.9158e+08)	1.3099e+10(2.5105e+08)	1.3008e+10(3.1363e+08)
	WFG3	5.4884e+09(2.4860e+08)	6.5318e+09(2.1390e+08)	8.3818e+09(1.8390e+08)	8.4812e+09(1.5373e+08)	8.1622e+09(1.5084e+08)	8.6691e+09(1.3316e+08)

UD methods in an independent manner to generate $2 \times N$ vectors. We initialize our final set of vectors with extreme points of the unit simplex. For each subset, we find iteratively the elements that optimize the UD measure defined by:

$$D(\boldsymbol{\lambda}) = \frac{1}{N} \sum_{k=1}^{N} \Big\{ \min_{1 \le j \le N, j \neq k} d(\lambda^j, \lambda^k) \Big\} \tag{1}$$

where d is the Euclidean distance between two reference vectors. In this proposal, a rate value r must be predefined to establish the number of vectors obtained by each method. Algorithm 1 shows the detailed steps of our proposed approach.

Algorithm 1: Our proposed method to generate distributed reference directions

Input: N: the number of reference directions
m: the number of objective functions
r: rate of the boundary reference directions
Output: $\boldsymbol{\lambda}_{MUD}$: A set of uniform distributed reference directions
$\boldsymbol{\lambda}_{UD}$: Compute $2 \times N$ reference directions applying the UD method
$\boldsymbol{\lambda}_{MSLD}$: Compute $2 \times N$ reference directions applying the MSLD method
Inialize $\boldsymbol{\lambda}_{MUD} \leftarrow \{(1, 0, \dots, 0), \dots, (0, \dots, 1)\}$, it includes all the boundary vertices of a unit simplex
while $|\boldsymbol{\lambda}_{MUD}| < (1 - r) * N$ **do**
  Find the element $\lambda^j \in \boldsymbol{\lambda}_{UD}$ with the optimal uniform design measure usign equation (1)
  Add this element to $\boldsymbol{\lambda}_{MUD}$
while $|\boldsymbol{\lambda}_{MUD}| < N$ **do**
  Find the element $\lambda^j \in \boldsymbol{\lambda}_{MSLD}$ with the optimal uniform design measure usign equation (1)
  Add this element to $\boldsymbol{\lambda}_{MUD}$

4 EXPERIMENTAL RESULTS

We compare the described techniques using the MOEA/D framework to solve MOPs defined in two test suites: the *Deb-Thiele-Laumanns-Zitzler (DTLZ)* and the *Walking Fish Group (WFG)*. We tested our approach with 3, 5, 7, and 10 objectives. The number of weight vectors defined were $\{120, 210, 210, 220\}$, respectively. The number of objective function evaluations per MOP was set to 50,000.

We performed 30 independent runs for each MOEA and problem instance. For comparing our results, we adopted the hypervolume indicator (HV)to assess both convergence and maximum spread. We established the following reference points: $(4, 4, \dots 4)^T$ for DTLZ1, DTLZ3, DTLZ5, $(4, 4, \dots 2m + 1)^T$ for DTLZ7 and $(3, 5, \dots, 2i + 1)^T$ for WFG1, WFG2, WFG3. Table 1 presents the mean and the standard deviation of hypervolume values that we obtained. In the 3-objective MOPs, all methods obtained similar hypervolume values. For MOPs with more than 3 objectives, the combined strategies (SLD + UD and MSLD + UD) outperform the sole use of SLD and UD. In general, our proposed MSLD + UD obtained competitive results in almost all MOP instances.

5 CONCLUSIONS AND FUTURE WORK

The results presented here confirm that is more beneficial the use of methods that cover a large area of objective space to deal with different Pareto front shapes. Moreover, it is important to maintain the extreme points of the unit simplex in order to reach higher HV values. As part of our future work, we are interested in studying other uniform design measures and in analyzing the effect of these methods in other MOEAs. We also plan to combine the use of adaptive reference vectors with methods to generate uniformly distributed vectors in a particular area of objective search space.

REFERENCES

[1] K. Deb and H. Jain. 2014. An Evolutionary Many-Objective Optimization Algorithm Using Reference-Point-Based Nondominated Sorting Approach, Part I: Solving Problems With Box Constraints. *Evolutionary Computation, IEEE Transactions on* 18, 4 (Aug 2014), 577–601.

[2] S. Jiang and S. Yang. 2017. A Strength Pareto Evolutionary Algorithm Based on Reference Direction for Multiobjective and Many-Objective Optimization. *IEEE Transactions on Evolutionary Computation* 21, 3 (June 2017), 329–346.

[3] Xiaoliang Ma, Yutao Qi, Lingling Li, Fang Liu, Licheng Jiao, and Jianshe Wu. 2014. MOEA/D with uniform decomposition measurement for many-objective problems. *Soft Computing* 18, 12 (01 Dec 2014), 2541–2564.

[4] Hoang Pham. 2006. *Handbook of Engineering Statistics* (first ed.). Springer, London.

[5] H. Scheffé. 1958. Experiments with Mixtures. *Journal of the Royal Statistical Society. Series B (Statistical Methodology)* 20 (1958), 344–360.

[6] Saúl Zapotecas Martínez, Víctor A. Sosa Hernández, Hernán Aguirre, Kiyoshi Tanaka, and Carlos A. Coello Coello. 2014. Using a Family of Curves to Approximate the Pareto Front of a Multi-Objective Optimization Problem. In *PPSN XIII, 13th International Conference*, Thomas Bartz-Beielstein, Jürgen Branke, Bogdan Filipič, and Jim Smith (Eds.). Springer. LNCS Vol. 8672, Ljubljana, Slovenia, 682–691.