A Parameterized Runtime Analysis of Randomized Local Search and Evolutionary Algorithm for Max l-Uncut*

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ABSTRACT

In the last few years, parameterized complexity has emerged as a new tool to analyze the running time of randomized local search algorithm. However, such analysis are few and far between. In this paper, we do a parameterized running time analysis of a randomized local search algorithm and a (1+1) EA for a classical graph partitioning problem, namely, Max l-Uncut, and its balanced counterpart Max Balanced l-Uncut. In Max l-Uncut, given an undirected graph $G = (V, E)$, the objective is to find a partition of $V(G)$ into l parts such that the number of uncut edges - edges within the parts - is maximized. In the last few years, Max l-Uncut and Max Balanced l-Uncut are studied extensively from the approximation point of view. In this paper, we analyze the parameterized running time of a randomized local search algorithm (RLS) for Max Balanced l-Uncut where the parameter is the number of uncut edges. RLS generates a solution of specific fitness in polynomial time for this problem. Furthermore, we design a fixed parameter tractable randomized local search and a (1 + 1) EA for Max l-Uncut and prove that they perform equally well.

CCS CONCEPTS

• **Theory of computation** → *Design and analysis of algorithms*;

KEYWORDS

Max l-Uncut, Max Balanced l-Uncut, Running time Analysis, Randomized Local Search, (1 + 1) EA

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1 INTRODUCTION

Bioinspired computing is a widely used method to deal with NP-hard combinatorial optimization problems. It is always interesting to analyze the convergence of these techniques theoretically. In the past few years, parameterized complexity has emerged as a successful tool to analyze the convergence of randomized local search algorithms and evolutionary algorithms. This approach has been successfully used for Minimum Vertex Cover [5], Maximum Leaf Spanning Tree [4], Euclidean Travelling Salesperson Problem [8], Makespan Scheduling [6] and Weighted Vertex Cover Problem [7]. In this paper, we explore this technique for Max l-Uncut. Max l-Uncut consists of partitioning of the vertex set of graph G into l parts such that number of uncut edges is maximized. An *uncut edge* is defined as an edge whose both the end points lie within the same part. In Max Balanced l-Uncut, the vertex set of graph G must be partitioned into l almost equal parts, i.e., for a partition $P = \{A_1, \cdots, A_l\}$ of $V(G)$, $||A_i| - |A_j|| \leq 1$, for all $i, j \in [l]$. This problem was motivated from the study of the homophily law of large scale networks [11]. The Max l-Uncut problem is the complement of well studied Min l-Cut problem. It is well known that Min l-Cut is polynomial time solvable when l is fixed [2] and NP-complete when l is given as input [1], though the balanced version of the problem is known to be NP-complete even for fixed l, where $l \geq 2$. Since the problems Min l-Cut and Max l-Uncut (Balanced Min l-Cut and Max Balanced l-Uncut) are complements of each other, the above results hold for Max l-Uncut as well. In the last few years, Max l-Uncut and Max Balanced l-Uncut have been studied extensively from the point of view of approximation algorithms. Ye and Zhang [10] developed a 0.602-approximation algorithm for Max Balanced 2-Uncut. Wu et al. [9] designed a 0.3456-approximation algorithm for Max Balanced 3-Uncut problem. Zhang et al. [11] proposed approximation algorithms for Max l-Uncut when l is given as input. They developed a randomized $\left(1 - \frac{l}{n}\right)^2$-approximation algorithm and a greedy $\left(1 - \frac{2(l-1)}{n}\right)$-approximation algorithm.

Our Contributions. For Max Balanced l-Uncut, we analyze the parameterized running time of a randomized local search algorithm (RLS), where the parameter is the number of uncut edges. We also study the fitness landscape for Max Balanced 2-Uncut. This problem satisfies Grover's wave equation [3] under the neighborhood operator defined by our RLS strategy which gives that Max Balanced 2-Uncut has elementary landscape. We also prove that RLS generates a solution of specific fitness in polynomial time for this problem, which is also an optimal solution for star graphs. Futhermore, we design a fixed parameter tractable randomized local search and a (1 + 1) EA for Max MultiPartite Uncut which deals with finding a partition of $V(G)$ into l parts where l is given as part of input such that number of uncut edges is maximized. These algorithms also obtain a solution of specific fitness in polynomial time.

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2 RANDOMIZED LOCAL SEARCH FOR MAX BALANCED 2-UNCUT

In this section, we present a randomized local search algorithm, RLS1, for MAX BALANCED 2-UNCUT and analyse its parameterized running time. In RLS1, given an undirected graph G, we start with a uniform random partition of vertex set $V(G)$ into two sets A and B such that $||A| - |B|| \leq 1$. Now, given a partition (A, B) of vertex set $V(G)$, we select two vertices, one each from A and B, uniformly at random and swap them.

We proved that the landscape induced by neighborhood defined by RLS1 and fitness function of MAX BALANCED 2-UNCUT is elementary. Moreover, RLS1 obtains solution of certain quality in $O(mn^2)$ time. In particular, we proved the following result.

LEMMA 2.1. RLS1 *finds a solution* P' *after an expected* $O(mn^2)$ *iterations such that*

$$f_{\text{uncut}}(P') \geq \begin{cases} \frac{m(n-2)}{2(n-1)} & \text{if } n \text{ is even} \\ \frac{m(n-2)}{2n-1} & \text{if } n \text{ is odd} \end{cases}$$

We have also shown that RLS1 is a fixed parameter tractable algorithm with respect to standard parameterization of MAX BALANCED 2-UNCUT. We define the parameterized version of MAX BALANCED 2-UNCUT as follows.

MAX BALANCED 2-UNCUT
Input: An undirected graph G and a non-negative integer k
Parameter: k
Goal: Find a partition $P = \{A, B\}$ of $V(G)$ where $||A| - |B|| \leq 1$ such that number of uncut edges in P is at least k

THEOREM 2.2. RLS1 *solves a parameterized instance* (G, k) *of* MAX BALANCED 2-UNCUT *in* $O(\max(mn^2, k^{O(k)}))$ *expected time. Futhermore, after* $O(\max(mn^3, nk^{O(k)}))$ *iterations,* RLS1 *solves parameterized instance of* MAX BALANCED 2-UNCUT *with constant probability.*

3 MAX BALANCED l-UNCUT

We have also analysed the parameterized running time of randomized local search, RLS2 (similar to RLS1) for MAX BALANCED l-UNCUT, when l is fixed.

THEOREM 3.1. RLS2 *solves a parameterized instance* (G, k) *of* MAX BALANCED l-UNCUT *in* $O(\max(mn^2, k^{O(k)}))$ *expected time. Futhermore, after* $O(\max(mn^3, nk^{O(k)}))$ *iterations,* RLS2 *solves parameterized instance of* MAX BALANCED l-UNCUT *with constant probability.*

4 MAX MULTIPARTITE UNCUT

In this section, we present a randomized local search and a simple evolutionary algorithm for MAX MULTIPARTITE UNCUT and analyse their parameterized running time.

Randomized Local Search for Max MultiPartite Uncut (RLS3**).** We start with a uniform random partition $P = \{A_1, \cdots, A_l\}$ of $V(G)$. Now in each iteration, select a vertex v uniformly at random and then choose a partition A_i to move v. We proved that RLS3 obtains a solution of certain quality in polynomial time and give the following lemma.

LEMMA 4.1. RLS3 *generates a partition* P *in* $O(nml)$ *time such that* $f_{\text{uncut}}(P) \geq \frac{m}{l}$.

Next we prove the running time of RLS3 for the standard parameterization of MAX MULTIPARTITE UNCUT and give the following result.

THEOREM 4.2. RLS3 *is an* FPT *algorithm which solves an instance* (G, l, k) *of* MAX MULTIPARTITE UNCUT *in* $O(\max(mnl, (kl^2)^{O(kl)}))$ *expected time. Futhermore,* RLS3 *solves parameterized instance of* MAX MULTIPARTITE UNCUT *with constant probability after* $O(\max(mn^2l, n(kl^2)^{O(kl)}))$ *iterations.*

(1+1) EA **for Max MultiPartite Uncut.** In this algorithm, every vertex has equal probability to move in some other partition. $(1+1)$ EA also generates a solution of specific fitness in $O(nml)$ time. We can show that $(1+1)$ EA is also FPT for MAX MULTIPARTITE UNCUT.

LEMMA 4.3. $(1+1)$ EA *generates a partition* P' *of* $V(G)$ *such that* $f_{\text{uncut}}(P') \geq \frac{m}{l}$ *after an expected* $O(nml)$ *iterations.*

THEOREM 4.4. $(1+1)$ EA *solves a parameterized instance* (G, l, k) *of* MAX MULTIPARTITE UNCUT *in* $O(\max(mnl, (kl^2)^{O(kl)}))$ *expected time. Futhermore,* $(1+1)$ EA *solves parameterized instance of* MAX MULTIPARTITE UNCUT *with constant probability after* $O(\max(mn^2l, n(kl^2)^{O(kl)}))$ *iterations.*

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