Incorporation of a decision space diversity maintenance mechanism into MOEA/D for multi-modal multi-objective optimization

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ABSTRACT

A Multi-objective optimization problem with several different Pareto optimal solution sets is defined as a multi-modal multi-objective optimization problem. Finding all the Pareto optimal solution sets for this type of problem can provide more options for the decision maker, which is important in some real-world situations. The Multi-objective evolutionary algorithm based on decomposition (MOEA/D) has been proved to perform well in various multi-objective problems but it does not perform well in finding all the Pareto optimal solution sets for multi-modal multi-objective optimization problems. In this paper, a MOEA/D variant is proposed to solve these problems. K solutions are assigned to each weight vector in the MOEA/D variant and the solutions are evaluated by not only the scalarizing function values but also the minimum distance from other solutions with the same weight vector and the average distance from the neighboring solutions in the same weight vector grid. Experimental results show that the MOEA/D variant performs much better than the original MOEA/D on the multi-modal distance minimization problems.

CCS CONCEPTS

• **Theory of computation → Evolutionary algorithms**;

KEYWORDS

Multi-modal multi-objective optimization, Decision space diversity, Evolutionary multi-objective optimization (EMO), MOEA/D

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1 INTRODUCTION

Some multi-objective optimization problem can have several different Pareto optimal sets. Put another way, at least two similar parts of feasible region map to the same part of the objective set. This type of problems is defined as Multi-modal multi-objective optimization problems[9]. Figure 1 describes an exemplary one. The three rhombuses in the feasible region correspond to the same rhombus in the objective set.

Solving multimodal multi-objective optimization problems is important and necessary. For a decision maker, it is never a bad thing to have plenty of options since there are always a lot of constraints in the real world. An exemplary real-world multi-modal multi-objective optimization problem is presented in section 3

There are a lot of efforts have been paid to modify evolutionary algorithms to solve multi-modal optimization problems[2, 4, 8, 12–14]. For multi-modal multi-objective optimization problems, Liang[9] introduced the niching method[11] into NSGA-II[3] to deal with them. In this paper, we incorporated a new diversity space mechanism into MOEA/D. We used multi-grid method[6] to separate its population and introduced two factors to force the solutions with the same weight vector to stay away from each other. The experimental results show that the new MOEA/D variant out performs the original MOEA/D in solving multimodal multi-objective optimization problems.

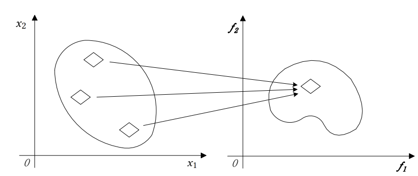

Figure 1: Multi-modal multi-objective optimization problem

This paper is organized as follows. Section 2 presents the decision space diversity maintenance mechanism of the MOEA/D variant. Section 3 examines the effects of the proposed idea. Section 4 concludes this paper.

2 THE PROPOSED DECISION SPACE DIVERSITY MAINTENANCE MECHANISM

MOEA/D has been proved to perform pretty well in multi-objective problems[1, 7, 10]. However, In a preliminary experiment, the poor performance of MOEA/D on multimodal multi-objective problems had been observed, which is shown in Figure 6. In order to solve the above problem, we modified the original MOEA/D. In this study, we adopted the PBI method (Weighted Sum and Tchebycheff are left for future research). Since in multi-modal multi-objective optimization problems, an objective vector can be mapped from several different solutions in decision space. The original MOEA/D assign each

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weight vector one solution, which can only obtain one of these solutions.

In order to find all these solutions, the number of solutions for each weight vector should at least be the same as the number of different Pareto sets. Thus, We assign K solutions to a weight vector. This can be viewed as using k grid structures of weight vectors.

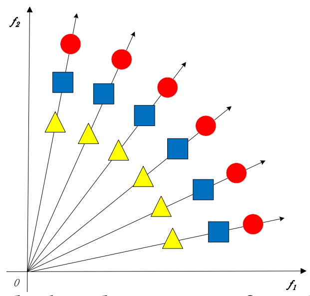

Figure 2: The k grid structures of weight vectors

Figure 2 shows an example of 3 grid structures of weight vectors. The triangles, squares, and circles represent solutions in decision space. "Solutions" with the same shape are in the same weight vector grid. Parent selection and solution replacement are performed in each weight vector grid independently in the same manner as the original MOEA/D. However, solutions with the same weight vector (These solutions are in different weight vector grids but correspond to the same weight vector) can be overlapped. In order to separate these overlapped solutions, we calculated two factors to force them to stay away from each other. The first factor is the minimum distance from other solutions with the same weight vector. We prefer solution who have a larger such distance. This preference will cause evolutionary pressure that makes solutions with the same weight vector away from each other, which hopefully can help us to single out each solution in different Pareto sets. The second factor is the average distance from neighboring solutions (solutions in the neighborhood defined in [15]) in the same weight vector grid. We prefer solution who have a lower such distance, which will make the solutions in a neighborhood of a weight vector grid more close to each other.

We use weighted sum method to combine these three factors. The evaluation function is as follows:

$$f_{eval} = w_1 g^{pbi} + w_2 d_{min} + w_3 d_{avg} \tag{1}$$

where g^{pbi} is the PBI function value, d_{min} is the minimum distance mentioned above, d_{avg} is the average distance mentioned previously, w_1, w_2, w_3 are weights of these three factors. w_2 is set to be a negative number while w_1 and w_3 are set to be positive numbers. We prefer solution with a smaller f_{eval} value, which force solutions to evolve to have a smaller d_{avg} and g^{pbi} but larger d_{min}.

The pseudo code of the proposed MOEA/D variant is shown in Algorithm 1. The space complexity of the proposed MOEA/D variant is the same as the original MOEA/D since they both maintain the population of the same size. For solving the test problem defined in section3, the time complexity for calculating d_{min} is $O(KD)$. The time complexity for calculating g^{pbi} is $O(MN_{building}D)$. M is the number of objectives, $N_{building}$ is the number of a particular type of buildings, and D is the dimensionality of the variable space. In the experiment, K and $N_{building}$ are both 4 and the distance calculation are in the same space whose dimensionality is 2. So, the time complexity for calculating d_{min} is dominated by the time complexity for calculating g^{pbi}. The time complexity for calculating d_{avg} is $O(TD)$. In the experiment, T is set to be 20, which is slightly larger than the product of M and $N_{building}$. We could set T to be the product of M and $N_{building}$, then the complexity of calculating d_{avg} is the same as the complexity of calculating g^{pbi}. Thus, the time complexity of the proposed MOEA/D variant is almost the same as the complexity of the original MOEA/D.

Algorithm 1 The proposed MOEA/D variant

Input: A MOP, a spread of evenly distributed weight vectors: $\Lambda = \{\lambda^1, \lambda^2, ..., \lambda^N\}$, N: The number of the weight vectors, MNI: max number of iteration, T:number of neighbours, K: number of weight vector grids

Output: The final population

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Calculate the Euclidean distance of every two different weight vectors and single out T closest weight vectors as neighbours of a weight vector. Store the indexes of neighbors of weight vector λ^i in B[i], B[i] = {i_1, i_2, ..., i_T}
Initialize the population X = {x^1, x^2, ..., x^N} randomly and calculated the respective object value F(x) for each solution x. Assign k solutions to each weight vector.
Initialize the reference point: set Z = {z_1, z_2, ..., z_m} by a problem-specific method.
for t = 1 → MNI do
    for i = 1 → N do
        for p = 1 → K do
            Reproduction: Randomly select two solutions x^l, x^r in the neighborhood of x_i in weight vector grid p to generate a new solution x_child
            Update of reference point: for each j ∈ {1, 2, ..., m}, if z_j > f_j(x_child), set z_j = f_j(x_child)
            Update of Neighboring Solutions in weight vector grid p: for each parental solution x_parental in the neighborhood, if f_eval(x_child) < f_eval(x_parental), replace x_parental with x_child
        end for
    end for
end for
```

3 EFFECTS OF THE PROPOSED IDEA

3.1 Test problems

We employed the test problem proposed by Ishibuchi[5]. There is a company who wants to build an apartment. The company wants to make the new apartment close to a hospital, a subway station, a primary school, and a gas station. The minimum distance from the apartment to any hospital, any subway station, any primary school, and any gas station are the four objectives.

Since there are a lot of hospitals, subway stations, primary schools, and gas stations in a city, it can form a distance minimization problem with several different Pareto sets. We call it multi-modal distance minimization problem(MMDMP), which is depicted in the left chart in Figure 4. The four squares are identical. In each vertex of a square, there is a building we are interested in. The triangle represents a hospital, the circle represents a subway station, the pentagon represents a gas station, and the pentacle represents a school. Any point in the chart is a potential place to build an apartment. Apparently, points in the four squares are nondominated to each other. All points out of the four squares are dominated by points in them. Apart from this basic problem, we also test our algorithm on its variants. In one variant shown in the middle chart, the size of each "Pareto box" is slightly different. In this case, the true Pareto set is the smallest box. In the other variant shown in the right chart, the shape of each "Pareto box" is slightly different. In this case, a part of each box is Pareto optimal.

3.2 Examine the effect of d_{min} and d_{avg}

Experimental setting:

We define a criterion called Pareto set coverage ratio to measure the decision space diversity of solutions. It is formulated as $p_c = n_c/n_e$ where p_c is the Pareto set coverage ratio, n_c is the number of covered Pareto sets, n_e is the number of existing Pareto sets.

We test three different settings of W in the modified MOEA/D. They are $W_1 = (1, 0, 0)$, $W_2 = (1, -1, 0)$, $W_3 = (1, -1, 0.05)$. W_1 means we do not incorporate d_{min} and d_{avg}. W_2 means we only incorporate d_{min} and W_3 means we incorporate both.

The max number of iteration is 80, T is 20, H is 10, K is 4, the distribution indexes in the simulated binary crossover(SBX) and the polynomial mutation are set to be 20, the crossover rate is 1. In a similar manner as Zhang & Li[15], Z_i is initialized to be nine-tenths of the lowest value of f_i found in the objective set of the initial population. The initial population for the three MOEA/D variants is identical. The random numbers used to select parents, used to determine whether mutate or not, and used for the SBX and the polynomial mutation are also identical for the three MOEA/D variants.

Since the three variants all converge very fast (in 2 or 3 iterations), we do not compare convergence in this paper. We only compare solutions in the final population in this paper. We run the algorithms 100 times and calculate the average Pareto set coverage ratio.

Experimental results:

Figure 5 shows the Pareto set coverage ratio of each iteration for different settings of w in the proposed MOEA/D on the MMDMP with four identical Pareto boxes, with Pareto boxes of slightly different size, and with Pareto boxes of slightly different shape from left to right.

For brevity, we call the modified MOEA/D with W_1 $MOEA/D_1$, the modified MOEA/D with W_2 $MOEA/D_2$, and the modified MOEA/D with W_3 $MOEA/D_3$. From the figure we can see that the Pareto set coverage ratio of solutions of $MOEA/D_2$ are much higher than solutions of $MOEA/D_1$, which means incorporating d_{min} improves the diversity of the solutions and thus help the MOEA/D variant find more Pareto sets. The Pareto set coverage ratio of solutions of $MOEA/D_3$ are slightly higher than solutions of $MOEA/D_2$ in MMDMP with four identical Pareto boxes. In MMDMP with four Pareto boxes of different size, the performance of $MOEA/D_2$ and $MOEA/D_3$ is similar. $MOEA/D_3$ perform worse than $MOEA/D_2$ in MMDMP with four Pareto boxes of different shapes. The effect of d_{avg} is not obvious.

3.3 Examine the effect of K

Experimental setting:

We test the performance of the proposed MOEA/D variant with different K on the above test problems. In order to have a fair comparison, we adjust H according to K to make the population size of the MOEA/D variants almost the same. The population size is 1120, 1092, and 1144 when k is 2, 3, and 4 respectively. W is $(1, -1, 0.05)$. All other settings are the same as in section 3.2.

Experimental results: Figure 3 shows that the Pareto set coverage ratio of the MOEA/D variant with a large K is higher than the MOEA/D variant with a smaller K. With a larger K, the MOEA/D variant can find more Pareto sets when the population size stays the same.

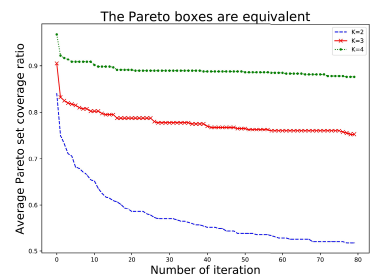

Figure 3: Results of MOEA/D variants with different K

3.4 Compare the original MOEA/D and the proposed MOEA/D variant

Experimental setting:

The population size of the MOEA/D variant and the original MOEA/D is 1144 and 1140 respectively. W of the proposed MOEA/D variant is $(1, -1, 0.05)$ Other settings are the same as in section 3.2.

Experimental results:

As showed in Figure 6, the average Pareto set coverage ratio of solutions of the proposed MOEA/D variant are much higher than the solutions of the original MOEA/D.

4 CONCLUSIONS

In this paper, we incorporated a decision space diversity maintenance mechanism into MOEA/D to solve multi-modal multi-objective optimization problems without increasing the space and time complexity. The proposed idea was examined through experiments on the test problems defined in section 3. The results in section 3.2 showed that incorporating d_{min} into the evaluation function could significantly increase the decision space diversity of solutions and helped the MOEA/D variant to find more different Pareto sets. The effect of incorporating d_{avg} is not apparent, which needs further study. The results in section 3.3 showed that the decision space diversity increase as K becomes larger. From results in Figure 6, we can see the proposed MOEA/D have a much higher performance in decision space diversity than the original MOEA/D.

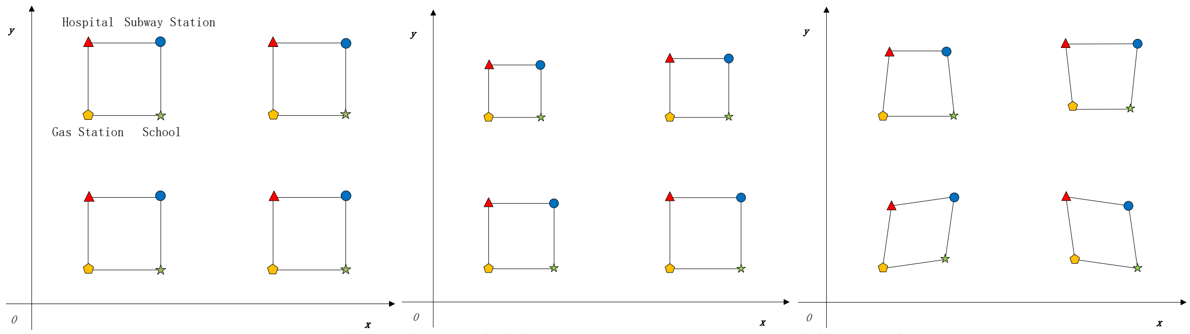

Figure 4: A sample multi-modal distance minimization problem and its variants

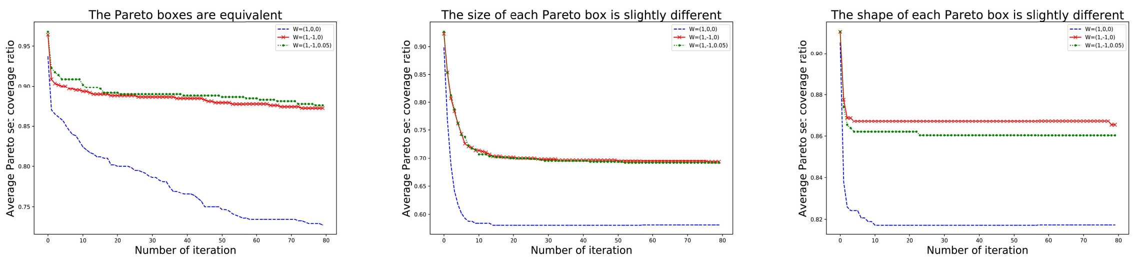

Figure 5: Results of the proposed MOEA/D variant with different W

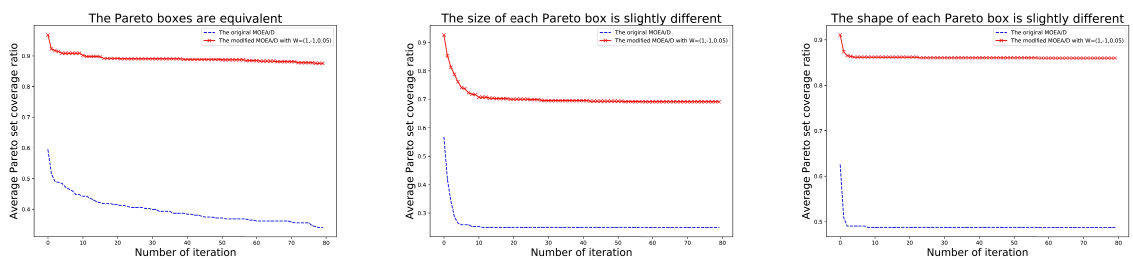

Figure 6: Results of the original MOEA/D and the proposed MOEA/D variant with W_3 on MMDMP with 4 Pareto boxes

It would be interesting to Modify the formula of the evaluation function, adjust the parameters, and test our algorithm on different test problems. All these are left for future research.

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