Search-Maps

Visualising and Exploiting the Global Structure of Computational Search Spaces

Gabriela Ochoa & Nadarajen Veerapen

University of Stirling
Stirling, Scotland, UK
{goc,nve}@cs.stir.ac.uk

http://gecco-2018.sigevo.org/

GECCO '18 Companion, July 15–19, 2018, Kyoto, Japan

ACM ISBN 978-1-4503-5764-7/18/07.
https://doi.org/10.1145/3205651.3207884

Instructors

- Gabriela Ochoa is a Senior Lecturer in Computing Science at the University of Stirling, Scotland. She holds a PhD from the University of Sussex, UK. Her research interests include evolutionary and heuristic search methods, with emphasis on autonomous search, hyper-heuristics, fitness landscape analysis and visualisation. She is associate editor of both Evolutionary Computation (MIT Press) and IEEE Transactions on Evolutionary Computation, and served as the EiC for GECCO 2017.

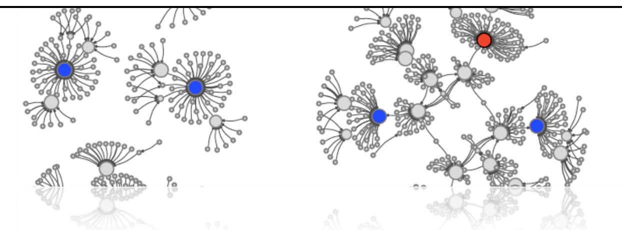
- Nadarajen Veerapen is a Research Fellow at the University of Stirling in Scotland. He holds a PhD in Computer Science from the University of Angers, France. His research interests include local search, hybrid methods, search-based software engineering and visualisation. He has served as Student Affairs Chair for GECCO 2017 and GECCO 2018 and has co-organised the workshop on Landscape-Aware Heuristic Search at PPSN 2016 and GECCO 2017 and GECCO 2018.

Outline

- Motivation and Background
  - Fitness landscapes
  - The notion of funnels
  - Complex networks
- Local Optima Networks
  - Definition of Nodes & Edges
  - Visualisation & Metrics
- Case Studies
  - Combinatorial optimisation
    - Binary: NK landscapes, number partitioning (NPP)
    - Permutation: TSP
  - Genetic Improvement
- Closing

Practical Sessions
- Sampling
- Visualisation
- Metrics

Download Materials
www.cs.stir.ac.uk/~goc/gecco2018tutorial



- Overall goal
- Fitness landscapes
- The notion of funnels
- Complex networks

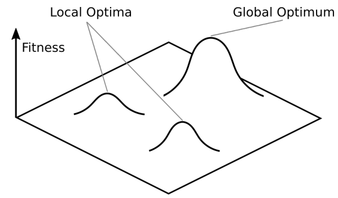
MOTIVATION AND BACKGROUND

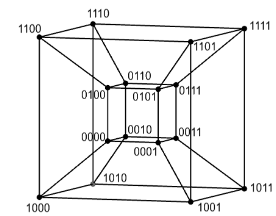
Overall goal

❖ To develop and establish a set of sampling methodologies, visualisation techniques and metrics to thoroughly characterise the *global structure* of computational search spaces.

❖ To lay the foundations for a new perspective to understand problem structure and improve heuristic search algorithms: *Search Space Cartography.*

Fitness landscapes





(S, N, f)

S Search space
N Neighbourhood structure
f Fitness function

Features of landscapes

Multimodality, ruggedness, deceptiveness & neutrality

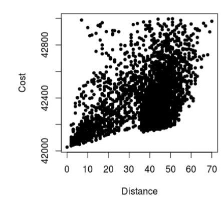
- No. of local optima
- Avg. size of local basins
- Avg. size of global basin
- Fitness-distance correlation
- Auto-correlation length
- Neutral degree
- …

The *big-valley* structure in combinatorial optimisation

❖ Several studies in the 90s. TSP (Boese et al, 1994), NK landscapes (Kauffman, 1993), graph bipartitioning (Merz & Freisleben, 1998) flowshop scheduling (Reeves, 1999)

❖ Distribution of local optima is not uniform. Clustered in a big-valley (*globally convex)* structure

❖ Many local optima, but easy to escape. Gradient at the coarse level leads to the global optimum.



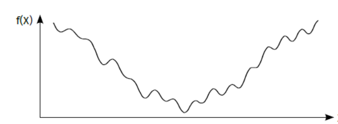


Fig. 1: Depiction of the 'big-valley' structure.

TSP: big-valley. Local optima confined to a small region

What is a Funnel?

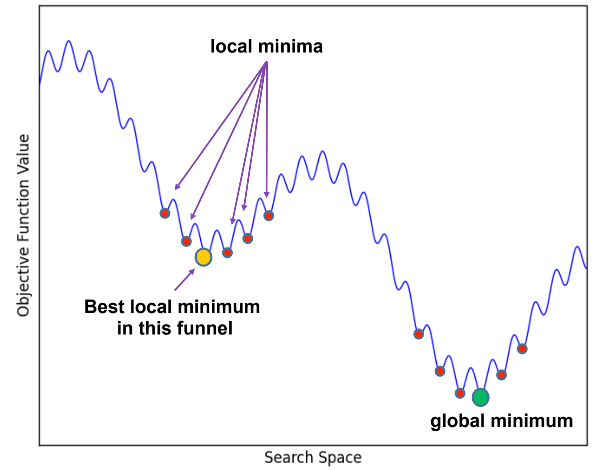
"A key concept that has arisen within the protein folding community is that of a *funnel* consisting of a set of downhill pathways that converge on a single low-energy minimum."

Doye, J. P. K., Miller, M. A., & Wales, D. J . The double-funnel energy landscape of the 38-atom Lennard-Jones cluster. *Journal of Chemical Physics*, 1999

Funnels in continuous optimisation
- Multilevel global structure (Locatelli, 2005)
- *Dispersion* metric (Lunacek & Whitley, 2006, 2008)
- Feature-based detection of (single) funnel structure (Kerschke et al., 2015)

Funnels in combinatorial optimisation
- Related to the big-valley (central-massif) hypothesis (previous slide)
- The big-valley re-visited (Hains, Whitley & Howe, 2011)
- Characterisation of funnels with Local Optima Networks (our contribution)

What is a Funnel?



Complex networks are everywhere!

"Behind each complex system, there is an intricate network that encodes the interactions between the system's components."
Albert-László Barabási, Network Science

Features of networks

Distance
- Number of links that make up the path between two points
- "Geodesic" = shortest path

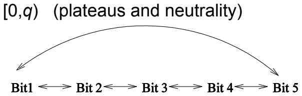
Topology (Degree distribution)
- Gives an idea of the spread in the number of links the nodes have
- $p(k)$ is the probability that a randomly selected node has k links

Cohesion
- Local: clustering coefficient or transitivity
- Global: components, community structure

NK landscapes (Kauffman, 93), NKq (Newman, 98)

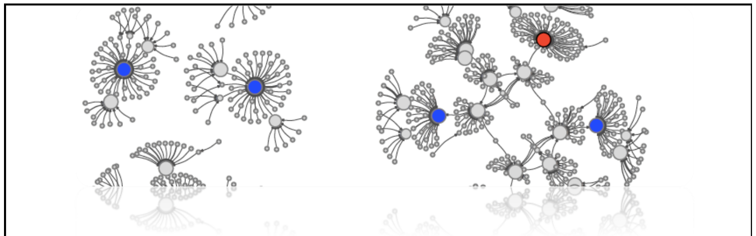
- Binary strings of length N
- Fitness function f: $B^N \rightarrow R^+$
- K $(0 \leq K < N)$ determines how many other bits in the string influence a given bit x_i
- Interacting bits can be **Adjacent** or **Random**
- Fitness contribution of each bit is:
  - Standard *NK* model: random real numbers [0,1]
  - Quantized *NKq* model: integer numbers [0,q) (plateaus and neutrality)

$$f(x) = \sum_{i=1}^{N} f_i(x|_{mask_i})$$



N=5, K = 2, Adjacent interaction

Sum of sub-functions.
$mask_i$: selects the $K+1$ bits that will be accessed by sub-function f_i



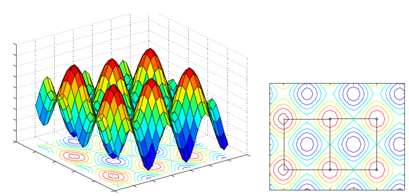
- Overview
- Definition of Nodes
- Definition of Edges: basin, escape, monotonic, crossover
- Visualisation & Metrics

LOCAL OPTIMA NETWORKS

Overview

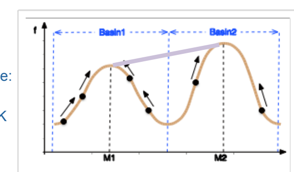
- Bring the tools of *complex networks* analysis to study the structure of combinatorial fitness landscapes
- Goal. Understand problem difficulty, design effective heuristic search algorithms
- Methodology. Extract a network that represents the landscape
  - Nodes. Local optima
  - Edges. Notion of adjacency/transition among local optima
- Conduct a network analysis
- Relate network features to search difficulty
- Exploit knowledge to design better algorithms

Local Optima Networks (LONs)



2D function landscape (left), and a contour plot of the local optima partition of space into basins of attraction (right). A simple regular network of six local maxima can be observed.

- Nodes. Local optima according to a hill-climbing heuristic
- Edges. Adjacency of basins. Transitions among optima.



- P. K. Doye. The network topology of a potential energy landscape: a static scale-free network. *Physical Review Letter*, 2002.
- G. Ochoa, M. Tomassini, S. Verel, and C. Darabos. A study of NK landscapes' basins and local optima networks. *GECCO* 2008

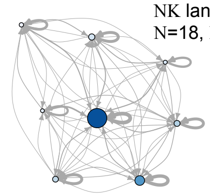
LON original model

- Space S, Neigborhood $N(s)$, fitness $f(s)$
- LON Model. Directed graph $\text{LON} = (L, E)$
- $h(s)$ stochastic operator that associates each solution s to its local optimum (Alg. 1)
- The basin of attraction of a local optimum $l_i \in L$ is the set $B_i = \{s \in S \mid h(s) = l_i\}$
- Nodes (L). A local optima is a solution l such that $\forall\ s \in N(s), f(s) \le f(l)$
- Basin Edges (E). Two local optima are connected if their basins of attraction intersect. At least on solution within a basin has a neighbour within the other basin.

Algorithm 1: Best-improvement local search

```
Choose initial solution s ∈ S
repeat
   choose s' ∈ N(s), f(s') = max_{x∈N(s)} f(x)
   if f(s) ≤ f(s') then
      s ← s'
   end if
until s is a Local optimum
```

NK landscape
N=18, K=2



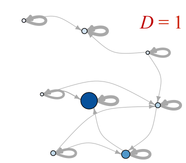
w_{ij} proportion of transitions from solutions $s \in Bi$ to solutions $s' \in Bj$

Escape edges

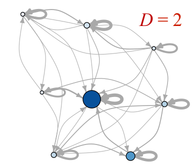
- Account for the chances of escaping a local optimum after a controlled mutation (e.g. 1 or 2 bit-flips in binary space) followed by hill-climbing
- Given a distance function d and integer value D, there is and edge e_{ij} between l_i and l_j f a solution s exists such that $d(s,l_i) \le D$ and $h(s) = l_j$
- w_{ij} cardinality of $\{s \in S \mid d(s,l_i) \le D \text{ and } h(s) = l_j\}$
- Sampled networks. There is an edge e_{ij} between l_i and l_j if l_j can be obtained after applying a *perturbation* to l_i followed by hill-climbing. Weights are estimated by the sampling process.

NK landscape
N=18, K=2

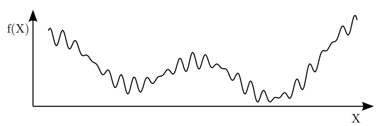
$D = 1$



$D = 2$



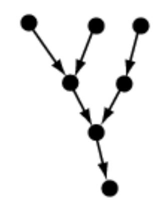
Characterisation of funnels



- Funnels can be loosely defined as groups of local optima, which are close in configuration space within a group, but well-separated between groups.
- A funnel conforms a coarse-grained gradient towards a low cost optimum.
- How to characterise funnels more rigorously using LONs?
  - Connected components. Funnels are sub-graphs, connected components within LONs. (EvoCOP, 2016)
  - Communities. Funnels are *communities* within LONs. (GECCO, 2016, 2017)
  - Monotonic sequences. Concept from energy landscapes. Conceptually sound characterisation, incorporating both grouping and coarse-grained gradient. (EvoCOP 2017, 2018; JoH 2017)

Characterisation of funnels with LONs

- Monotonic edges. Keep only non-deteriorating edges $l_1 \rightarrow l_2$, if $f(l_2) \le f(l_1)$
- Monotonic sequence. Path of connected local optima $l_1 \rightarrow l_2 \rightarrow l_3 \ldots \rightarrow l_s$ $f(l_i) \le f(l_{i-1})$
- Sink. Natural end of the sequence, when there is no adjacent improving local optima
- Funnel. Aggregation of all monotonic sequences ending at the same point (sink). Basin of attraction level of local optima



Sink. Node without outgoing edges

S set of sinks

```
i ← 0
for s ∈ S do
    fbasin[i] ← breadthFirstSearch(LON, s)
    fbsize[i] ← length(fbasin[i])
    i ← i + 1
end
```

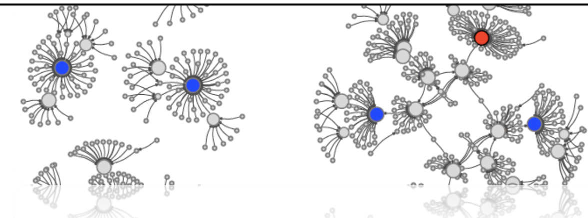
Complex network tools

Visualisation

- Force directed layout
  - Position nodes in 2D
  - Edges of similar length
  - Minimise crossings
  - Exhibit symmetries
- Example algorithms
  - Fruchterman & Reingold
  - Kamada & Kawai
- Software packages
  - R igraph
  - Gephi

Metrics

- Network metrics
  - Number of nodes
  - Number of edges (density)
  - Number of global optima
  - Weight of self-loops
  - Avg. fitness of local optima
  - Number of connected components
  - Avg. path length to a global optimum
  - Centrality (PageRank) of global optima
  - Clustering coefficient
- Funnel metrics
  - Number of funnels (sinks)
  - Normalised size of global funnel(s)
  - Normalised incoming strength (weighted degree) of global sink(s)

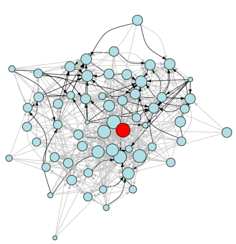


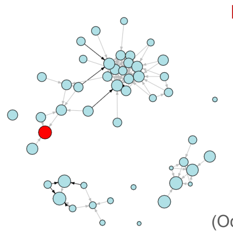
- NK landscapes, Grey-box optimisation & Tunnelling Crossover
- Number partitioning phase transition & multiple funnels
- TSP and multiple funnels
- Exploiting knowledge of the global structure
- Genetic improvement landscapes

CASE STUDIES

Crossover network model (XLON)

- Partition Crossover (PX), deterministic and greedy
- NKq landscapes q=100; K={2, 3} and N ={20, 25, 30}
- Fast extraction of all local optima using Grey-box optimisation (k-bounded additive functions).





Example XLON:

- N = 20, K = 2
- 2^{20} = 1,048,576
- Adjacent: 60 nodes, 1 component
- Random: 50 nodes, 7 components

(Ochoa, Chicano, Tinos, Whitley. GECCO 2015)

XLON

Definition

Graph (V, E_{PX}) where nodes are local optima end edges link parents to offspring via partition crossover

Construction

```
Output: V (set of local optima)
1: V ← ∅
2: for x ∈ B^n do
3:   if S_i(x) ≤ 0 for all 1 ≤ i ≤ n then
4:     V ← V ∪ {x}
5:   end if
6: end for
```

1. Local optima identification

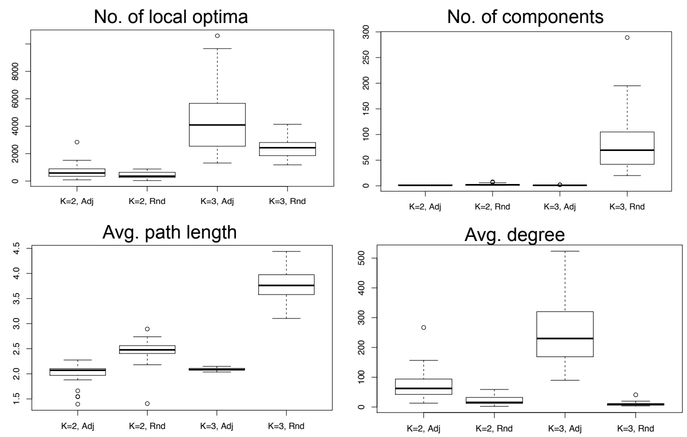
- *Score* $S_i(x)$ is the change in fitness from x to solution flipping bit i
- x is a local optimum if all $S_i(x)$ are lower than or equal to zero
- Efficient incremental calculation of *Score*. Overall complexity $O(2^N)$

```
Input: V
Output: XLON = G(V, E_PX)
1: for {x,y} ⊆ V do {All pairs of local optima}
2:   w ← PartitionCrossover(x,y)
3:   z ← HillClimber (w)
4:   if z ≠ x and z ≠ y then
5:     E_PX ← E_PX ∪ {(x,z),(y,z)}
6:   end if
7: end for
```

2. Network construction

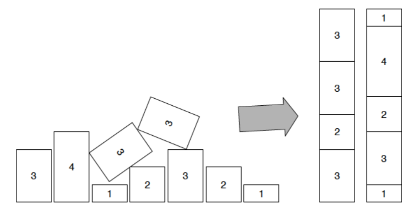
- All *x*, *y* pairs $nv*(nv-1)/2$
- PX and fast deterministic HC
- If z different to parents, two edges (x,z) and (y,z) are added to the network

Results N = 30, q=100, 30 replicas



Number Partitioning (NPP)

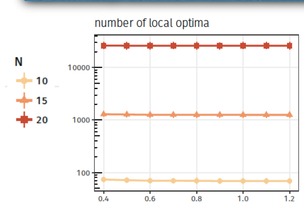
- Given a set of n positive integers $A=\{a_1, a_2, ..,a_n\}$, drawn at random from the set $\{1, 2, .., M\}$, find a disjoint partition (S_1, S_2) of A such that the discrepancy D between their sums is minimised
- A partition is perfect if $D = 0$, where $D = |\,\Sigma_{S1}\, a_i - \Sigma_{S2}\, a_i\,|$
- Easy-hard phase transition, $k = \log_2(M)/n$

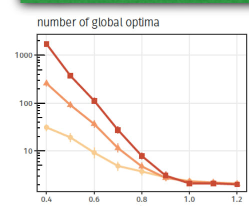


NPP fitness landscape

What features of the fitness landscape are responsible for the widely different behaviours?

Most fitness landscape metrics are insensitive/oblivious to the easy/hard phase transition!

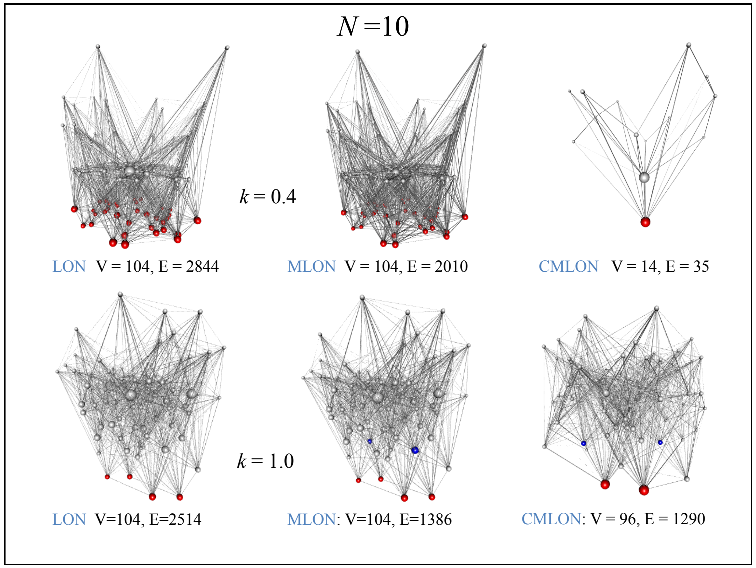


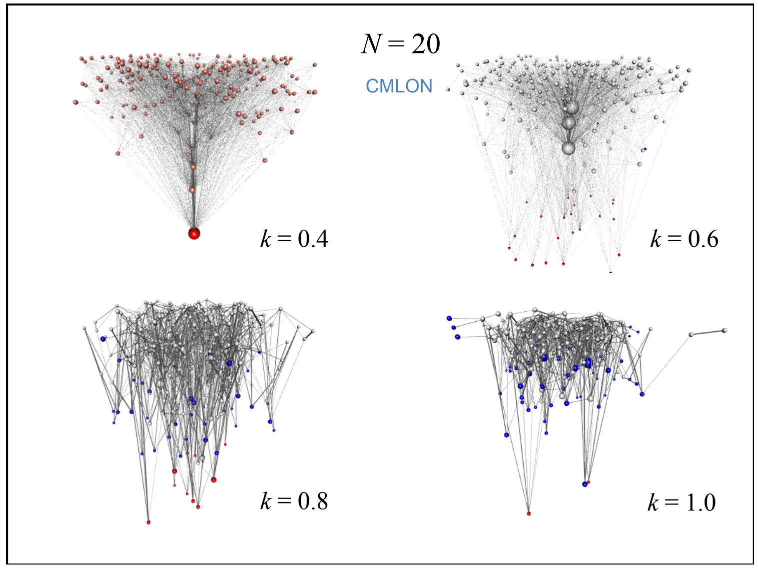


- Stadler, P., Hordijk, W., & Fontanari, J. (2003). Phase transition and landscape statistics of the number partitioning problem. *Physical Review E*
- K. Alyahya, J. Rowe (2014). Phase Transition and Landscape Properties of the Number Partitioning Problem. *EvoCOP*.

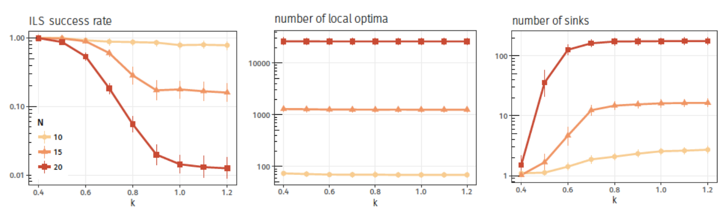
Methodology

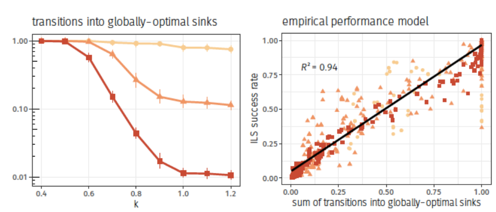
- Full enumeration and extraction of LONs
- $N = \{10, 15, 20\}$, k in $[0.4, 1.2]$ step 0.1
- 30 instances for each N and k
- LON. 1-flip local search, 2-flip perturbation $(D = 2)$
- MLON. Monotonic LON, worsening edges pruned
- CMLON. compressed MLON, LON plateaus contracted in a single node
- Empirical search performance: ILS success rate





LON metrics & Search Performance





(Ochoa, Veerapen, Daolio, Tomassini. EvoCOP 2017)

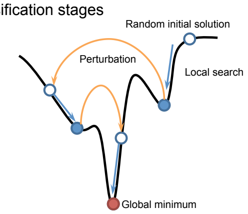
Travelling Salesman Problem (TSP)

- A prominent combinatorial optimisation problem
- Given n cities and the pairwise distance between them: what is the shortest possible route that visits each city and returns to the origin city?
- After over 50 years of intense study maintains its theoretical and practical relevance
- Successful exact solver: Concorde (Applegate et al., 2006)
- Successful heuristic solvers
  - Chained-LK. Iterated local search using Lin-Kernighan heuristic and *double-bridge* perturbation (Martin, Otto, Felten, 1992)
  - LKH. Improved implementation of Lin-Kernighan heuristic (Helsgaun, 2000,2009)
  - EAX. Evolutionary algorithm with edge exchange crossover (Nagata and Kobayashi, 2013)

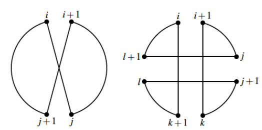
LON definition and sampling

Data: I, TSP instance
Result: L, set of local optima,
E, set of escape edges
$L \leftarrow \{\}$; $E \leftarrow \{\}$
for $i \leftarrow 1$ **to** 1000 **do**
  $s_{start} \leftarrow$ initialSolution()
  $s_{start} \leftarrow \text{LK}(s_{start})$
  $L \leftarrow L \cup \{s_{start}\}$
  while $j < 10000$ **do**
    $s_{end} \leftarrow \text{applyKick}(s_{start})$
    $s_{end} \leftarrow \text{LK}(s_{end})$
    $j \leftarrow j + 1$
    if $Objective(s_{end}) \leq Objective(s_{start})$ **then**
      $L \leftarrow L \cup \{s_{end}\}$
      $E \leftarrow E \cup \{(s_{start}, s_{end})\}$
      $s_{start} \leftarrow s_{end}$
      $j \leftarrow 0$
    end
  end
end

Chained Lin-Kernighan
(Martin, Otto, Felten, 1992)
- Form of Iterated Local Search
- Diversification & Intensification stages

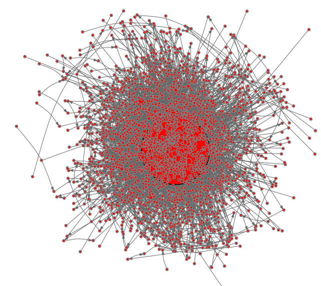


- Nodes. Lin-Kernighan
- Edges. Double-bridge

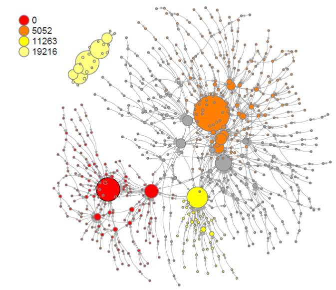


(a) 2-exchange (b) Double-bridge

TSP Synthetic Instances



C755 Clustered Cities
Funnels: 1, Success: 100%

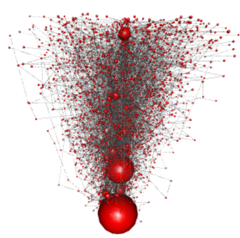


E755 Uniform Cities
Funnels: 4, Success: 13%

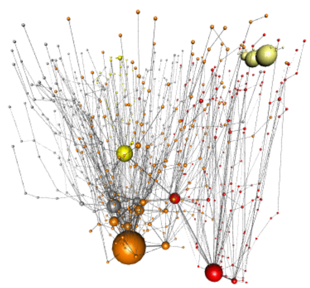
DIMACS random instances

(Ochoa & Veerapen, JoH 2017)

TSP Synthetic Instances



C755 Clustered Cities
Funnels: 1, Success: 100%

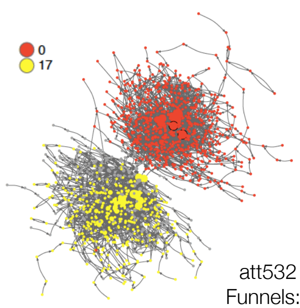


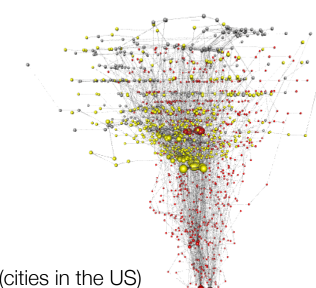
E755 Uniform Cities
Funnels: 4, Success: 13%

DIMACS random instances

Same layout, 3D projection where z coordinate is fitness

TSPLIB City Instance att532





att532 (cities in the US)
Funnels: 2, Success: 44%

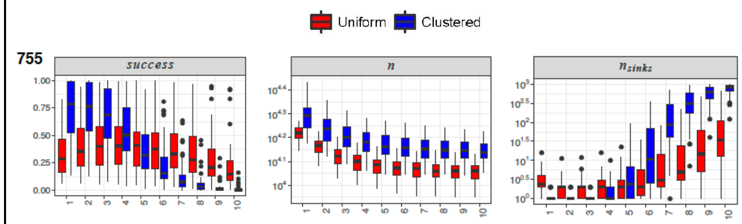
2D layout and 3D projection where z coordinate is fitness

Exploiting knowledge of the global structure

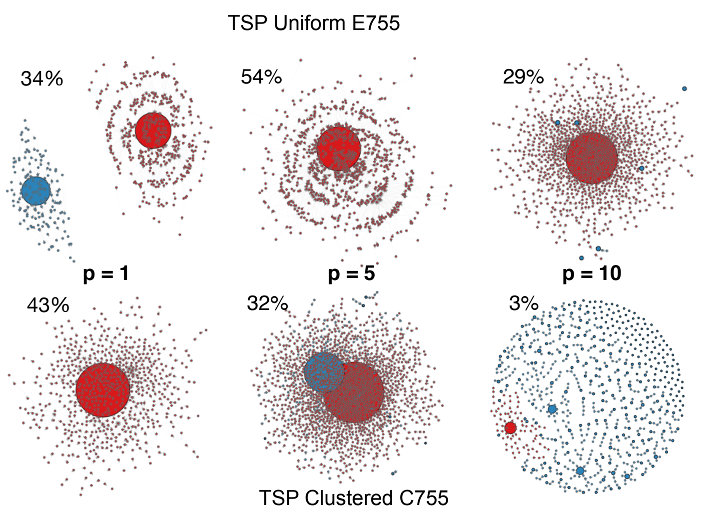
- ❖ Instances of several combinatorial optimisation problems have a multi-funnel structure
- ❖ Sub-optimal funnels act as traps to the search process
- ❖ Can we devise mechanisms for escaping sub-optimal funnels?
  - Restarts
  - Stronger perturbation in ILS implementations
  - Crossover

Increasing perturbation strength

- Chained-LK, Perturbation: 1 to10 double-bridge kicks
- TSP synthetic instances DIMACS: Uniform & Clustered
- Sizes 506, 755, 1010



(McMenemy, Veerapen & Ochoa. EvoCOP 2018)

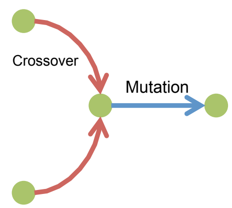


Other types of edges

- ❖ The LON model is not restricted to basin transition edges or escape edges.
- ❖ The model can also accommodate more than one type of edge.
- ❖ One example are LONs for Hybrid Evolutionary Algorithms.

LONs for Hybrid EAs

- Consider a Hydrid EA which incorporates a local search component to generate local optima.
- Two types of edges
  - Crossover (followed by local search)
  - Mutation (followed by local search)




```
P ← popInit()
while termination condition is not satisfied do
    Q(1) ← bestSolution(P)
    for i ← 2 to maxPop do
        (p1, p2) ← selection(P)
        Q(i) ← crossover(p1, p2); s ← 3opt(Q(i))
        LO ← LO ∪ {s}; E ← E ∪ {(p1, s), (p2, s)}
        if crossover did not improve the solutions then
            best ← chooseBest(p1, p2)
            Q(i) ← doubleBridgeMutation(best); s ← 3opt(Q(i))
            LO ← LO ∪ {s}; E ← E ∪ {(best, s)}
        end
    end
    if best sol. did not improve in last 20 gen. then  Q ← immigration(P)
    P ← Q
end
```

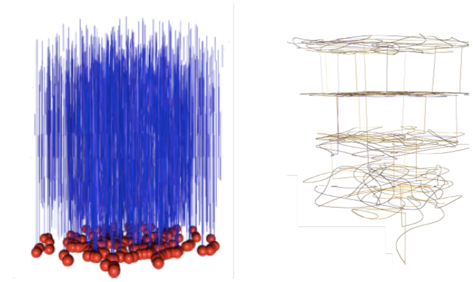
(Veerapen, Ochoa, Tinós, Whitley. PPSN 2016)

Contrasting LONs from two solving methods

- Hybrid GA vs ILS

Asymmetric TSP
Instance rbg323 LONs

Only edges and global optima are plotted.

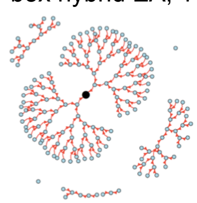


Hybrid GA
Partition Crossover (PX)
Success: 100%

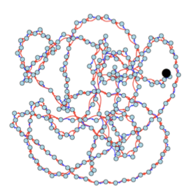
Chained LK
Success: 0%

Contrasting LONs from two solving methods

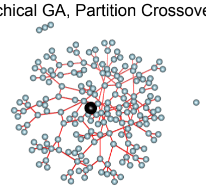
- Grey-box hybrid EA, 1 million variables NK

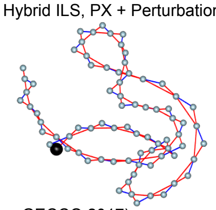


Hierarchical GA, Partition Crossover (PX)



Hybrid ILS, PX + Perturbation





(Chicano, Whitley, Ochoa, Tinos. GECCO 2017)

Genetic Improvement of Software

- **Genetic improvement (GI)** uses automated search to find improved versions of existing software
- GI is different from Genetic Programming since it modifies existing code
- It is not necessary to use Genetic Programming
- Other methods such a Genetic Algorithms may be used
- Local Search is used in this case study

Program Search Test Bench

- Introduce random mutations to a bug free-program
- Try to recover a version passing all test cases (Competent programmer hypothesis, DeMillo et al., 1978)
- Mutations restricted to Comparison (<, <=, ==, !=, >=, >) and Boolean operators (&&, ||)
- Objective function: Minimise number of failed test cases

Input 1	Input 2	Input 3	Expected Output	Output	Failed
1	1	2	3	3	FALSE
1	2	1	3	4	TRUE
1	2	2	1	1	FALSE

Program Search Test Bench

- Mutations of comparison operators (<, <=, ==, !=, >=, >)
- Mutations of Boolean operators (&&, ||)
- Representation: vector of integers

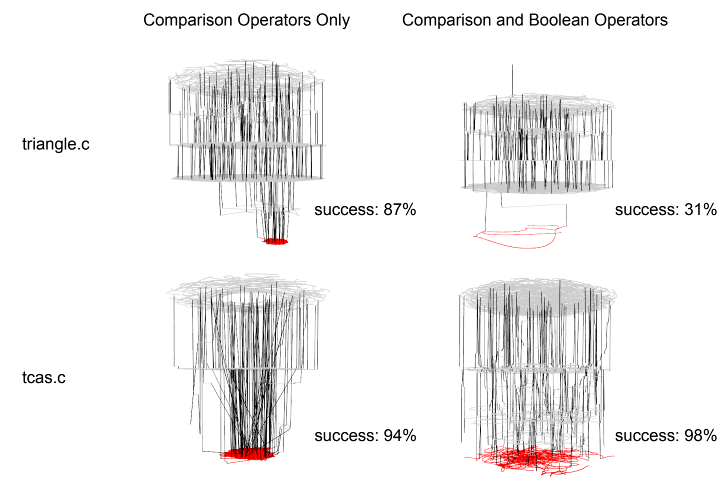
2	2	0	4	6	7

```
if ( side1 == side2 ) {
      triang = triang + 1 ;
}
if ( side1 == side3 ) {
      triang = triang + 2 ;
}
```

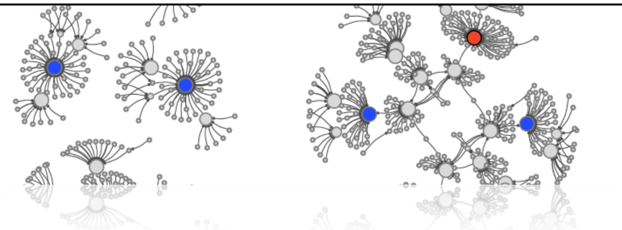
Program Search Test Bench

- Program and search space characteristics

	triangle.c	tcas.c
Lines of code	40	135
Number of comparison operators	17	14
Number of Boolean operators	7	16
Number of input parameters	3	12
Number of output values	1	3
Number of test cases	14	1578
Size of search space with comparison operators only	1.69×10^{13}	7.84×10^{10}
Size of search space with comparison and Boolean operators	2.17×10^{15}	5.14×10^{15}



(Langdon, Veerapen, Ochoa. EuroGP 2017)
(Veerapen, Daolio, Ochoa. GECCO comp. 2017)



- More accessible (visual) approach to heuristic understanding
- Rigorous characterisation of funnels
- Global structure impacts search
- New code available to assist researchers

CLOSING

References (1)

- K. Alyahya and J. E. Rowe, “Phase Transition and Landscape Properties of the Number Partitioning Problem,” in *Evolutionary Computation in Combinatorial Optimisation*, C. Blum and G. Ochoa, Eds. Springer Berlin Heidelberg, 2014, pp. 206–217.
- D. L. Applegate, R. E. Bixby, V. Chvátal, and W. J. Cook, *The Traveling Salesman Problem: A Computational Study*. Princeton University Press, 2006.
- A.-L. Barabási and M. Pósfai, *Network Science*. Cambridge University Press, 2016.
- K. D. Boese, A. B. Kahng, and S. Muddu, “A new adaptive multi-start technique for combinatorial global optimizations,” *Operations Research Letters*, vol. 16, no. 2, pp. 101–113, Sep. 1994.
- R. A. DeMillo, R. J. Lipton, and F. G. Sayward, “Hints on Test Data Selection: Help for the Practicing Programmer,” *Computer*, vol. 11, no. 4, pp. 34–41, Apr. 1978.
- J. P. K. Doye, “Network Topology of a Potential Energy Landscape: A Static Scale-Free Network,” *Phys. Rev. Lett.*, vol. 88, no. 23, p. 238701, May 2002.
- J. P. K. Doye, M. A. Miller, and D. J. Wales, “The double-funnel energy landscape of the 38-atom Lennard-Jones cluster,” *The Journal of Chemical Physics*, vol. 110, no. 14, pp. 6896–6906, Apr. 1999.
- T. M. J. Fruchterman and E. M. Reingold, “Graph drawing by force-directed placement,” *Softw: Pract. Exper.*, vol. 21, no. 11, pp. 1129–1164, Nov. 1991.

References (2)

- I. Gent and T. Walsh, “Phase Transitions and Annealed Theories: Number Partitioning as a Case Study,” in *Proceedings of the 12th European Conference on Artificial Intelligence (ECAI-96)*, 1996, pp. 170–174.
- D. R. Hains, L. D. Whitley, and A. E. Howe, “Revisiting the big valley search space structure in the TSP,” *J Oper Res Soc*, vol. 62, no. 2, pp. 305–312, Feb. 2011.
- K. Helsgaun, “An effective implementation of the Lin–Kernighan traveling salesman heuristic,” *European Journal of Operational Research*, vol. 126, no. 1, pp. 106–130, Oct. 2000.
- K. Helsgaun, “General k-opt submoves for the Lin–Kernighan TSP heuristic,” *Math. Prog. Comp.*, vol. 1, no. 2–3, pp. 119–163, Jul. 2009.
- S. Herrmann, M. Herrmann, G. Ochoa, and F. Rothlauf, “Shaping Communities of Local Optima by Perturbation Strength,” in *Proceedings of the Genetic and Evolutionary Computation Conference*, New York, NY, USA, 2017, pp. 266–273.
- S. Herrmann, G. Ochoa, and F. Rothlauf, “Communities of Local Optima As Funnels in Fitness Landscapes,” in *Proceedings of the Genetic and Evolutionary Computation Conference 2016*, New York, NY, USA, 2016, pp. 325–331.
- T. Kamada and S. Kawai, “An algorithm for drawing general undirected graphs,” *Information Processing Letters*, vol. 31, no. 1, pp. 7–15, Apr. 1989.
- S. A. Kauffman, *The Origins of Order: Self-organization and Selection in Evolution*. Oxford University Press, 1993.

References (3)

- P. Kerschke, M. Preuss, S. Wessing, and H. Trautmann, “Detecting Funnel Structures by Means of Exploratory Landscape Analysis,” in *Proceedings of the 2015 Annual Conference on Genetic and Evolutionary Computation*, New York, NY, USA, 2015, pp. 265–272.
- W. B. Langdon, N. Veerapen, and G. Ochoa, “Visualising the Search Landscape of the Triangle Program,” in *Genetic Programming*, 2017, vol. 10196, pp. 96–113.
- M. Locatelli, “On the Multilevel Structure of Global Optimization Problems,” *Comput Optim Applic*, vol. 30, no. 1, pp. 5–22, Jan. 2005.
- M. Lunacek and D. Whitley, “The Dispersion Metric and the CMA Evolution Strategy,” in *Proceedings of the 8th Annual Conference on Genetic and Evolutionary Computation*, New York, NY, USA, 2006, pp. 477–484.
- M. Lunacek, D. Whitley, and A. Sutton, “The Impact of Global Structure on Search,” in *Parallel Problem Solving from Nature – PPSN X*, G. Rudolph, T. Jansen, N. Beume, S. Lucas, and C. Poloni, Eds. Springer Berlin Heidelberg, 2008, pp. 498–507.
- O. Martin, S. W. Otto, and E. W. Felten, “Large-step markov chains for the TSP incorporating local search heuristics,” *Operations Research Letters*, vol. 11, no. 4, pp. 219–224, May 1992.
- P. McMenemy, N. Veerapen, and G. Ochoa, “How Perturbation Strength Shapes the Global Structure of TSP Fitness Landscapes,” in *Evolutionary Computation in Combinatorial Optimization*, 2018, pp. 34–49.

References (4)

- P. Merz and B. Freisleben, "Memetic Algorithms for the Traveling Salesman Problem," *Complex Systems*, vol. 13, no. 4, pp. 297–345, 2001.
- Y. Nagata and S. Kobayashi, "A Powerful Genetic Algorithm Using Edge Assembly Crossover for the Traveling Salesman Problem," *INFORMS Journal on Computing*, vol. 25, no. 2, pp. 346–363, May 2013.
- M. E. J. Newman and R. Engelhardt, "Effects of selective neutrality on the evolution of molecular species," *Proceedings of the Royal Society of London B: Biological Sciences*, vol. 265, no. 1403, pp. 1333–1338, Jul. 1998.
- G. Ochoa, F. Chicano, R. Tinós, and D. Whitley, "Tunnelling Crossover Networks," in *Proceedings of the 2015 Annual Conference on Genetic and Evolutionary Computation*, New York, NY, USA, 2015, pp. 449–456.
- G. Ochoa, M. Tomassini, S. Vérel, and C. Darabos, "A Study of NK Landscapes' Basins and Local Optima Networks," in *Proceedings of the 10th Annual Conference on Genetic and Evolutionary Computation*, New York, NY, USA, 2008, pp. 555–562.
- G. Ochoa and N. Veerapen, "Deconstructing the Big Valley Search Space Hypothesis," in *Evolutionary Computation in Combinatorial Optimization*, F. Chicano, B. Hu, and P. García-Sánchez, Eds. Springer International Publishing, 2016, pp. 58–73.
- G. Ochoa and N. Veerapen, "Mapping the global structure of TSP fitness landscapes," *J Heuristics*, pp. 1–30, May 2017.

References (5)

- G. Ochoa, N. Veerapen, F. Daolio, and M. Tomassini, "Understanding Phase Transitions with Local Optima Networks: Number Partitioning as a Case Study," in *Evolutionary Computation in Combinatorial Optimization*, 2017, vol. 10197, pp. 233–248.
- C. R. Reeves, "Landscapes, operators and heuristic search," *Annals of Operations Research*, vol. 86, pp. 473–490, Jan. 1999.
- P. F. Stadler, W. Hordijk, and J. F. Fontanari, "Phase transition and landscape statistics of the number partitioning problem," *Phys. Rev. E*, vol. 67, no. 5, p. 056701, May 2003.
- N. Veerapen, F. Daolio, and G. Ochoa, "Modelling Genetic Improvement Landscapes with Local Optima Networks," in *Proceedings of the Genetic and Evolutionary Computation Conference Companion*, New York, NY, USA, 2017, pp. 1543–1548.
- N. Veerapen, G. Ochoa, R. Tinós, and D. Whitley, "Tunnelling Crossover Networks for the Asymmetric TSP," in *Parallel Problem Solving from Nature – PPSN XIV*, J. Handl, E. Hart, P. R. Lewis, M. López-Ibáñez, G. Ochoa, and B. Paechter, Eds. Springer International Publishing, 2016, pp. 994–1003.