Extension of Weighted Empirical Distribution And Group-based Adaptive Differential Evolution For Joint Chance Constrained Problems

Kiyoharu Tagawa
Kindai University
Higashi-Osaka, Osaka, Japan
tagawa@info.kindai.ac.jp

ABSTRACT

There are two types of Chance Constrained Problems (CCPs), namely Separate CCP (SCCP) and Joint CCP (JCCP). This paper extends the optimization method for solving SCCP and applies it to JCCP. By using Bonferroni inequality, JCCP is stated with the Cumulative Distribution Function (CDF) of uncertain function value. Then Weighted Empirical CDF (W_ECDF) is used to approximate the CDF in JCCP. A new Adaptive Differential Evolution (ADE) combined with W_ECDF is also contrived for solving both CCPs.

CCS CONCEPTS

• **Mathematics of computing** → *Mathematical optimization*;

KEYWORDS

Chance constrained problem, CDF, differential evolution

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1 INTRODUCTION

There are two types of CCPs, namely SCCP and JCCP [1]. An optimization method [2] has been proposed to solve SCCP efficiently without using the time-consuming Monte Carlo simulation. In this paper, in order to solve JCCP as well as SCCP, the above optimization method is improved about the following three points: 1) JCCP is formulated as a deterministic optimization problem by using Bonferroni inequality [1] and the CDF approximated by W_ECDF. 2) Previous W_ECDF [2] is extended to consider the correlation between random variables. 3) A new ADE combined with W_ECDF is proposed for solving CCPs efficiently.

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2 PROBLEM FORMULATION

Let $\boldsymbol{x} \in \boldsymbol{X} \subseteq \Re^D$, $\boldsymbol{X} = [\underline{x}_j,\, \overline{x}_j]^D$, $j = 1, \cdots, D$ be a vector of decision variables, or a solution. Uncertainties are given by a vector of random variables $\boldsymbol{\xi} \in \Xi$ with a support $\Xi \subseteq \Re^K$. By using measurable functions $g_m : \boldsymbol{X} \times \Xi \to \Re$ and a required sufficiency level $\alpha \in (0, 1)$, JCCP is stated as

$$\left[\begin{array}{ll} \min\limits_{\boldsymbol{x}\in\boldsymbol{X}} & g_0(\boldsymbol{x}) \\ \text{sub. to} & \Pr\left(\begin{array}{c} \forall \boldsymbol{\xi} \in \Xi :\, g_m(\boldsymbol{x},\, \boldsymbol{\xi}) \le 0, \\ m = 1, \cdots, M \end{array}\right) \ge \alpha \end{array}\right. \tag{1}$$

where $\Pr(A)$ is the probability that an event A will occur.

Since $\boldsymbol{\xi} \in \Xi$ is stochastic, $g_m(\boldsymbol{x},\, \boldsymbol{\xi}) \in \Re$ becomes a random variable too. The CDF of $g_m(\boldsymbol{x},\, \boldsymbol{\xi}) \in \Re$ is defined as

$$F_m(\boldsymbol{x},\, \gamma) = \Pr(\forall \boldsymbol{\xi} \in \Xi :\, g_m(\boldsymbol{x},\, \boldsymbol{\xi}) \le \gamma) \tag{2}$$

where the CDF of $g_m(\boldsymbol{x},\, \boldsymbol{\xi})$ depends on $\boldsymbol{x} \in \boldsymbol{X}$.

Bonferroni inequality [1] is stated as

$$\Pr(A_1 \wedge \cdots \wedge A_M) \ge \sum_{m=1}^{M} \Pr(A_m) - M + 1. \tag{3}$$

From (2) and (3), JCCP in (1) is written as

$$\left[\begin{array}{ll} \min\limits_{\boldsymbol{x}\in\boldsymbol{X}} & g_0(\boldsymbol{x}) \\ \text{sub. to} & \displaystyle\sum_{m=1}^{M} F_m(\boldsymbol{x},\, 0) - M + 1 \ge \alpha. \end{array}\right. \tag{4}$$

The probability distribution of $\boldsymbol{\xi} \in \Xi$ in CCP is usually known [1]. However, in real-world applications, $g_m(\boldsymbol{x},\, \boldsymbol{\xi})$ in (1) is too complex to derive the CDF in (2) analytically.

3 APPROXIMATION OF CDF

3.1 Weighted Empirical CDF (W_ECDF)

In order to take samples $\boldsymbol{\xi}^n \in \Xi$ from its support uniformly, we use a set of uniformly distributed samples $\boldsymbol{u}^n \in \boldsymbol{S}$, $\Xi \subseteq \boldsymbol{S}$ instead of $\boldsymbol{\xi}^n \in \Xi$. The indicator function is defined as

$$\mathbb{1}(g_m(\boldsymbol{x},\, \boldsymbol{u}^n) \le \gamma) = \begin{cases} 1; & \text{if } g_m(\boldsymbol{x},\, \boldsymbol{u}^n) \le \gamma \\ 0; & \text{otherwise.} \end{cases} \tag{5}$$

Let $f : \Xi \to [0, \infty)$ be the Probability Density Function (PDF) of $\boldsymbol{\xi}^n \in \Xi$. From $\boldsymbol{u}^n \in \boldsymbol{S}$, W_ECDF is made as

$$\mathbb{F}_m(\boldsymbol{x},\, \gamma) = \frac{1}{W} \sum_{n=1}^{N} f(\boldsymbol{u}^n)\, \mathbb{1}(g_m(\boldsymbol{x},\, \boldsymbol{u}^n) \le \gamma) \tag{6}$$

where $W = f(\boldsymbol{u}^1) + \cdots + f(\boldsymbol{u}^n) + \cdots + f(\boldsymbol{u}^N)$ [2].

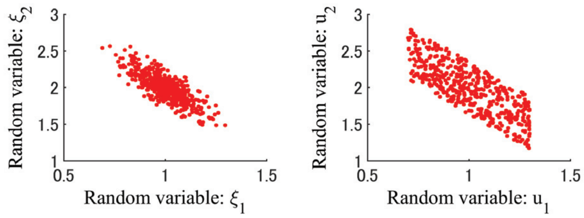

Figure 1: Comparison of $\boldsymbol{\xi}^n \in \boldsymbol{\Xi}$ and $\boldsymbol{u}^n \in \boldsymbol{S}$

Let $\tilde{\mathbb{F}}_m(\boldsymbol{x}, \gamma)$ be a smoothed W_ECDF. Thereby, JCPP in (4) is written with a correction level $\beta \geq \alpha$ as

$$\left[\begin{array}{ll} \min\limits_{\boldsymbol{x} \in \boldsymbol{X}} & g_0(\boldsymbol{x}) \\ \text{sub. to} & J_P(\boldsymbol{x}) = \sum_{m=1}^{M} \tilde{\mathbb{F}}_m(\boldsymbol{x}, 0) - M + 1 \geq \beta. \end{array}\right. \tag{7}$$

3.2 Support for W_ECDF

We define the support $\boldsymbol{S} \subseteq \Re^K$ by a mean vector $\boldsymbol{\theta} \in \Re^K$ and a covariance matrix $\boldsymbol{V}$ of uniform distribution as

$$\boldsymbol{u} = (u_1, \cdots, u_K) \sim \mathcal{U}(\boldsymbol{S}) = \mathcal{U}(\boldsymbol{\theta}, \boldsymbol{V}). \tag{8}$$

First of all, we decide the range $[\underline{u}_j, \overline{u}_j] \subseteq \Re$ that covers the whole range of the random variable $\xi_j \in \Re$. Then, from $\theta_j = (\overline{u}_j + \underline{u}_j)/2$, we have $\boldsymbol{\theta} = (\theta_1, \cdots, \theta_K)$. We suppose that the correlation matrix $\boldsymbol{R}$ of $\boldsymbol{\xi} \in \boldsymbol{\Xi}$ is known. By using $\boldsymbol{R}$ and a diagonal matrix $\boldsymbol{B} = [s_j]$ composed from the standard deviation $s_j = (\overline{u}_j - \underline{u}_j)/\sqrt{12}$, we have $\boldsymbol{V} = \boldsymbol{B}\,\boldsymbol{R}\,\boldsymbol{B}$.

Cholesky decomposition $\boldsymbol{V} = \boldsymbol{L}\,\boldsymbol{L}^T$ in (8) provides a lower triangular matrix $\boldsymbol{L}$. Let $\varepsilon_j^n \in \Re$, $j = 1, \cdots, K$ be a set of samples of the mutually independent random variables $\varepsilon_j \in [-\sqrt{3}, \sqrt{3}]$ following the standard uniform distribution. The samples $\boldsymbol{u}^n \in \boldsymbol{S}$ of $\boldsymbol{u} \sim \mathcal{U}(\boldsymbol{S})$ are generated as

$$\boldsymbol{u}^n = \boldsymbol{L}\,\boldsymbol{\varepsilon}^n + \boldsymbol{\theta}, \; n = 1, \cdots, N \tag{9}$$

where $\boldsymbol{\varepsilon}^n = (\varepsilon_1^n, \cdots, \varepsilon_K^n)$ and $\varepsilon_j^n \in [-\sqrt{3}, \sqrt{3}]$.

Figure 1 compares samples $\boldsymbol{\xi}^n = (\xi_1^n, \xi_2^n) \in \boldsymbol{\Xi}$ following a multivariate normal distribution with $\boldsymbol{u}^n = (u_1^n, u_2^n) \in \boldsymbol{S}$ where the support $\boldsymbol{S} \subseteq \Re^2$ is defined as noted above.

4 GROUP-BASED ADE (JADE2G)

ADEGL [3] is an extended JADE [4] for solving unconstrained optimization problems. Inspired by ADEGL, we propose a new ADE for solving CCP, which is called JADE based on 2 Groups (JADE2G). The individuals $\boldsymbol{x}_i \in \boldsymbol{P}_t$ of population are divided into two groups, namely feasible ones and infeasible ones. Let $\phi(\boldsymbol{x}) = \max\{\beta - J_P(\boldsymbol{x}), \; 0\}$ be the constraint violation of JCCP in (7). The scale factor $SF_i \in [0, 1]$ is generated by two Cauchy distributions as

$$SF_i \sim \left\{\begin{array}{ll} \mathcal{C}(\mu_{SF1}, \sigma_{SF}); & \text{if } \phi(\boldsymbol{x}_i) = 0 \\ \mathcal{C}(\mu_{SF2}, \sigma_{SF}); & \text{if } \phi(\boldsymbol{x}_i) > 0 \end{array}\right. \tag{10}$$

where $\sigma_{SF} = 0.1$. Two locations are initialized as $\mu_{SF1} = 0.5$ and $\mu_{SF2} = 0.8$. Thereafter, the values of μ_{SF1} and μ_{SF2} are updated in each group according to the rule of JADE.

Table 1: Experimental results

ρ	N	$g_0(\boldsymbol{x}_b)$	$\widehat{\Pr}(A)$	β
-0.8	30	11.508	0.941	0.939
0.0	50	12.151	0.927	0.932
$+0.8$	60	12.763	0.930	0.946

The crossover rate $CR_i \in [0, 1]$ of $\boldsymbol{x}_i \in \boldsymbol{P}_t$ is generated by using two Normal distributions as

$$CR_i \sim \left\{\begin{array}{ll} \mathcal{N}(\mu_{CR1}, \sigma_{CR}^2) & \text{if } \phi(\boldsymbol{x}_i) = 0 \\ \mathcal{N}(\mu_{CR2}, \sigma_{CR}^2) & \text{if } \phi(\boldsymbol{x}_i) > 0 \end{array}\right. \tag{11}$$

where $\sigma_{CR} = 0.1$. Two means are initialized as $\mu_{CR1} = 0.5$ and $\mu_{CR2} = 0.8$. They are also updated in each group.

5 NUMERICAL EXPERIMENT

The proposed method was applied to the following JCCP:

$$\left[\begin{array}{ll} \min\limits_{\boldsymbol{x}} & g_0(\boldsymbol{x}) = 2\,x_1 + 2\,x_2 + 3\,x_3^2 + x_4^2 \\ \text{sub. to} & \Pr(g_1(\boldsymbol{x}, \boldsymbol{\xi}) \leq 0, \; g_2(\boldsymbol{x}, \boldsymbol{\xi}) \leq 0) \geq \alpha \\ & 0.5 \leq x_1 \leq 1.5, \; 0 \leq x_2 \leq 1.5 \\ & 0 \leq x_3 \leq 2.5, \; 0 \leq x_4 \leq 0.5 \end{array}\right. \tag{12}$$

where $\alpha = 0.9$ and $g_m(\boldsymbol{x}, \boldsymbol{\xi})$, $m = 1, 2$ are given as

$$\left(\begin{array}{l} q(x_j, \xi_j) = \xi_j - x_j\,(1 - \exp(-\xi_j/x_j)), \; j = 1, 2 \\ g_1(\boldsymbol{x}, \boldsymbol{\xi}) = 2\,q(x_1, \xi_1) + 2\,q(x_2, \xi_2) - x_3 - x_4 \\ g_2(\boldsymbol{x}, \boldsymbol{\xi}) = 2\,q(x_1, \xi_1) - x_4 \end{array}\right. \tag{13}$$

A 2-dimensional normal distribution is used in (12) as

$$\boldsymbol{\xi} = (\xi_1, \xi_2) \sim \mathcal{N}_2(1, 2, 0.1^2, 0.2^2, \rho) \tag{14}$$

where ρ is the correlation coefficient between ξ_1 and ξ_2.

Table 1 shows the experimental results averaged over 20 runs. The empirical probability $\widehat{\Pr}(A)$ is also calculated with the Monte Carlo simulation [2]. Since $\widehat{\Pr}(A) \geq \alpha$ holds, the solution $\boldsymbol{x}_b \in \boldsymbol{X}$ obtained by JADE2G is feasible.

6 CONCLUSIONS

The optimization method based on JADE2G and W_ECDF is applicable not only to JCCP but also to SCCP. Even though the normal distribution was used in experiments, it can be replaced by any distribution if its PDF is known.

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