Supplementary Material: Robust Multi-Modal Optimisation

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CCS CONCEPTS

• **Theory of computation → Evolutionary algorithms**; • **Computing methodologies → Uncertainty quantification**;

KEYWORDS

Robust optimisation; uncertain optimisation; multi-modal optimisation.

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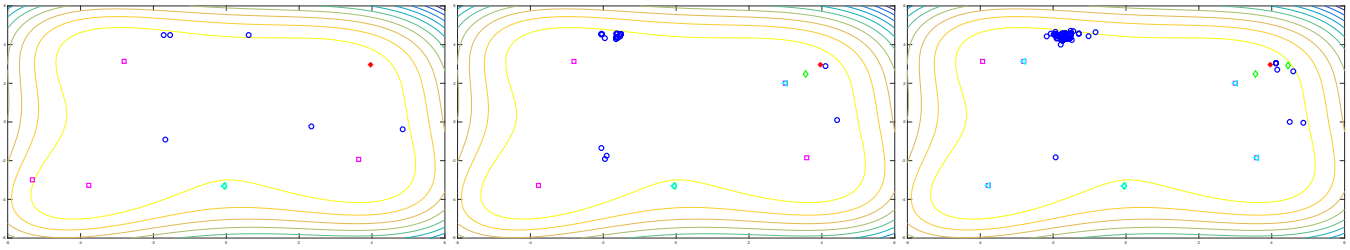
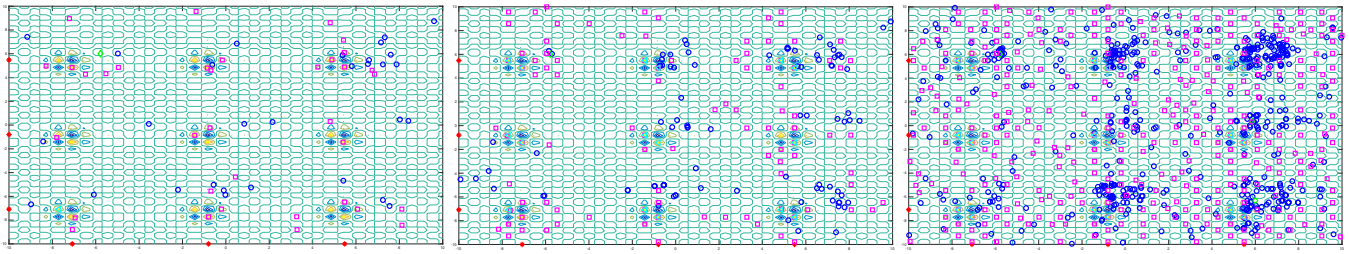
Table 1: f_{eff} of the best returned $\hat{f}_{\text{eff}}$ presented as median (MAD). Highest value for each problem in each snapshots marked in bold. Results that are significantly better than all other results are underlined and marked in red. Note that all values are rounded. The difference between the best median and values equal to it due to rounding is shown between square brackets.

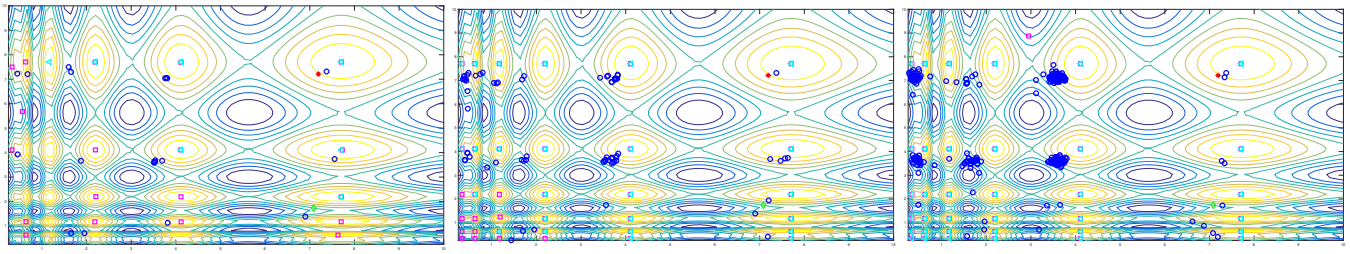
			MaxEval/100				MaxEval/10				MaxEval			
		max f_{eff}	NMMSO	NMMSO+ASA	RSCMSA	RSCMSA+ASA	NMMSO	NMMSO+ASA	RSCMSA	RSCMSA+ASA	NMMSO	NMMSO+ASA	RSCMSA	RSCMSA+ASA
TP1	d= 2	1.89	**1.72** (0.13)	1.68 (0.13)	1.11 (0.05)	1.11 (0.05)	1.76 (0.00)	1.80 (0.04)	**1.83** (0.00)	1.73 (0.12)	1.76 (0.00)	**1.80** (0.04)	1.76 (0.00)	1.79 (0.10)
	d= 5	4.73	**3.96** (0.16)	3.86 (0.17)	2.86 (0.25)	2.78 (0.28)	**4.32** (0.17)	4.23 (0.09)	4.28 (0.18)	4.15 (0.20)	4.47 [7.00E-04] (0.06)	4.34 (0.13)	**4.47** (0.07)	4.32 (0.14)
	d= 10	9.47	7.53 (0.24)	**7.56** (0.28)	5.69 (0.42)	5.69 (0.42)	**8.09** (0.29)	7.81 (0.30)	7.23 (0.32)	7.54 (0.32)	**8.71** (0.22)	8.49 (0.16)	8.71 [5.39E-03] (0.18)	8.55 (0.13)
TP2	d= 2	-0.83	-1.76 (0.11)	**-1.64** (0.35)	-1.68 (0.25)	-1.68 (0.25)	-2.17 (0.02)	-1.54 (0.38)	-2.17 (0.00)	**-1.50** (0.37)	-2.17 (0.00)	-1.50 (0.30)	-2.17 (0.00)	**-1.44** (0.38)
	d= 5	-2.08	-4.10 (0.22)	**-3.52** (0.37)	-4.80 (0.54)	-4.76 (0.51)	-4.60 (0.30)	-3.80 (0.67)	-4.92 (0.29)	**-3.78** (0.49)	-4.93 (0.11)	-4.21 (0.39)	-4.74 (0.23)	**-3.92** (0.30)
	d= 10	-4.17	-8.02 (0.44)	**-7.48** (0.38)	-8.57 (0.78)	-8.57 (0.78)	-8.35 (0.33)	**-7.73** (0.52)	-9.86 (0.28)	-8.55 (0.47)	-9.19 (0.34)	-7.77 (0.70)	-9.58 (0.48)	**-7.44** (0.55)
TP3	d= 2	0.68	**0.59** (0.03)	0.59 [1.09E-04] (0.03)	0.24 (0.13)	0.24 (0.13)	0.60 [1.10E-03] (0.00)	0.58 (0.03)	**0.60** (0.00)	0.60 [5.62E-03] (0.02)	**0.60** (0.00)	0.59 (0.03)	0.60 [3.67E-06] (0.00)	0.60 [6.40E-03] (0.02)
	d= 5	0.68	**0.55** (0.03)	0.53 (0.04)	0.32 (0.05)	0.33 (0.05)	**0.59** (0.01)	0.59 [4.89E-03] (0.02)	0.56 (0.06)	0.52 (0.04)	0.59 [1.34E-03] (0.01)	0.59 [4.95E-03] (0.02)	**0.59** (0.01)	0.59 [7.92E-04] (0.02)
	d= 10	0.68	**0.52** (0.03)	0.51 (0.02)	0.29 (0.07)	0.29 (0.07)	**0.57** (0.03)	0.55 (0.02)	0.51 (0.01)	0.47 (0.02)	0.59 (0.01)	0.58 (0.01)	**0.61** (0.01)	0.58 (0.02)
TP4	d= 2	0.42	0.37 (0.01)	**0.39** (0.01)	0.15 (0.12)	0.15 (0.12)	0.35 (0.00)	**0.40** (0.01)	0.38 (0.00)	0.39 (0.01)	0.35 (0.00)	0.40 (0.01)	0.35 (0.00)	**0.41** (0.00)
	d= 5	0.42	**0.38** (0.01)	0.38 [2.91E-03] (0.01)	0.15 (0.04)	0.15 (0.04)	0.36 (0.01)	0.38 [6.16E-03] (0.01)	0.38 [4.35E-03] (0.01)	**0.38** (0.01)	0.35 (0.00)	0.38 [1.31E-04] (0.01)	0.36 (0.00)	**0.38** (0.01)
	d= 10	0.42	**0.38** (0.01)	0.36 (0.02)	0.13 (0.04)	0.13 (0.04)	0.38 [6.54E-04] (0.00)	**0.38** (0.01)	0.27 (0.03)	0.27 (0.03)	0.37 (0.00)	**0.38** (0.00)	0.37 (0.00)	0.38 [1.31E-03] (0.00)
TP5		112.00	84.67 (0.02)	**89.18** (8.09)	73.01 (7.41)	73.01 (7.41)	84.67 (0.00)	**92.27** (3.86)	84.67 (0.00)	86.34 (7.23)	84.67 (0.00)	**85.09** (5.07)	84.67 (0.00)	84.65 (5.20)
TP6		0.60	0.31 (0.00)	**0.53** (0.06)	0.34 (0.20)	0.34 (0.20)	0.31 (0.00)	**0.55** (0.05)	0.31 (0.00)	0.54 (0.04)	0.31 (0.00)	**0.43** (0.10)	0.31 (0.00)	0.42 (0.09)
TP7		0.80	0.46 (0.00)	**0.69** (0.06)	0.13 (0.12)	0.13 (0.12)	0.46 (0.00)	**0.70** (0.06)	0.46 (0.00)	0.65 (0.13)	0.46 (0.00)	**0.70** (0.04)	0.46 (0.00)	0.68 (0.07)
TP8		176.45	78.61 (22.08)	**163.04** (7.27)	-95.81 (179.62)	-95.81 (179.62)	70.10 (11.10)	**169.03** (5.62)	59.00 (22.20)	147.43 (11.56)	81.20 (51.38)	170.26 (4.45)	81.20 (51.38)	**175.54** (0.82)
TP9		0.95	-0.99 (0.01)	**0.33** (0.05)	-1.13 (0.87)	-1.13 (0.87)	-0.77 (0.22)	**0.33** (0.04)	-0.99 (0.00)	0.16 (0.16)	-0.56 (0.00)	**0.34** (0.03)	-0.77 (0.22)	0.33 (0.04)
TP10		3.75	-0.24 (0.01)	**0.07** (0.55)	-0.24 (0.00)	-0.06 (0.25)	-0.24 (0.00)	**0.04** (0.37)	-0.24 (0.00)	0.03 (0.26)	-0.24 (0.00)	**0.22** (0.40)	-0.24 (0.00)	0.03 (0.31)
TP11		0.93	0.37 (0.20)	**0.91** (0.02)	0.37 (0.20)	0.88 (0.04)	0.28 (0.16)	**0.93** (0.01)	0.17 (0.20)	0.92 (0.01)	0.17 (0.20)	**0.92** (0.01)	0.06 (0.15)	0.92 [1.18E-03] (0.01)
TP12		-5.46	-13.35 (0.00)	**-5.98** (0.32)	-13.35 (0.00)	-6.22 (0.54)	-13.35 (0.00)	-5.94 (0.36)	-13.35 (0.00)	**-5.81** (0.27)	-13.35 (0.00)	**-5.99** (0.26)	-13.35 (0.00)	-6.26 (0.42)
TP13		-5.46	-5.46 [3.76E-07] (0.00)	-5.81 (0.23)	**-5.46** (0.00)	-5.85 (0.26)	**-5.46** (0.00)	-5.81 (0.23)	-5.46 [4.33E-12] (0.00)	-5.83 (0.23)	**-5.46** (0.00)	-5.81 (0.23)	-5.46 [4.33E-12] (0.00)	-5.80 (0.23)



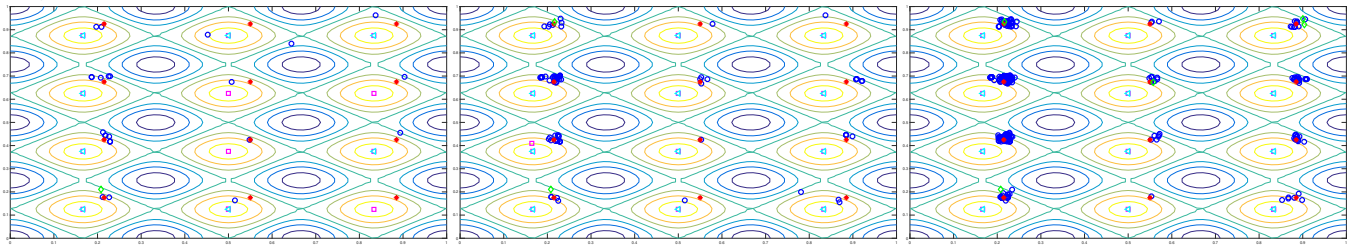
(a) TP8, (1) 10^{-2}×maximum evaluations (b) TP8, (2) 10^{-1}×maximum evaluations (c) TP8, maximum evaluations



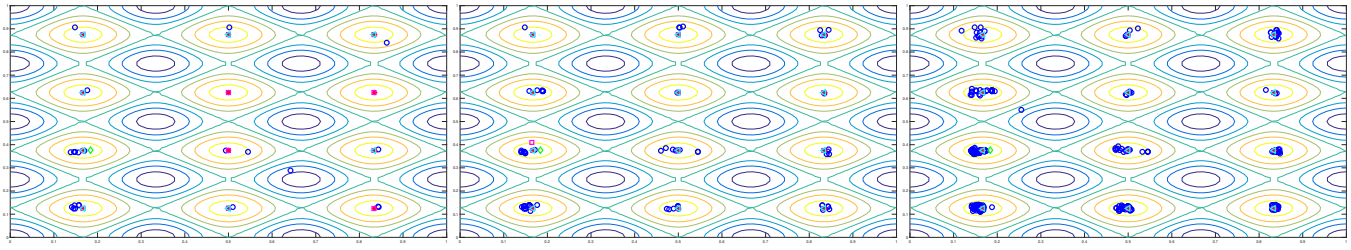
(d) TP10, (1) 10^{-2}×maximum evaluations (e) TP10, (2) 10^{-1}×maximum evaluations (f) TP10, maximum evaluations



(g) TP11, (1) 10^{-2}×maximum evaluations (h) TP11, (2) 10^{-1}×maximum evaluations (i) TP11, maximum evaluations



(j) TP12, (1) 10^{-2}×maximum evaluations (k) TP12, (2) 10^{-1}×maximum evaluations (l) TP12, maximum evaluations



(m) TP13, (1) 10^{-2}×maximum evaluations (n) TP13, (2) 10^{-1}×maximum evaluations (o) TP13, maximum evaluations

Figure 1: Snapshots of the runs at different time intervals for all the algorithms. The figure shows snapshots of one run per problem with all the algorithms starting from the same initial starting conditions (same random seed). The figures shows the original landscape contour plots with the robust optima indicated with a red star. The results for RS-CMSA is shown in cyan triangle, RS-CMSA+ASA in green diamond, NNMSO in magenta square, and NNMSO+ASA in blue circle.

Table 2: PR $\epsilon = 10^{-2}$. Highest value for each problem in each snapshots marked in bold.

		MaxEval/100				MaxEval/10				MaxEval			
		NMMSO(1)	NMMSO+ASA(1)	RSCMSA(1)	RSCMSA+ASA(1)	NMMSO(2)	NMMSO+ASA(2)	RSCMSA(2)	RSCMSA+ASA(2)	NMMSO(3)	NMMSO+ASA(3)	RSCMSA(3)	RSCMSA+ASA(3)
TP1	d= 2	**0.07**	0.00	0.00	0.00	**0.73**	0.13	0.20	0.17	**1.00**	0.40	0.23	0.20
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.27**	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP2	d= 2	**0.03**	**0.03**	0.00	0.00	0.00	**0.30**	0.00	0.03	0.00	**0.60**	0.00	0.17
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP3	d= 2	**0.10**	**0.10**	0.00	0.00	**0.87**	0.53	0.00	0.07	**1.00**	0.90	0.00	0.13
	d= 5	0.00	0.00	0.00	0.00	**0.13**	0.00	0.00	0.00	**0.40**	0.10	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP4	d= 2	0.00	**0.01**	0.00	0.00	0.02	**0.26**	0.00	0.03	0.00	**0.53**	0.00	0.21
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.01**	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP5		**1.00**	0.10	0.00	0.00	**1.00**	0.17	0.00	0.00	**1.00**	0.37	0.00	0.00
TP6		0.02	**0.28**	0.02	0.02	0.03	**0.72**	0.02	0.03	0.01	**0.99**	0.02	0.09
TP7		0.10	**0.23**	0.00	0.00	0.00	**0.63**	0.00	0.13	0.00	**0.93**	0.00	0.13
TP8		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.03**
TP9		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP10		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP11		0.00	**0.30**	0.00	**0.30**	0.00	**0.77**	0.00	0.67	0.00	**0.97**	0.00	0.90
TP12		0.00	**0.01**	0.00	0.00	0.00	**0.07**	0.00	0.00	0.00	**0.26**	0.00	0.00
TP13		**0.99**	0.03	0.64	0.00	**1.00**	0.12	**1.00**	0.00	**1.00**	0.31	**1.00**	0.00

Table 3: PR $\epsilon = 10^{-3}$. Highest value for each problem in each snapshots marked in bold.

		MaxEval/100				MaxEval/10				MaxEval			
		NMMSO(1)	NMMSO+ASA(1)	RSCMSA(1)	RSCMSA+ASA(1)	NMMSO(2)	NMMSO+ASA(2)	RSCMSA(2)	RSCMSA+ASA(2)	NMMSO(3)	NMMSO+ASA(3)	RSCMSA(3)	RSCMSA+ASA(3)
TP1	d= 2	**0.03**	0.00	0.00	0.00	**0.53**	0.07	0.20	0.00	**1.00**	0.07	0.23	0.00
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.13**	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP2	d= 2	0.00	0.00	0.00	0.00	0.00	**0.03**	0.00	0.00	0.00	**0.07**	0.00	0.03
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP3	d= 2	**0.03**	0.00	0.00	0.00	**0.70**	0.03	0.00	0.00	**1.00**	0.20	0.00	0.00
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.37**	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP4	d= 2	0.00	0.00	0.00	0.00	0.00	**0.02**	0.00	0.00	0.00	**0.08**	0.00	0.03
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP5		**0.93**	0.07	0.00	0.00	**1.00**	0.13	0.00	0.00	**1.00**	0.10	0.00	0.00
TP6		0.01	**0.09**	0.01	0.01	0.01	**0.23**	0.01	0.01	0.00	**0.59**	0.01	0.02
TP7		0.03	**0.13**	0.00	0.00	0.00	**0.20**	0.00	0.07	0.00	**0.57**	0.00	0.07
TP8		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP9		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP10		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP11		0.00	0.00	0.00	**0.07**	0.00	**0.17**	0.00	**0.17**	0.00	**0.53**	0.00	0.30
TP12		0.00	**0.00**	0.00	0.00	0.00	**0.01**	0.00	0.00	0.00	**0.04**	0.00	0.00
TP13		**0.94**	0.00	0.64	0.00	**1.00**	0.02	**1.00**	0.00	**1.00**	0.03	**1.00**	0.00

Table 4: PR $\epsilon = 10^{-4}$. Highest value for each problem in each snapshots marked in bold.

		MaxEval/100				MaxEval/10				MaxEval			
		NMMSO(1)	NMMSO+ASA(1)	RSCMSA(1)	RSCMSA+ASA(1)	NMMSO(2)	NMMSO+ASA(2)	RSCMSA(2)	RSCMSA+ASA(2)	NMMSO(3)	NMMSO+ASA(3)	RSCMSA(3)	RSCMSA+ASA(3)
TP1	d= 2	0.00	0.00	0.00	0.00	**0.23**	0.00	0.17	0.00	**0.97**	0.00	0.20	0.00
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP2	d= 2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.03**
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP3	d= 2	0.00	0.00	0.00	0.00	**0.07**	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP4	d= 2	0.00	0.00	0.00	0.00	0.00	**0.01**	0.00	0.00	0.00	**0.03**	0.00	0.00
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP5		**0.77**	0.03	0.00	0.00	**1.00**	0.07	0.00	0.00	**1.00**	0.03	0.00	0.00
TP6		0.00	**0.04**	0.01	0.01	0.00	**0.08**	0.01	0.01	0.00	**0.31**	0.01	0.01
TP7		**0.03**	**0.03**	0.00	0.00	0.00	**0.10**	0.00	0.03	0.00	**0.33**	0.00	0.03
TP8		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP9		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP10		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP11		0.00	0.00	0.00	0.00	0.00	**0.07**	0.00	0.00	0.00	**0.20**	0.00	0.00
TP12		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.00**	0.00	0.00
TP13		**0.75**	0.00	0.64	0.00	**1.00**	0.00	**1.00**	0.00	**1.00**	0.01	**1.00**	0.00

Table 5: PR $\epsilon = 10^{-5}$. Highest value for each problem in each snapshots marked in bold.

		MaxEval/100				MaxEval/10				MaxEval			
		NMMSO(1)	NMMSO+ASA(1)	RSCMSA(1)	RSCMSA+ASA(1)	NMMSO(2)	NMMSO+ASA(2)	RSCMSA(2)	RSCMSA+ASA(2)	NMMSO(3)	NMMSO+ASA(3)	RSCMSA(3)	RSCMSA+ASA(3)
TP1	d= 2	0.00	0.00	0.00	0.00	**0.10**	0.00	0.07	0.00	**0.97**	0.00	0.10	0.00
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP2	d= 2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP3	d= 2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP4	d= 2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP5		**0.77**	0.03	0.00	0.00	**1.00**	0.07	0.00	0.00	**1.00**	0.03	0.00	0.00
TP6		0.00	**0.01**	**0.01**	**0.01**	0.00	**0.05**	0.01	0.01	0.00	**0.09**	0.01	0.01
TP7		0.00	0.00	0.00	0.00	0.00	**0.03**	0.00	0.00	0.00	**0.07**	0.00	0.00
TP8		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP9		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP10		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP11		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP12		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP13		0.42	0.00	**0.64**	0.00	**1.00**	0.00	**1.00**	0.00	**1.00**	0.00	**1.00**	0.00

Table 6: SR $\epsilon = 10^{-1}$. Highest value for each problem in each snapshots marked in bold.

		NMMSO(1)	NMMSO+ASA(1)	RSCMSA(1)	RSCMSA+ASA(1)	NMMSO(2)	NMMSO+ASA(2)	RSCMSA(2)	RSCMSA+ASA(2)	NMMSO(3)	NMMSO+ASA(3)	RSCMSA(3)	RSCMSA+ASA(3)
TP1	d= 2	**0.47**	0.30	0.00	0.00	**1.00**	0.97	0.70	0.43	**1.00**	**1.00**	0.80	0.57
	d= 5	0.00	0.00	0.00	0.00	**0.10**	0.00	0.00	0.00	**0.87**	0.07	0.03	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP2	d= 2	0.13	**0.30**	0.00	0.00	0.03	**0.80**	0.00	0.23	0.00	**1.00**	0.00	0.43
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.07**	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP3	d= 2	**0.87**	0.73	0.03	0.03	**1.00**	**1.00**	0.97	0.80	**1.00**	**1.00**	**1.00**	0.87
	d= 5	**0.43**	**0.43**	0.00	0.00	**1.00**	0.93	0.50	0.27	**1.00**	**1.00**	0.97	0.97
	d= 10	**0.17**	0.03	0.00	0.00	**0.63**	0.33	0.03	0.00	**1.00**	**1.00**	0.77	0.80
TP4	d= 2	**0.07**	0.03	0.00	0.00	**0.97**	0.23	0.03	0.00	**1.00**	0.57	0.30	0.00
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP5		**1.00**	0.20	0.00	0.00	**1.00**	0.40	0.00	0.00	**1.00**	0.77	0.00	0.00
TP6		0.00	**0.53**	0.00	0.00	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.03
TP7		0.13	**0.83**	0.00	0.00	0.03	**1.00**	0.00	0.30	0.07	**1.00**	0.00	0.70
TP8		0.00	0.00	0.00	0.00	0.00	**0.03**	0.00	0.00	0.00	**0.17**	0.00	0.10
TP9		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP10		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP11		0.00	**0.97**	0.00	0.83	0.00	**1.00**	0.00	0.90	0.03	**1.00**	0.00	0.93
TP12		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.10**	0.00	0.00
TP13		**0.97**	0.00	0.00	0.00	**1.00**	0.00	**1.00**	0.00	**1.00**	0.13	**1.00**	0.00

Table 7: SR $\epsilon = 10^{-2}$. Highest value for each problem in each snapshots marked in bold.

		NMMSO(1)	NMMSO+ASA(1)	RSCMSA(1)	RSCMSA+ASA(1)	NMMSO(2)	NMMSO+ASA(2)	RSCMSA(2)	RSCMSA+ASA(2)	NMMSO(3)	NMMSO+ASA(3)	RSCMSA(3)	RSCMSA+ASA(3)
TP1	d= 2	**0.07**	0.00	0.00	0.00	**0.73**	0.13	0.20	0.17	**1.00**	0.40	0.23	0.20
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.27**	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP2	d= 2	**0.03**	**0.03**	0.00	0.00	0.00	**0.30**	0.00	0.03	0.00	**0.60**	0.00	0.17
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP3	d= 2	**0.10**	**0.10**	0.00	0.00	**0.87**	0.53	0.00	0.07	**1.00**	0.90	0.00	0.13
	d= 5	0.00	0.00	0.00	0.00	**0.13**	0.00	0.00	0.00	**0.40**	0.10	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP4	d= 2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.03**	0.00	0.00
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP5		**1.00**	0.10	0.00	0.00	**1.00**	0.17	0.00	0.00	**1.00**	0.37	0.00	0.00
TP6		0.00	0.00	0.00	0.00	0.00	**0.27**	0.00	0.00	0.00	**0.93**	0.00	0.00
TP7		0.10	**0.23**	0.00	0.00	0.00	**0.63**	0.00	0.13	0.00	**0.93**	0.00	0.13
TP8		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.03**
TP9		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP10		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP11		0.00	**0.30**	0.00	**0.30**	0.00	**0.77**	0.00	0.67	0.00	**0.97**	0.00	0.90
TP12		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP13		**0.93**	0.00	0.00	0.00	**1.00**	0.00	**1.00**	0.00	**1.00**	0.00	**1.00**	0.00

Table 8: SR $\epsilon = 10^{-3}$. Highest value for each problem in each snapshots marked in bold.

		NMMSO(1)	NMMSO+ASA(1)	RSCMSA(1)	RSCMSA+ASA(1)	NMMSO(2)	NMMSO+ASA(2)	RSCMSA(2)	RSCMSA+ASA(2)	NMMSO(3)	NMMSO+ASA(3)	RSCMSA(3)	RSCMSA+ASA(3)
TP1	d= 2	**0.03**	0.00	0.00	0.00	**0.53**	0.07	0.20	0.00	**1.00**	0.07	0.23	0.00
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.13**	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP2	d= 2	0.00	0.00	0.00	0.00	0.00	**0.03**	0.00	0.00	0.00	**0.07**	0.00	0.03
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP3	d= 2	**0.03**	0.00	0.00	0.00	**0.70**	0.03	0.00	0.00	**1.00**	0.20	0.00	0.00
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.37**	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP4	d= 2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP5		**0.93**	0.07	0.00	0.00	**1.00**	0.13	0.00	0.00	**1.00**	0.10	0.00	0.00
TP6		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.03**	0.00	0.00
TP7		0.03	**0.13**	0.00	0.00	0.00	**0.20**	0.00	0.07	0.00	**0.57**	0.00	0.07
TP8		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP9		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP10		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP11		0.00	0.00	0.00	**0.07**	0.00	**0.17**	0.00	**0.17**	0.00	**0.53**	0.00	0.30
TP12		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP13		**0.50**	0.00	0.00	0.00	**1.00**	0.00	**1.00**	0.00	**1.00**	0.00	**1.00**	0.00

Table 9: SR $\epsilon = 10^{-4}$. Highest value for each problem in each snapshots marked in bold.

		NMMSO(1)	NMMSO+ASA(1)	RSCMSA(1)	RSCMSA+ASA(1)	NMMSO(2)	NMMSO+ASA(2)	RSCMSA(2)	RSCMSA+ASA(2)	NMMSO(3)	NMMSO+ASA(3)	RSCMSA(3)	RSCMSA+ASA(3)
TP1	d= 2	0.00	0.00	0.00	0.00	**0.23**	0.00	0.17	0.00	**0.97**	0.00	0.20	0.00
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP2	d= 2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.03**
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP3	d= 2	0.00	0.00	0.00	0.00	**0.07**	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP4	d= 2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP5		**0.77**	0.03	0.00	0.00	**1.00**	0.07	0.00	0.00	**1.00**	0.03	0.00	0.00
TP6		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP7		**0.03**	**0.03**	0.00	0.00	0.00	**0.10**	0.00	0.03	0.00	**0.33**	0.00	0.03
TP8		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP9		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP10		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP11		0.00	0.00	0.00	0.00	0.00	**0.07**	0.00	0.00	0.00	**0.20**	0.00	0.00
TP12		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP13		**0.10**	0.00	0.00	0.00	**1.00**	0.00	**1.00**	0.00	**1.00**	0.00	**1.00**	0.00

Table 10: SR $\epsilon = 10^{-5}$. Highest value for each problem in each snapshots marked in bold.

		NMMSO(1)	NMMSO+ASA(1)	RSCMSA(1)	RSCMSA+ASA(1)	NMMSO(2)	NMMSO+ASA(2)	RSCMSA(2)	RSCMSA+ASA(2)	NMMSO(3)	NMMSO+ASA(3)	RSCMSA(3)	RSCMSA+ASA(3)
TP1	d= 2	0.00	0.00	0.00	0.00	**0.10**	0.00	0.07	0.00	**0.97**	0.00	0.10	0.00
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP2	d= 2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP3	d= 2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP4	d= 2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	d= 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP5		**0.77**	0.03	0.00	0.00	**1.00**	0.07	0.00	0.00	**1.00**	0.03	0.00	0.00
TP6		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP7		0.00	0.00	0.00	0.00	0.00	**0.03**	0.00	0.00	0.00	**0.07**	0.00	0.00
TP8		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP9		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP10		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP11		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP12		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TP13		**0.03**	0.00	0.00	0.00	**1.00**	0.00	**1.00**	0.00	**1.00**	0.00	**1.00**	0.00

Table 11: The p-value of the two-sided Wilcoxon signed rank test comparing each snapshot results of NMMSO and NMMSO+ASA. P-values ≤ 0.05 are marked in bold.

		best f_{eff}			mean AE			PR $\epsilon = 10^{-1}$			PR $\epsilon = 10^{-2}$		
		(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
TP1	d= 2	0.73	0.70	0.70	**0.00**	**0.00**	**0.00**	0.30	1.00	1.00	0.50	**0.00**	**0.00**
	d= 5	**0.03**	**0.02**	**0.00**	**0.00**	**0.00**	**0.00**	1.00	0.25	**0.00**	1.00	1.00	**0.01**
	d= 10	0.61	**0.03**	**0.00**	**0.01**	**0.00**	**0.00**	1.00	1.00	1.00	1.00	1.00	1.00
TP2	d= 2	**0.01**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	0.18	**0.00**	**0.00**	1.00	**0.00**	**0.00**
	d= 5	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	1.00	1.00	0.50	1.00	1.00	1.00
	d= 10	0.05	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	1.00	1.00	1.00	1.00	1.00	1.00
TP3	d= 2	0.19	0.31	**0.04**	**0.00**	**0.00**	**0.00**	0.39	1.00	1.00	1.00	**0.01**	0.25
	d= 5	0.15	0.70	0.98	**0.00**	**0.00**	**0.00**	1.00	0.50	1.00	1.00	0.12	**0.02**
	d= 10	**0.04**	**0.04**	0.10	**0.00**	**0.00**	**0.00**	0.22	**0.03**	1.00	1.00	1.00	1.00
TP4	d= 2	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.01**	**0.00**	**0.00**	1.00	**0.00**	**0.00**
	d= 5	0.69	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.02**	**0.03**	0.37	1.00	1.00	**0.00**
	d= 10	**0.03**	0.45	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	0.21	1.00	1.00	1.00
TP5		0.20	**0.04**	0.53	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.02**	**0.00**	**0.00**	**0.00**
TP6		**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
TP7		**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	0.22	**0.00**	**0.00**
TP8		**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	1.00	1.00	0.06	1.00	1.00	1.00
TP9		**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	1.00	1.00	1.00	1.00	1.00	1.00
TP10		**0.03**	**0.04**	**0.00**	**0.00**	**0.00**	**0.00**	1.00	1.00	1.00	1.00	1.00	1.00
TP11		**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
TP12		**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	0.12	**0.00**	**0.00**
TP13		**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**

Table 12: The p-value of the two-sided Wilcoxon signed rank test comparing each snapshot results of RS-CMSA and RS-CMSA+ASA. P-values ≤ 0.05 are marked in bold.

		best f_{eff}			mean AE			PR $\epsilon = 10^{-1}$			PR $\epsilon = 10^{-2}$		
		(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
TP1	d= 2	1.00	**0.03**	0.15	1.00	**0.00**	**0.00**	1.00	0.06	0.12	1.00	1.00	1.00
	d= 5	0.58	0.59	**0.04**	0.50	**0.00**	**0.00**	1.00	1.00	1.00	1.00	1.00	1.00
	d= 10	1.00	0.08	**0.02**	1.00	**0.00**	**0.00**	1.00	1.00	1.00	1.00	1.00	1.00
TP2	d= 2	1.00	**0.00**	**0.00**	1.00	**0.00**	**0.00**	1.00	**0.02**	**0.00**	1.00	1.00	0.06
	d= 5	0.86	**0.00**	**0.00**	0.60	**0.00**	**0.00**	1.00	1.00	1.00	1.00	1.00	1.00
	d= 10	1.00	**0.00**	**0.00**	1.00	**0.00**	**0.00**	1.00	1.00	1.00	1.00	1.00	1.00
TP3	d= 2	1.00	0.42	0.35	1.00	**0.00**	**0.00**	1.00	0.06	0.12	1.00	0.50	0.12
	d= 5	0.38	0.06	0.63	0.38	**0.00**	**0.00**	1.00	0.09	1.00	1.00	1.00	1.00
	d= 10	1.00	**0.00**	0.10	1.00	**0.00**	**0.00**	1.00	1.00	1.00	1.00	1.00	1.00
TP4	d= 2	1.00	**0.00**	**0.00**	1.00	**0.00**	**0.00**	1.00	0.25	**0.05**	1.00	0.25	**0.00**
	d= 5	0.30	0.89	**0.00**	**0.04**	**0.00**	**0.00**	1.00	1.00	**0.00**	1.00	1.00	1.00
	d= 10	1.00	0.36	**0.00**	1.00	**0.00**	**0.00**	1.00	0.11	**0.00**	1.00	1.00	1.00
TP5		1.00	0.05	0.16	1.00	**0.00**	**0.00**	1.00	1.00	1.00	1.00	1.00	1.00
TP6		1.00	**0.00**	**0.00**	1.00	**0.00**	**0.00**	1.00	**0.00**	**0.00**	1.00	1.00	**0.01**
TP7		1.00	**0.00**	**0.00**	1.00	**0.00**	**0.00**	1.00	**0.00**	**0.00**	1.00	0.12	0.12
TP8		1.00	**0.00**	**0.00**	1.00	**0.00**	**0.00**	1.00	1.00	0.25	1.00	1.00	1.00
TP9		1.00	**0.00**	**0.00**	1.00	**0.00**	**0.00**	1.00	1.00	1.00	1.00	1.00	1.00
TP10		**0.01**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	1.00	1.00	1.00	1.00	1.00	1.00
TP11		**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
TP12		**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	0.12	0.06	**0.02**	1.00	1.00	1.00
TP13		**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**