Pareto dominance-based MOEAs on Problems with Difficult Pareto Set Topologies

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ABSTRACT

Despite the extensive application of multi-objective evolutionary algorithms (MOEAs) to solve multi-objective optimization problems (MOPs), understanding their working principles is still open to research. One of the most popular and successful MOEA approaches is based on Pareto dominance and its relaxed version, Pareto ε-dominance. However, such approaches have not been sufficiently studied in problems of increased complexity. In this work, we study the effects of the working mechanisms of the various components of these algorithms on test problems with difficult Pareto set topologies. We focus on separable unimodal and multimodal functions with 2, 3, and 4 objectives, all having difficult Pareto set topologies. Our experimental study provides some interesting and useful insights to understand better Pareto dominance-based MOEAs.

CCS CONCEPTS

• **Theory of computation → Evolutionary algorithms**;

KEYWORDS

Working principles of evolutionary computing, Multi-objective optimization, Selection, Recombination operators, Differential Evolution.

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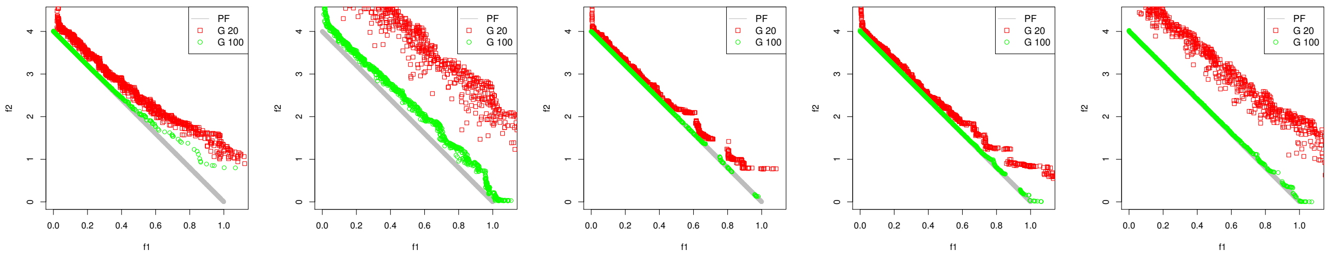
1 INTRODUCTION

In this paper, we aim to get a deeper understanding of the various components of dominance- and ε-dominance-based MOEAs on problems with complex Pareto set (PS) topologies. To do so, we study the performance of two well-established MOEAs, namely the dominance-based NSGA-II [3] and the ε-dominance-based AεSεH [1, 2], using distinct crossover operators and parent selection to produce five different algorithm configurations. We use ZCAT test problems [4], which includes a vast number of characteristics that are commonly seen in real-world problems, such as difficult PS topologies, complicated Pareto fronts (PF), multi-modality, non-separability, deceptiveness, and bias. By monitoring the size of first front, we try to distinguish algorithm's search in different stages so we can evaluate the impact of each algorithm feature into performance. The experimental study presented in this work, shows some interesting and useful conclusion to understand better Pareto dominance-based MOEAs.

2 ALGORITHMS IN COMPARISON AND EXPERIMENTAL SETUP

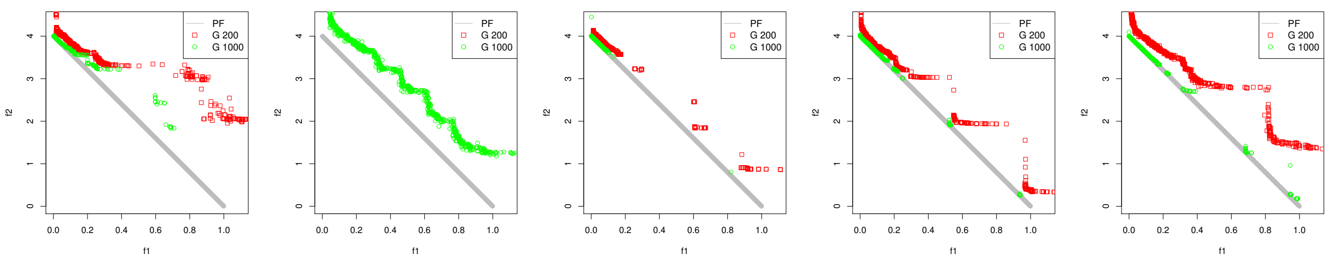
In this study, five different algorithm configurations are compared: NSGA-II-SBX, NSGA-II-DE, AεSεH-SBX, AεSεH-DE^t and AεSεH-DE. An adaptation of Differential Evolution (DE) was implemented to use it as a crossover operator, so the traditional steady state replacement of DE was substituted for a generational replacement compatible with both algorithms. In case of AεSεH, two distinct methods for parent selection were implemented. In AεSεH-DE each solution in the population is selected as a parent, and in case of AεSεH-DE^t a tournament is performed between solutions from the same neighborhood to select parents.

In total, 4 from all 20 ZCAT problems were tested, namely ZCAT3, ZCAT4, ZCAT8, and ZCAT9, with Unimodal & Separable and Multimodal & Separable instances. The number of objectives vary from m = 2 to 4 and set the total number of variables to n = 10 × m as suggested by [4].



(a) NSGA-II-SBX (b) NSGA-II-DE (c) AϵSϵH-SBX (d) AϵSϵH-DE^t (e) AϵSϵH-DE

Figure 1: Population from the best run of each algorithm over ZCAT8 unimodal instance.



(a) NSGA-II-SBX (b) NSGA-II-DE (c) AϵSϵH-SBX (d) AϵSϵH-DE^t (e) AϵSϵH-DE

Figure 2: Population from the best run of each algorithm over ZCAT8 multimodal instance.

We run each algorithm 30 times using the same set of seeds. The maximal number of generations is set 100 for unimodal problems, and 1000 for multimodal. Crossover settings are p_c = 1.0, p_{cv} = 0.5, and mutation rate is p_m = 1/n. When DE operator was used, F = 0.5. In case of SBX crossover, η_c = 15. In both cases, polynomial mutation is set to η_m = 20. Population size is set to $|\mathcal{P}|$ = 1000, and the reference neighborhood size for AϵSϵH is set to 20 individuals.

3 EXPERIMENTAL RESULTS AND DISCUSSION

In this work, we carefully evaluate algorithms performance using unimodal and multimodal separable ZCAT problems.

Figures 1 and 2 present results for the best run of each algorithm in terms of hypervolume value. By analyzing the results, there is a trade-off between algorithms using SBX and DE, while SBX produces a faster convergence towards the Pareto Front (PF), DE is capable of finding better distributed solutions in detriment of convergence. This conclusion can be verified by comparing NSGA-II-SBX and NSGA-II-DE.

Results for ZCAT8 suggest that neighborhood parent selection may increase convergence of solutions. When comparing algorithms NSGA-II-SBX and AϵSϵH-SBX, solutions produced by AϵSϵH are closer to the PF than NSGA-II for the same generation. This faster approach to the PF may lead AϵSϵH to shift the search from convergence stage to diversity stage, producing a better distributed set of solutions. When comparing the size of first front, AϵSϵH-SBX reaches a number equal to the population size faster than NSGA-II-SBX. As pointed out in [2], this can be an indicator for phase transition between conversion to distribution. Therefore, the algorithm can produce a better coverage of PF as it is operating in diversity stage. The inclusion of tournament selection with DE, AϵSϵH-DE^t, makes the algorithm convergence faster compared with AϵSϵH-DE.

4 CONCLUSION

From our analysis on the simulations, some interesting conclusions can be taken from multi-objective problems with complicated Pareto set topologies for unimodal and multimodal problems. One conclusion is that neighborhood parent selection in objective space can produce an improvement on the convergence of solutions, contributing to a faster transition from dominance-based stage to diversity-based, and by consequence improving algorithm PF coverage. Another finding is that differential evolution operator can improve the search ability of the algorithms by finding solutions better distributed across the objective space.

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