Trust-region based Algorithms with Low-budget for Multi-objective Optimization

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ABSTRACT

In many practical multi-objective optimization problems, evaluations of objectives and constraints are computationally time-consuming because they require expensive simulations of complicated models. In this paper, we propose a metamodel-based multi-objective evolutionary algorithm to make a balance between error uncertainty and progress. In contrast to other trust region methods, our method deals with multiple trust regions. These regions can grow or shrink in size according to the deviation between metamodel prediction and high-fidelity evaluation. We introduce a performance indicator based on hypervolume to control the size of the trust regions. We compare our results with a standard metamodel-based approach without trust region and a multi-objective evolutionary algorithm. The results suggest that our trust region based methods can effectively solve test and real-world problems using limited solution evaluations with increased accuracy.

KEYWORDS

Trust Region, Multi-objective, Metamodel, Surrogate Assisted, Optimization

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1 INTRODUCTION

Most of the real-world problems involve time-consuming experiments and simulations that cause optimization to be increasingly expensive. To face this challenge and to reduce the computational cost, metamodels as approximations of exact models are used for the optimization task.

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Although most existing methods are directed towards proposing more accurate metamodels or introducing efficient search schemes [4–6], there is a need for managing error uncertainty of one particular under-performing metamodel during optimization. A better management of a metamodel can, not only restrain the model from becoming worse, but also boost the performance by recognizing the inherent complexity of search regions.

In this paper, we introduce a trust region concept for multi-objective optimization to reduce model error uncertainty during the metamodel-based optimization. This allows a continuous convergence towards the Pareto front instead of relying completely on assumptions of the metamodel from the first iteration on.

2 METHODOLOGY

The overall algorithm follows the metamodel-based optimization procedure. We propose several modifications of the classical trust region concept [1] to make it applicable for multi-objective optimization. An outline of the algorithm including the trust region adaption method is provided in the following.

An initial population with n individuals is generated using Latin hypercube sampling. Afterwards, for each generation k a metamodel is built using all existing solutions and NSGA-II [3] with a population size of n is executed to find non-dominated solutions using only metamodel predictions. For this metamodel-based optimization the search space is reduced to be in trust region δ_k^p for all solutions p. We set the initial trust region $\delta_0^p = 0.75\Delta_{max}$, where $\Delta_{max} = \sqrt{n}$ is the largest diagonal of an n-dimensional unit hypercube. All distances are calculated in the normalized space.

After evaluating the obtained solutions using the high-fidelity evaluation function, we update the trust region δ_{k+1}^p using the hypervolume performance indicator. Each point in the archive A and each new infill point q found in a trust region of $p \in A$ needs to have a trust region assigned for the next generation. Therefore, we compute the ratio between actual and predicted improvement and adjust the trust radii accordingly. The predicted hypervolume is calculated by the objective values evaluated in model space using $\hat{F}(.)$. We include all archive points A as a common ground for computation. We calculate the performance indicator $PI_{HV}(A, q)$ for every infill point q. Since larger values indicate better hypervolume, we use negative of the hypervolume. Improvement of hypervolume becomes

Table 1: Experimental results providing GD and IGD values for 7 test problems.

Problem/Method	NSGA-II		M1-2		HV-TR	
	IGD	GD	IGD	GD	IGD	GD
ZDT1	0.38105	0.44105	0.0116	0.0109	**0.0036755**	**0.0017905**
	p=1.8267e-04	p=1.8267e-04	p=1.7861e-04	p=2.4613e-04	-	-
ZDT2	1.0245	0.74535	0.00995	0.00755	**0.0018445**	**0.001053**
	p=1.8267e-04	p=1.8165e-04	p=1.8165e-04	p=1.8165e-04	-	-
ZDT3	0.31	0.41875	0.0132	0.0076	**0.0055315**	**0.001846**
	p=1.8267e-04	p=1.8267e-04	p=0.1618	p=4.3095e-04	-	-
ZDT6	4.7585	4.557	1.5778	**2.27535**	**0.40195**	4.634
	p=1.8267e-04	p=1.8165e-04	p=1.8267e-04	-	-	p=1.8267-e04
BNH	0.42055	0.16845	0.49435	0.139	**0.1508**	**0.08714**
	p=1.8267e-04	p=1.8267e-04	p=1.8267e-04	p=1.8267e-04	-	-
SRN	1.227	1.827	0.65905	**0.72755**	**0.39795**	**0.76495**
	p=1.8267e-04	p=2.4613e-04	p=1.8267e-04	-	-	p=0.7337
Welded Beam	**0.2722**	**0.10115**	0.9685	1.7787	**0.2749**	0.78545
	-	-	p=1.8267e-04	p=0.0036	p=0.8501	p=3.2984e-04

$$PI_{HV}(A,q) = \frac{HV(F(A) \cup F(q)) - HV(F(A))}{HV(F(A) \cup \hat{F}(q)) - HV(F(A))}. \quad (1)$$

The metric above does not consider whether solutions are feasible or not. For this reason, we use constrained violation performance indicator $PI_{CV}(p,q)$ if both solutions are infeasible. In this case, the performance improvement r is calculated as described in the classical single-objective trust region method [1] by using CV as a function value. If one solution is feasible and the other not, the fact of feasibility determines the value of r.

$$r = \begin{cases} PI_{HV}(A,q), & \text{if both } p \text{ and } q \text{ feasible,} \\ r_2 + \epsilon, & \text{if } p \text{ infeasible, } q \text{ feasible,} \\ r_1 - \epsilon, & \text{if } p \text{ feasible, } q \text{ infeasible,} \\ PI_{CV}(p,q), & \text{otherwise.} \end{cases} \quad (2)$$

Here $\epsilon > 0 \in \mathcal{R}$ is a small positive number. The pre-defined positive constants $0 < r_1 < r_2$ are the hyper-parameters that regulate expansion and contraction of the trust regions. After calculating the performance ratio r between p and q, we set the trust radius of p for generation $k+1$ by the following rule.

$$\delta^p_{k+1} = \begin{cases} c_1 \delta^p_k & \text{if } r < r_1 \\ \min\{c_2 \delta^p_k, \Delta_{max}\} & \text{if } r > r_2 \\ \delta^p_k & \text{otherwise} \end{cases} \quad (3)$$

The positive constants $0 < c_1 < 1$ and $c_2 > 1$ control the size of subsequent trust radius. The parameter Δ_{max} is the largest allowed trust radius for solutions. We assign the trust radius of q to be $\delta^q_k = \min_{p \in T(q)} \delta^p_{k+1}$ where q lies inside the trust regions of $T(q)$.

After updating the trust regions, the next generation is executed by building a new metamodel using all existing solutions and optimizing inside trust regions using the predictions only.

3 RESULTS

In this section, we present experimental results obtained by running three different optimization algorithms. We compare the proposed algorithm HV-TR with M1-2 [2], which works similar to HV-TR but without the trust region, and the state-of-the-art multi-objective evolutionary method NSGA-II [3].

Median IGD values of 10 runs of 7 test problems are presented in Table 1. The statistical significance of each algorithm is computed from the best performing algorithm and the p-value is shown in the table. The table demonstrates that trust region methods perform usually better than non-trust region based methods for *most* of the problems with limited solution evaluations. The differences in the IGD and GD values between trust and non-trust region methods are remarkable – one to three orders of magnitude better. Only in the case of welded beam design problem, NSGA-II performs significantly better in terms of GD.

4 CONCLUSION

In this paper, we make three main contributions: firstly, we have introduced trust region concept in multi-objective population-based method; secondly, we have proposed a performance indicator based on hypervolume to adapt trust regions; thirdly, we have presented a scheme for handling constrained problems. We have tested our method in two objective constrained and unconstrained test and real-world problems.

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