Envy based fairness in hedonic games

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ABSTRACT

Hedonic games are coalition formation games where agents have hedonic preferences for coalition structures. The main focus of hedonic games has been on notion of stability. In this paper, however, we consider envy based fairness in hedonic games. We investigate emptiness of envy-free coalition structures and summarize the relationship with core stability.

CCS CONCEPTS

• **Computing methodologies → Cooperation and coordination**; Multi-agent systems; • **Applied computing** → *Economics*;

KEYWORDS

Hedonic games, fairness

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1 INTRODUCTION

Forming an effective coalition is a key capability of self-intersted agents and therefore coalition formation problems have become an important research topic in multi-agent systems. Hedonic games are coalition formation games where agents have hedonic preferences for coalition structures. In existing work, notions of stability for coalition structure have been studeid and several solution concepts such as Nash stability, individual stability, and core stability have been introduced [2–4]. More recently, computational issues, e.g., developing algorithms for computing stable coalition structures have been conducted [1].

Instead of stability, we can consider fairness as a desirable property for coalition structures. Envy-freeness is one of fairness criteria used in fair allocation problems. In hedonic games, an agent have envy toward another if the former prefer the latter's coalition to the former's coalition and a coalition structure said to be envy-free if no agents have envy in the coalition structure.

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In this paper, we focus on envy-free coalition structures. As a first step, we investigate the relationship between envy-freeness and core stability. There can be no envy-free coalition structures in general. Thus, we introduce a new notion of fairness called justified envy freeness to have a non-empty set of fair coalition structures. We also investigate the relationship between justified envy freeness and core stability.

2 MODEL

Let $N = \{1, 2, \ldots, n\}$ be a set of agents and $S \subseteq N$ be a coalition. A coalition structure π is a partition of agents N. More specifically, coalition structure $\pi = \{S_1, S_2, \ldots\}$ satisfies the following condition:

$$\forall S_i, S_j \ (i \neq j),\ S_i \cap S_j = \emptyset,\ \bigcup_{S_i \in \pi} S_i = N.$$

Each agent i belongs to its coalition denoted as $\pi(i)$ and let $\mathcal{N}_i$ be a set of coalitions that include agent i. $\succ = (\succ_1, \succ_2, \ldots, \succ_n)$ is a preference profile of agents. $\succ_i$ spesifis the preference relation of agent i over $\mathcal{N}_i$. A hedonic game is given as a tuple $(N, \succeq)$.

Example 1. *Let there be three agents $N = \{a, b, c\}$ and the preference of each agent is given as follows:*

- $\{a, b\} \succ_a \{a, c\} \succ_a \{a\} \succ_a \{a, b, c\}$,
- $\{a, b\} \succ_b \{b, c\} \succ_b \{b\} \succ_b \{a, b, c\}$,
- $\{a, c\} \succ_c \{b, c\} \succ_c \{c\} \succ_c \{a, b, c\}$.

In this example, both agent a and b prefer their coalition $\{a, b\}$ to other coalitions and agent c prefers coalition $\{a, c\}$ to other coalitoins. The main question in hedonic games is what coalition strucutre should be formed under the preference profile. For example, the grand coalition $\{\{a, b, b\}\}$ might not be formed because no agent prefers that coalition. Also, if the coalition sturucutre $\{\{a\}, \{b\}, \{c\}\}$ were formed, each agent would have an incentive to deviate from the coalition structure and form better coalitions. In general, a desirable coalition strucuter for all agents is not obvious and we have to utilize some criteria to define such coalition structures.

Solution concepts capture notons of desireble coalition structures. In this paper, we consider core stability, envy-freeness, and justified envy freeness. Let us introduce the definitions of these solution concepts.

Definition 1 (core stability). *We say that a coalition $S \subseteq N$ blocks a coalition structure π, if each agent prefers S to her current coalition $\pi(i)$ in the coalition structure π. A coalition structure that includes no blocking coalition is said to be in the core.*

Definition 2 (envy-freeness). *We say that agent i has envy toward agent j ($\pi(i) \neq \pi(j)$) if the following condition holds:*

$$(\pi(j) \setminus \{j\}) \cup \{i\} \succ_i \pi(i)$$

A coalition structure where no agents have envy is said to be envy-free.

Definition 3 (Justified envy freeness). *Assume agent i has envy toward agent j in coalition structure π. We say that the envy is justified if the following condition holds for any agent $k \in \pi(j) \setminus \{j\}$:*

$$(\pi(j) \setminus \{j\}) \cup \{i\} \succ_k \pi(j)$$

A coalition structure where no agents have justified envy is said to be justified envy free.

3 EMPTINESS OF ENVY-FREE AND JUSTIFIED ENVY FREE COALITION STRUCUTRES

In this section, we investigate emptiness issues of fair coalition structures. By definition, coalition structures that only contain the grand coalition (coalition of all agents) or single-agent coalitions, more formally, $\{\{N\}\}$ or $\{\{1\},\{2\},\ldots,\{n\}\}$, are envy-free because no agent have envy. Thus, in this paper, we assume that these coalition structures are prohibited from formaing to avoid senseless cases.

For emptiness of envy-free coalition structures, we have an example where a set of envy-free coalition structure is empty.

Example 2. *Let us cosider again the game given in Example 1. Since coalition strictures $\{\{N\}\}$ and $\{\{a\},\{b\},\{c\}\}$ are prohibited, coalition $\{a,b,c\}$ is removed from agents' preferences and the following theree coalition structures can be formed:*

$$\{\{a,b\},\{c\}\},\{\{a,c\},\{b\}\},\{\{b,c\},\{a\}\}.$$

In each of the above three coalition structures, there exist an agent who has envy toward another agent. For example, in coalition structure $\{\{a,b\},\{c\}\}$, agent c has envy toward a and b. Smilary, agent b has envy toward c in $\{\{a,c\},\{b\}\}$ and agent a has envy toward c in $\{\{b,c\},\{a\}\}$. Therefore, there exists no envy-free coalition strucutre in the game.

Justified envy freeness inherits the negative result from envy-freeness. We have an example where a set of justified envy free coalition structure is empty.

Example 3. *Let there be three agents $N = \{a,b,c\}$ and the preference of each agent is given as follows:*

- $\{a,b\} \succ_a \{a,c\} \succ_a \{a\}$,
- $\{b,c\} \succ_b \{a,b\} \succ_b \{b\}$,
- $\{a,c\} \succ_c \{b,c\} \succ_a \{c\}$.

In this example, it can be shown that there exist an agent who has justified envy in all possible coalition structures. Thus, there exists no justified envy free coalition strucutre in the game.

4 RELATIONSHIP WITH CORE STABILITY

In this section, we consider the relationship between envy based fairness and core stability. First, we show that envy-freeness does not imply core stability and vice versa.

Example 4. *Let there be three agents $N = \{a,b,c\}$ and the preference of each agent is given as follows:*

- $\{a\} \succ_a \{a,b\} \succ_a \{a,c\}$,
- $\{a,b\} \succ_b \{b\} \succ_b \{b,c\}$,
- $\{c\} \succ_c \{a,c\} \succ_c \{b,c\}$.

Let us consider coalition strucutre $\{\{a,b\},\{c\}\}$. In this case, agents a and b cannot have envy by the definition and agent c does not have any envy since c prefers her single-agent coalition $\{c\}$ to other coalitions. Thus, the coalition strucutre $\{\{a,b\},\{c\}\}$ is envy-free. However, since coalition $\{a\}$ blocks the coalition strucutre, it is not in the core.

Then, let us cosider again the game given in Example 1. In this game, coalition structure $\{\{a,b\},\{c\}\}$ is in the core. However, agent c has envy toward a and b as shown in Example 2 and thus the coalition structure is not envy-free.

We show that justified envy freeness does not imply core stability but core stability implies justified envy freeness.

Example 5. *Let there be four agents $N = \{a,b,c,d\}$ and the preference of each agent is given as follows:*

- $\{a,b,c\} \succ_a \{a,b\} \succ_a \{a,c\} \succ_a \{a,b,d\} \succ_a \{a,c,d\} \succ_a \{a\} \succ_a \{a,d\}$,
- $\{a,b,c\} \succ_b \{a,b\} \succ_b \{b,c\} \succ_b \{a,b,d\} \succ_b \{b,c,d\} \succ_b \{b,d\} \succ_b \{b\}$,
- $\{a,b,c\} \succ_c \{a,c\} \succ_c \{a,c,d\} \succ_c \{c,d\} \succ_c \{b,c\} \succ_c \{b,c,d\} \succ_c \{c\}$,
- $\{a,d\} \succ_d \{c,d\} \succ_d \{b,d\} \succ_d \{a,c,d\} \succ_d \{a,b,d\} \succ_d \{b,c,d\} \succ_d \{d\}$.

In this example, the following three coalition structures are justified envy free:

$$\{a,b,c\},\{d\}\},\{\{a,b\},\{c,d\}\},\{\{a,b\},\{c\},\{d\}\}$$

Only coalition structure $\{\{a,b,c\},\{d\}\}$ is in the core. Thus, justified envy freeness does not imply core stability

Theorem 4.1. *For any coalition structure π in the core, that coalition structure is always justified envy free.*

Proof. Assume that agent i has justified envy toward another agent j in coalition structure π that is in the core. Since agent i's envy is justified, for any $k \in \pi(j) \setminus \{j\}$, $(\pi(j) \setminus \{j\}) \cup \{i\} \succ_k \pi(j)$ holds. Thus, agent i and all other agents $k \in \pi(j) \setminus \{j\}$ prefer $(\pi(j) \setminus \{j\}) \cup \{i\}$ to their current coalitions, which means $(\pi(j) \setminus \{j\}) \cup \{i\}$ blocks the coalition structure π. This contradicts that the coalition structure π is in the core. Therefore it is shown that, for any coalition structure π in the core, that coalition structure is always justified envy free. ◻

5 CONCLUSION

In this paper, we investigate emptiness of envy based fair coalition strucutres and the relashionship with core stability. Future work include that analysing the computatinal complexity of checking emptiness of such coalition strucutres, developing an algorithm to find a fair coalition strucutre, and so on.

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