Studying MOEAs Dynamics and their Performance using a Three Compartmental Model

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ABSTRACT

The road to a better design of multi- and many-objective evolutionary algorithms requires a deeper understanding of their behavior. A step on this road has recently been taken with the proposal of compartmental models to study population dynamics. In this work, we push this step further by introducing a new set of features that we link with algorithm performance. By tracking the number of newly discovered Pareto Optimal (PO) solutions, the previously-found PO solutions and the remaining non-PO solutions, we can track the algorithm progression. By relating these features with a performance measure, such as the hypervolume, we can analyze their relevance for algorithm comparison. This study considers out-of-the-box implementations of recognized multi- and many-objective optimizers belonging to popular classes such as conventional Pareto dominance, extensions of dominance, indicator, and decomposition based approaches. In order to generate training data for the compartmental models, we consider multiple instances of MNK-landscapes with different numbers of objectives.

CCS CONCEPTS

• **Theory of computation → Evolutionary algorithms**;

KEYWORDS

Empirical study, Working principles of evolutionary computing, Genetic algorithms, Multi-objective optimization

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1 INTRODUCTION

In this work we focus on a modeling tool that tracks the populational dynamics of MOEAs proposed in [4]. We use a three compartments configuration, with features related to performance taken from the generational search assessment indices in [1, 3]. Our aim is to introduce a new set of features that can be link to algorithm performance, so that we can make an estimate of how well it will perform by running the model instead of the full algorithm. We track the relationship between newly discovered Pareto Optimal (PO) solutions, PO solutions that have been seen before, and the remaining non-PO solutions, including currently non-dominated ones, *per* generation. The model usage is exemplified using representative optimizers with different approaches on problem instances generated using enumerable MNK-landscapes.

2 METHODOLOGY

2.1 Compartmental Model

The model splits the population into three non-overlapping groups, or compartments. At each generation t, x_t, y_t and z_t represent the proportion of the population that belongs to each compartment, which at all times fulfills $1 = x_t + y_t + z_t$. Representing the model in a time discrete manner would give the following equation:

$$\begin{cases} x_{t+1} = (1-(\alpha+\beta))x_t + \bar{\alpha}y_t + \bar{\beta}z_t \\ y_{t+1} = \alpha x_t + (1-(\bar{\alpha}+\gamma))y_t + \bar{\gamma}z_t \\ z_{t+1} = \beta x_t + \gamma y_t + (1-(\bar{\beta}+\bar{\gamma}))z_t \\ 1 = x_t + y_t + z_t \end{cases} \tag{1}$$

where α and β are coefficients that represent the loss in x_t which becomes a gain for y_t and z_t, respectively. $\bar{\alpha}$ and γ represent the loss in y_t which becomes a gain for x_t and z_t, respectively. Finally, $\bar{\beta}$ and $\bar{\gamma}$ represent the loss in z_t.

2.2 Model Fitting

We define the problem of finding a good set of parameters that gives an optimal fit between the model estimate $\bar{X}$ and the real data X as an optimization problem, where we minimize the mean square error $mse = \frac{1}{n}\sum_{i=1}^{n}(\bar{X}-X)^2$ between these values. To do so, we use the Covariance Matrix Adaptation Evolutionary Strategy (CMA-ES), a single-objective numerical optimizer, and set the mse between $\bar{X}$ and X as the function to be minimized.

Table 1: Algorithm comparison by Acc. PO Solutions (model feature) and Hypervolume (performance metric)

Algorithm	Found PO	Predicted PO	HV	Rank_{PO}	Rank_{HV}
NSGA-II	148.033	142.500	0.375640	**3**	**3**
AϵSϵH	148.667	150.651	0.375904	**1**	**1**
IBEA$_{HV}$	136.567	134.788	0.375140	**4**	**4**
MOEA/D	144.667	145.473	0.375757	**2**	**2**

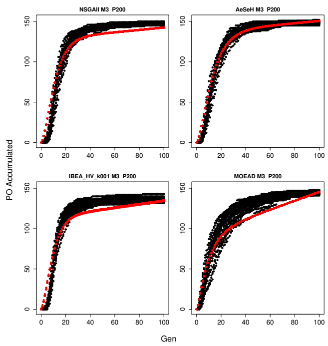

Figure 1: Model estimation vs real PO Accumulated data.

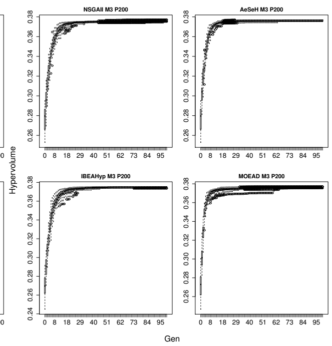

Figure 2: Accumulated joint non-dominated sets HV.

2.3 Algorithms and Test Problem

The test problems used are MNK-landscapes [2], a multi-objective problem generator, where the parameter K controls the ruggedness of the landscape, M is the number of objectives, and N the number of variables. In this study, we randomly generate instances with K = 1 bit, N = 20 bits and M = 3, 4, 5, 6 objectives.

Models are created over data collected from five different representative multi- and many-objective optimization algorithms, Non-dominated Sorting Genetic Algorithm-II (NSGA-II), Adaptive ϵ-Sampling ϵ-Hood (AϵSϵH), Indicator Based EA (IBEA) with the ϵ-indicator and hypervolume (HV) indicator, and MOEA based on Decomposition (MOEA/D).

Each algorithm was run 30 times setting the number of generations to 100. The number of solutions belonging to each tracked feature were collected, including during the initial population. For each number of objectives, we also consider various population sizes between 50 and 5600. We build a model, per configuration, for each algorithm.

3 FEATURES VS PERFORMANCE METRICS

Once the fitting process is done (omitted here due space restrictions), the obtained parameters in conjunction with the model can be employed in order to have an overall idea of algorithm performance. While this can be done in a sense by using only the Accumulated PO solutions as a performance measure, for which an example of the prediction made by our model is seen in Figure 1, we also relate it to the hypervolume, a commonly-used performance metric that gives the multi-dimensional area of the objective space that is dominated by a non-dominated set and enclosed by a reference point set to all zeros. To do so, we first compute the accumulated set of non-dominated solutions found until each generation, for each given run, followed by the computation of the hypervolume for each of these sets. The results for the 3-objective and population 200 case are reported in Figure 2.

Comparing Figures 1 and 2, we can see that the trend and growth rate in both seems similar for each algorithm. From generation 1 to 30, we see an exponential growth and then a slowdown from this point on of the hypervolume-value and the number of true PO solutions found. Although we cannot directly say how much the value of the hypervolume will go up with each newly-found PO solution, since this model only focuses on their numbers, we still can grasp the link between these two measures, that seem to scale together at least for some generations.

We also calculate the correlation between the hypervolume-values for each configuration and the number of PO solutions found until a given generation using Spearman's correlation coefficient. For the example shown, the coefficients are 0.897, 0.786, 0.844, 0.795 with $p < 2.2 \times 10^{-16}$ for NSGA-II, AϵSϵH, IBEA$_{HV}$ and MOEA/D, respectively, which supports our initial assumptions. These results illustrates the usefulness of our model to estimate the accumulated number of PO solutions in order to have an idea of the overall hypervolume convergence profile, and whether it will keep improving or will start to slow down. Table 1 shows the ranking of the algorithms according to the predicted Accumulated PO solutions and the hypervolume. Here is represented only one case of 13 configurations (population size, number of objectives). For seven out 13 cases the rank coincides, while in other four cases there is a partial coincidence.

From the results, although we cannot strongly affirm that the Acc. PO feature has the same weight as a comparison done with the HV values, the partial orderings given by this feature still prove useful, because it can provide us a good idea of the algorithm performance before actually running it.

4 CONCLUSION

In this work, we extended the model proposed in [4] by considering another set of features and by successfully relating them with algorithm performance. The proposed model divides the population into newly-discovered Pareto optimal solutions, Pareto optimal discovered at previous generation, and non-Pareto optimal solutions. Since it tracks absolutely new solutions, the model is aware of the accumulated Pareto optimal solutions at each generation, which can be used as a progress indicator. By relating our features with algorithm performance, we were able to show a high correlation between the predicted accumulated number of Pareto optimal solutions and the hypervolume of the joint set of non-dominated solutions identified at each given generation.

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