Benchmarking the PSA-CMA-ES on the BBOB Noiseless Testbed

Kouhei Nishida
Interdisciplinary Graduate School of Science and Technology, Shinshu Univercity
17st208e@shinshu-u.ac.jp

Youhei Akimoto
Faculty of Engineering, Information and Systems
University of Tsukuba
akimoto@cs.tsukuba.ac.jp

ABSTRACT

We evaluate the CMA-ES with population size adaptation mechanism (PSA-CMA-ES) on the BBOB noiseless testbed. On one hand, the PSA-CMA-ES with a simple restart strategy shows performance competitive with the best 2009 portfolio on most well-structured multimodal functions. On the other hand, it is not effective on weakly-structured multimodal functions. Moreover, on most unimodal functions, the scale-up of performance measure w.r.t. the dimension tends to be worse than the default CMA-ES, implying that the population size is adapted greater than needed on the unimodal functions. To improve performance on unimodal functions and weakly-structured multimodal functions, we additionally propose a restart strategy for the PSA-CMA-ES. The proposed strategy consists of three search regimes. The resulted restart strategy shows improved performance on unimodal functions and weakly-structured multimodal functions with a little compromise in the performance on well-structured multimodal functions. The overall performance is competitive to the BIPOP-CMA-ES.

CCS CONCEPTS

• **Computing methodologies → Continuous space search**;

KEYWORDS

Benchmarking, Black-box optimization, Covariance matrix adaptation, population size adaptation

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1 INTRODUCTION

The covariance matrix adaptation evolution strategy (CMA-ES) [4, 10, 11] is a stochastic and comparison-based search algorithm for continuous optimization. It maintains the multivariate normal distribution to converge into the optimum. Thanks to the covariance matrix adaptation, it can effectively solve difficult problems such as ill-conditioned and non-separable problems.

The CMA-ES is a quasi parameter free algorithm. That is, all the strategy parameters used in the CMA-ES don't have to be tuned for each problem. However it is well-known that a larger population size helps to find a better solution on relatively well-structured multimodal functions [9]. It is also important to set the initial step-size to a relatively small value compared to the search interval when solving weakly-structured multimodal functions [3]. Additionally, it is also empirically known that a large population size sometime leads to a convergence into a dominant but sub-optimal solution on some weakly-structured functions.

The BIPOP restart strategy [3] tackles these difficulties by interlacing two search regimes — one is global search by increasing the population size, the other is local search by using a relatively small population size and a relatively small step-size. This restart strategy works well on both well-structured and weakly-structured multimodal functions. However, when the BIPOP-CMA-ES is applied to highly multimodal functions, where a large population size is needed, some runs have to be wasted for the population size to get large enough since the population size increases by only the factor of 2 at each restart.

The PSA-CMA-ES [13] is a variant of the CMA-ES that incorporates the adaptation mechanism of the population size. It adapts the population size online based on the accuracy of the update of the distribution parameters, i.e., the mean vector and the covariance matrix of the multivariate normal distribution. In [13], the PSA-CMA-ES has been evaluated on unimodal and well-structured multimodal functions in noiseless and noisy scenarios. The results revealed that this algorithm works relatively well on well-structured multimodal functions without tuning the population size in advance. However, on unimodal functions, this algorithm keeps the population size a little larger than the default value, therefore, it wastes more function evaluations. Moreover, we concern that the PSA-CMA-ES may increase the population size inefficiently on specific functions whose global landscape looks flat or random, like some weakly-structured functions, due to the population size adaptation mechanism based on the accuracy of the parameter update.

In this paper, we evaluate the PSA-CMA-ES on the BBOB noiseless testbed. Additionally, to tackle the above-mentioned issues, we propose a novel restart strategy for the PSA-CMA-ES. We compare the PSA-CMA-ES with the proposed restart strategy, the PSA-CMA-ES with a simple restart, and the BIPOP-CMA-ES.

2 PSA-CMA-ES

The CMA-ES maintains the multivariate normal distribution parameterized by the mean vector $\boldsymbol{m}$, the step-size σ, and the covariance matrix $\boldsymbol{C}$. The distribution is adapted iteratively by sampling from the distribution and updating it based on the samples $\boldsymbol{x}_i$ (for $i = 1, 2, \ldots, \lambda$) and their ranking information. It also utilizes the

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movement information of the mean vector that is quantified by the evolution paths, $\boldsymbol{p}_\sigma, \boldsymbol{p}_c$, to accelerate the adaptation.

The PSA-CMA-ES [13] is a variant of the CMA-ES that adapts the population size during the optimization. The population size λ is adapted based on the estimated accuracy of the update of the normal distribution parameters. The accuracy is quantified on the basis of the length of the evolution path $\boldsymbol{p}_\theta$ on the parameter space of the normal distribution that is normalized w.r.t. the Fisher metric. If the parameter update is regarded as insufficiently accurate, the population size is increased, and vice versa. As a result, the population size is adapted so that the estimated accuracy of the parameter update is kept to have enough level. See [13] for more detail. A pseudo-code is provided in Algorithm 1.

The symbols that appear and are not explained explicitly in Algorithm 1 are listed as follows:

- **n:** the dimension of the objective function;
- **$\boldsymbol{x}_{i:\lambda_\mathrm{r}}$:** the i-th best solution of λ_r solutions;
- **h_σ:** the Heaviside function that equals to 1 if $\|\boldsymbol{p}_\sigma\| < \left(1.4 + \frac{2}{n+1}\right) \chi_n \sqrt{\gamma_\sigma}$, otherwise $h_\sigma = 0$;
- **χ_n:** the expected norm of the n-variate standard normal distribution. We use the approximated value $\chi_n \approx \sqrt{n}(1 - 1/(4n) + 1/(21n^2))$;
- **$\mathrm{vech}(\boldsymbol{A})$:** the vector consisting of upper triangle elements of the symmetric matrix $\boldsymbol{A}$;
- **$\sigma^*(\lambda_\mathrm{r})$:** the scaling factor of the optimal standard deviation derived in [1], whose approximated value below is used in our implementation,

$$\sigma^*(\lambda) = \frac{c \cdot n \cdot \mu_\mathrm{w}}{n - 1 + c^2 \cdot \mu_\mathrm{w}} \quad , \tag{1}$$

where $c = -\sum_{i=1}^{\lambda} \mathrm{w}_i \mathbb{E}[\mathcal{N}_{i:\lambda}]$ is the weighted average of the expected value of the normal order statistics from λ population.

We call the algorithm ignoring Line 22-32 in Algorithm 1 the default CMA-ES. This algorithm is almost the same as the original CMA-ES [4].

Algorithm 1: PSA-CMA-ES

input : $\boldsymbol{m} \in \mathbb{R}^n, \sigma \in \mathbb{R}_+$

set : $c_m = 1$, $\alpha = 1.4$, $\beta = 0.4$, $\lambda_\mathrm{min} = \lambda_\mathrm{def}$, $\lambda_\mathrm{max} = \infty$, $\boldsymbol{C} = \boldsymbol{I}, \boldsymbol{p}_c = \boldsymbol{0}, \boldsymbol{p}_\sigma = \boldsymbol{0}, \boldsymbol{p}_\theta = \boldsymbol{0}, \gamma_c = 0, \gamma_\sigma = 0, \gamma_\theta = 0, \lambda = \lambda_r = \lambda_\mathrm{def}$

1 **while** *not terminate* **do**

2 `// (re-)compute parameters depending on` λ

3 $\mu \leftarrow \lfloor \lambda_r / 2 \rfloor$

4 $w_i \leftarrow \frac{\log(\mu+0.5) - \log i}{\sum_{i=1}^{\mu}(\log(\mu+0.5) - \log i)} \quad (i = 1, \ldots, \mu)$

5 $w_i \leftarrow 0 \quad (i = \mu + 1, \ldots, \lambda_r)$

6 $\mu_\mathrm{eff} \leftarrow 1 / \sum_{i=1}^{\lambda_r} w_i^2$

7 $c_\sigma \leftarrow (\mu_\mathrm{eff} + 2)/(n + \mu_\mathrm{eff} + 5)$

8 $d_\sigma \leftarrow 1 + 2\max(0, \sqrt{(\mu_\mathrm{eff} - 1)/(n+1)} - 1) + c_\sigma$

9 $c_c \leftarrow (4 + \mu_\mathrm{eff}/n)/(n + 4 + 2\mu_\mathrm{eff}/n)$

10 $c_1 \leftarrow 2/((n + 1.3)^2 + \mu_\mathrm{eff})$

11 `// perform a CMA-ES iteration`

12 $\boldsymbol{x}_i \sim \boldsymbol{m} + \sigma \mathcal{N}(\boldsymbol{0}, \boldsymbol{C}) \quad$ for $i = 1, \ldots, \lambda_r$

13 $\boldsymbol{m}' \leftarrow \boldsymbol{m}$, $\boldsymbol{C}' \leftarrow \boldsymbol{C}$, $\sigma' \leftarrow \sigma$ `// keep old values`

14 $\boldsymbol{dm} \leftarrow c_m \sum_{i=1}^{\lambda_r} w_i (\boldsymbol{x}_{i:\lambda_r} - \boldsymbol{m})$

15 $\boldsymbol{m} \leftarrow \boldsymbol{m} + \boldsymbol{dm}$

16 $\boldsymbol{p}_\sigma \leftarrow (1 - c_\sigma)\boldsymbol{p}_\sigma + \sqrt{c_\sigma(2 - c_\sigma)\mu_\mathrm{eff}}\, (\boldsymbol{C})^{-\frac{1}{2}} \frac{\boldsymbol{dm}}{\sigma}$

17 $\boldsymbol{p}_c \leftarrow (1 - c_c)\boldsymbol{p}_c + h_\sigma \sqrt{c_c(2 - c_c)\mu_\mathrm{eff}} \frac{\boldsymbol{dm}}{\sigma}$

18 $\gamma_\sigma \leftarrow (1 - c_\sigma)^2 \gamma_\sigma + c_\sigma(2 - c_\sigma)$

19 $\gamma_c \leftarrow (1 - c_c)^2 \gamma_c + h_\sigma c_c (2 - c_c)$

20 $\sigma \leftarrow \sigma \exp\left(\frac{c_\sigma}{d_\sigma}\left(\frac{\|\boldsymbol{p}_\sigma\|}{\chi_n} - \sqrt{\gamma_\sigma}\right)\right)$

21 $\boldsymbol{C} \leftarrow \boldsymbol{C} + c_1 \left(\boldsymbol{p}_c (\boldsymbol{p}_c)^T - \gamma_c \boldsymbol{C}\right) + c_\mu \sum_{i=1}^{\lambda} w_i \left(\left(\boldsymbol{x}_{i:\lambda} - \boldsymbol{m}'\right)\left(\boldsymbol{x}_{i:\lambda} - \boldsymbol{m}'\right)^T - \boldsymbol{C}\right)$

22 `// update evolution path and its factor`

23 $\boldsymbol{d\theta} \leftarrow \left(\boldsymbol{dm}, \mathrm{vech}\left((\sigma)^2 \boldsymbol{C} - (\sigma')^2 \boldsymbol{C}'\right)\right)$

24 $\boldsymbol{p}_\theta \leftarrow (1 - \beta)\boldsymbol{p}_\theta + \sqrt{\beta(2 - \beta)} \frac{\boldsymbol{\mathcal{I}}_\theta^{\frac{1}{2}} \boldsymbol{d\theta}}{\mathbb{E}[\|\boldsymbol{\mathcal{I}}_\theta^{\frac{1}{2}} \boldsymbol{d\theta}\|^2]^{\frac{1}{2}}}$

25 $\gamma_\theta \leftarrow (1 - \beta)^2 \gamma_\theta + \beta(2 - \beta)$

26 `// update population size`

27 $\lambda \leftarrow \lambda \exp\left(\beta\left(\gamma_\theta - \frac{\|\boldsymbol{p}_\theta\|^2}{\alpha}\right)\right)$

28 $\lambda \leftarrow \min(\max(\lambda, \lambda_\mathrm{min}), \lambda_\mathrm{max})$

29 $\lambda_r' \leftarrow \lambda_\mathrm{r}$ `// keep old population size`

30 $\lambda_r \leftarrow \mathrm{round}(\lambda)$

31 `// step-size correction`

32 $\sigma \leftarrow \sigma \frac{\sigma^*(\lambda_\mathrm{r})}{\sigma^*(\lambda_\mathrm{r}')}$

33 **end**

3 RESTART STRATEGY

In BIPOP-CMA-ES [3], the CMA-ES with the default population size runs at first. After that, the BIPOP scheme is considered and the global search with incrementation of the population size or the local search by using a relatively small population size and a relatively small step-size is selected based on budgets for them and executed.

In our restart strategy, thanks to the population size adaptation mechanism, tuning of the population size is not needed. However, when the PSA-CMA-ES is applied to the weakly-structured multimodal functions, whose global structure looks nearly random or flat, it sometime keeps increasing the population size. In such a situation, increasing population size is less effective, therefore, we set the maximum population size $\lambda_\mathrm{max} = 512 \cdot \lambda_\mathrm{def}$. This is the value that the BIPOP-CMA-ES sets as the largest population size. Intuitively, this value may be excessively large, and it will be meaningful to investigate the effect of the maximum population size. We leave this part as a future work.

The restart strategy for the PSA-CMA-ES processes three search regimes as follows:

1st run: default CMA-ES. In the first run, we execute the CMA-ES with the default population size without the population size adaptation. In this run, we set the initial step-size to a sufficiently large value, i.e., $\sigma^0 = 2$. If the objective function is unimodal, we expect it solve the problem with this run.

2nd run: PSA-CMA-ES. If the first run is terminated, we apply the PSA-CMA-ES in the next run since the objective function is considered to be a multimodal function. The initial step-size is set to the same value as the first run. If the objective function is a well-structured multimodal function, we hope that it can solve the problem with a reasonably high success probability.

Additional runs: PSA-CMA-ES with relatively small step-size. If the second run is also terminated, we consider the objective function is probably a weakly-structured multimodal function. For such a function, the relatively small initial step-size is effective. Therefore, we initialize the step-size as

$$\sigma^0 \sim 2 \times 10^{-2\mathcal{U}[0,1]}. \tag{2}$$

This is the configuration used in the BIPOP strategy [3].

4 ALGORITHM VARIANTS

We evaluate two algorithms as follows:

PSA-CMA-ES: The PSA-CMA-ES without the restart strategy described in Sec. 3. The PSA-CMA-ES is applied to all runs and the initial step-size is $\sigma^0 = 2$ for all runs. There is no upper bound for the population size.

PSA-CMA-ESwRS: The PSA-CMA-ES with the restart strategy described in Sec. 3. At the first run, the default CMA-ES with default population size is applied. The initial step-size for the first run is $\sigma^0 = 2$. At the second run, the PSA-CMA-ES with the initial step-size $\sigma^0 = 2$ is applied. After that, the initial step-size is sampled by (2) and the PSA-CMA-ES runs. The population size is upper bounded by $\lambda_{\max} = 512 \cdot \lambda_{\text{def}}$.

5 EXPERIMENTAL PROCEDURE

For each (re-)start, we initialize the mean vector $\boldsymbol{m} \sim \mathcal{U}[-4, 4)^D$. A single run is terminated when the algorithm reaches the target function value or one of the termination conditions is satisfied. We employ the termination conditions of the BIPOP-CMA-ES [3], replacing λ by $\lambda^0 = \lambda_{\text{def}}$ or λ_{r}^t at a t-th iteration as follows:

`MaxIter` $= 100 + 50(D+3)^2/\sqrt{\lambda^0}$ is the maximum number of iterations in each run of CMA-ES.

`TolHistFun` $= 10^{-12}$**:** the range of the best function values during the last $10 + \lceil 30D/\lambda^0 \rceil$ iterations is smaller than `TolHistFun`.

`EqualFunVals`**:** in more than $1/3^{\text{rd}}$ of the last D iterations the objective function value of the best and the k-th best solution are identical, that is $f(\boldsymbol{x}_{1:\lambda_{\text{r}}^t}) = f(\boldsymbol{x}_{k:\lambda_{\text{r}}^t})$, where $k = 1 + \lceil 0.1 + \lambda^0/4 \rceil$.

`TolX`$= 10^{-12}$**:** all components of $\boldsymbol{p}_c^t$ and all square roots of diagonal components of $\boldsymbol{C}^t$, multiplied by σ^t/σ^0, are smaller than `TolX`.

`TolUpSigma`$= 10^{20}$**:** $\sigma^t/\sigma^0 >$ `TolUpSigma` $\sqrt{l^t}$, where l^t is the largest eigenvalue of $\boldsymbol{C}^t$, indicates a mismatch between σ increase and decrease of all eigenvalues in $\boldsymbol{C}$. In this, rather untypical, case the progression of the strategy is usually very low and a restart is indicated.

`Stagnation`**:** the median of the 20 newest values is not smaller than the median of the 20 oldest values, respectively, in the two arrays containing the best function values and the median function values of the last $\lceil 0.2t + 120 + 30D/\lambda^0 \rceil$ iterations.

`ConditionCov`**:** the condition number of $\boldsymbol{C}^t$ exceeds 10^{14}.

`NoEffectAxis`**:** $\boldsymbol{m}^t$ remains numerically constant when adding $0.1\sigma^t\sqrt{l^t}\boldsymbol{v}^t$, where l^t is the $1 + (t \mod D)$-largest eigenvalue of $\boldsymbol{C}^t$ and $\boldsymbol{v}^t$ is the corresponding normalized eigenvector.

`NoEffectCoor`**:** any element of $\boldsymbol{m}^t$ remains numerically constant when adding $0.2\sigma^t\boldsymbol{l}^t$, where elements of $\boldsymbol{l}^t$ are the square root of the diagonal elements of $\boldsymbol{C}^t$.

Restarts are launched until the algorithm reaches the target function value or the number of function call is over $10^6 D$.

6 CPU TIMING

In order to evaluate the CPU timing of the algorithm, we have run the PSA-CMA-ES and the PSA-CMA-ESwRS on the function f_8 with restarts for a maximum budget equal to $400(D+2)$ function evaluations according to [12]. The Python code was run on a Mac Intel(R) Core(TM) i5-7267U CPU @ 3.1GHz with 1 processor and 2 cores. The time per function evaluation for dimensions 2, 3, 5, 10, 20, 40 equals 1.8×10^{-4}, 2.1×10^{-4}, 1.8×10^{-4}, 1.4×10^{-4}, 9.6×10^{-5}, and 6.8×10^{-5} seconds respectively for the PSA-CMA-ES, 1.8×10^{-4}, 3.4×10^{-4}, 3.5×10^{-4}, 3.3×10^{-4}, 3.1×10^{-4}, and 3.0×10^{-4} seconds respectively for the PSA-CMA-ESwRS.

7 RESULTS

Results from experiments according to [12] and [5] on the benchmark functions given in [2, 8] are presented in Figures 1, 2 and 3 and in Tables 1 and 2. The experiments were performed with the old BBOB code version 15.03 to compare BIPOP-CMA-ES, the plots were produced with version 2.2 of COCO [7].

The **average runtime (aRT)**, used in the figures and tables, depends on a given target function value, $f_{\text{t}} = f_{\text{opt}} + \Delta f$, and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach f_{t}, summed over all trials and divided by the number of trials that actually reached f_{t} [6, 14]. **Statistical significance** is tested with the rank-sum test for a given target Δf_{t} using, for each trial, either the number of needed function evaluations to reach Δf_{t} (inverted and multiplied by -1), or, if the target was not reached, the best Δf-value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

8 DISCUSSION

8.1 PSA-CMA-ES

Unimodal functions. From Figure 1, on some unimodal functions (f_1, f_2, f_6, f_8-f_{12} and f_{14}), we observe that the higher the dimension, the relatively worse the aRT compared to the BIPOP-CMA-ES. Given that the unimodal functions are solved in the first run with

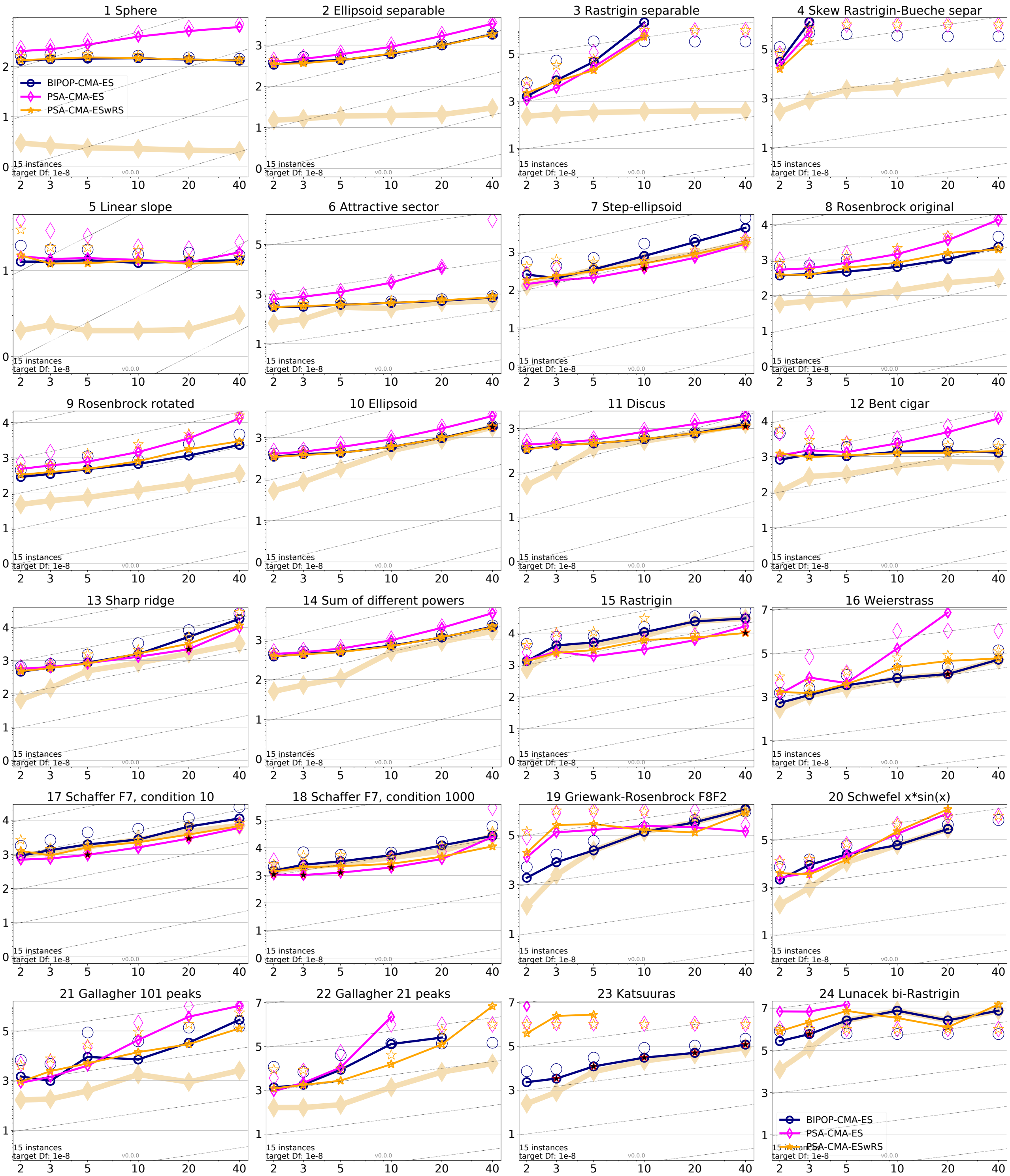

Figure 1: Average running time (aRT in number of f-evaluations as $\log_{10}$ value), divided by dimension for target function value 10^{-8} versus dimension. Slanted grid lines indicate quadratic scaling with the dimension. Different symbols correspond to different algorithms given in the legend of f_1 and f_{24}. Light symbols give the maximum number of function evaluations from the longest trial divided by dimension. Black stars indicate a statistically better result compared to all other algorithms with $p < 0.01$ and Bonferroni correction number of dimensions (six). Legend: ○: BIPOP-CMA-ES, ◇: PSA-CMA-ES, ⋆: PSA-CMA-ESwRS

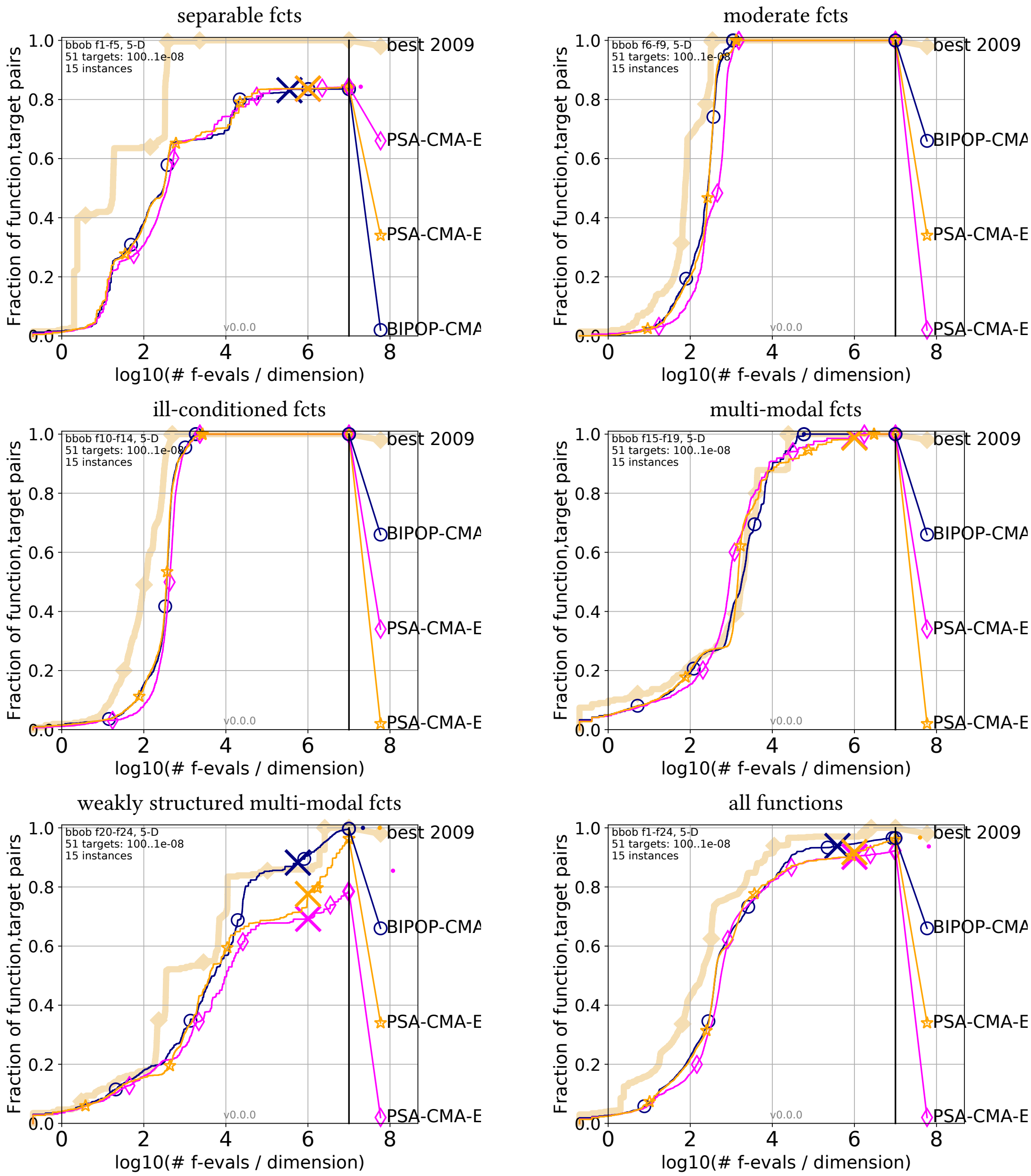

Figure 2: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 51 targets with target precision in $10^{[-8..2]}$ for all functions and subgroups in 5-D. As reference algorithm, the best algorithm from BBOB 2009 is shown as light thick line with diamond markers.

the default population size in the BIPOP-CMA-ES, this implies that the population size in the PSA-CMA-ES is kept at a greater value than the default value and its saturated value increases as the dimension increases. This may be because that the hyper-parameter of the PSA mechanism is set to a constant independent of the dimension. It is necessary to investigate the hyper-parameter setting of the PSA mechanism depending on the dimension in the future work.

Well-structured multimodal functions. We observe that the PSA-CMA-ES outperforms the best 2009 portfolio on the well-structured multimodal functions excluding f_{16} and f_{19} from Figure 1. On f_{16}, if the initial step-size is not small compared to the search interval, the accuracy of the update is likely to be regarded as insufficient due to the repetitive landscape with non-unique global optima. As a result, the population size is increased and the step-size adaptation becomes slow. Then the population size is adapted to be excessively

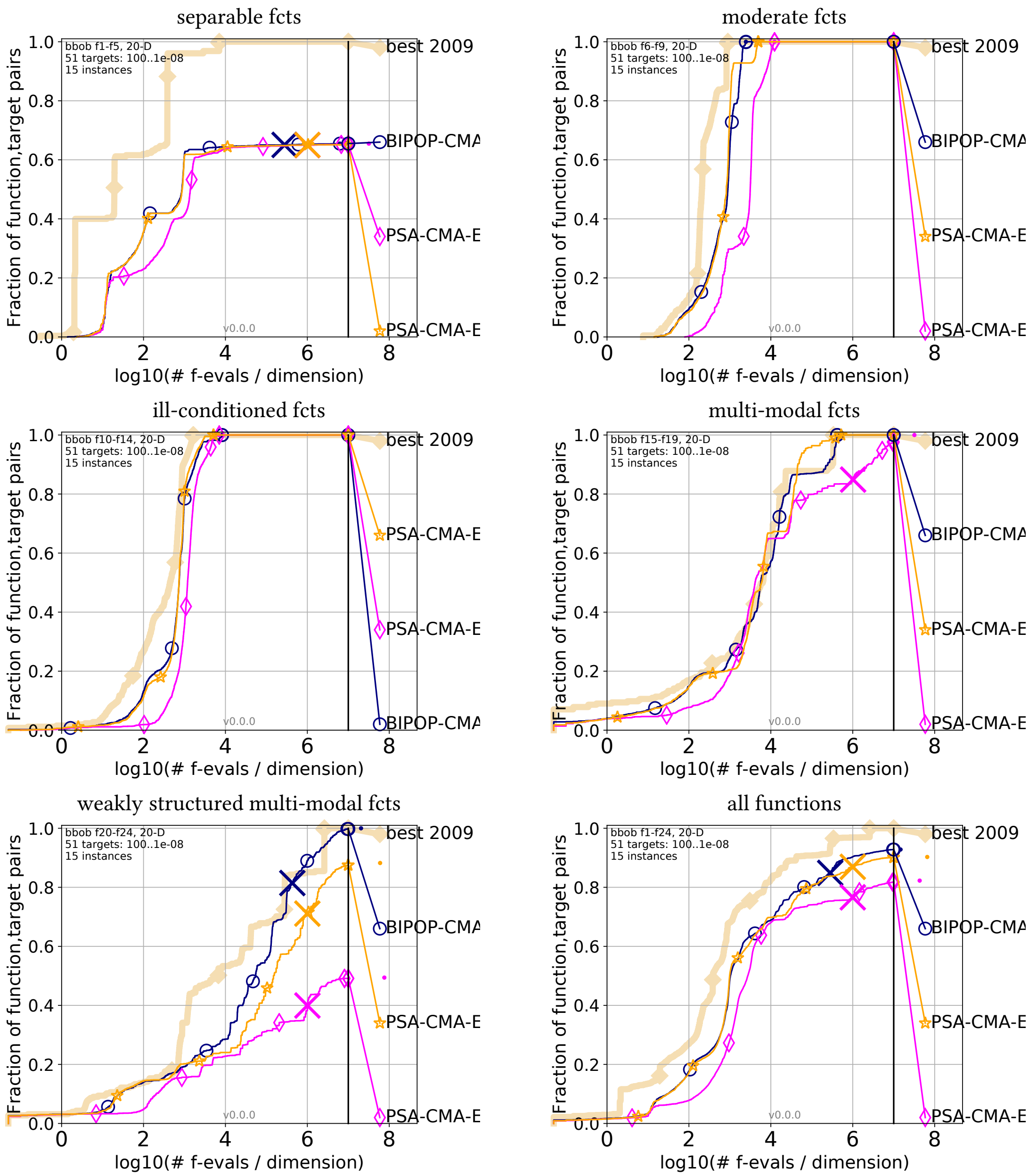

Figure 3: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 51 targets with target precision in $10^{[-8..2]}$ for all functions and subgroups in 20-D. As reference algorithm, the best algorithm from BBOB 2009 is shown as light thick line with diamond markers.

large and it gets harder for the covariance matrix to converge. A simple idea to avoid this phenomenon is to provide the upper bound for the population size.

Weakly-structured multimodal functions. With the simple restart strategy, the PSA mechanism but is not effective for weakly-structured multimodal functions. Once the step-size is large, the global landscape of a function looks random, resulting in increasing the population size. The PSA-CMA-ES then tends to increase the step-size as well. However, to solve weakly structured multimodal functions, the step-size needs to be sufficiently small. Therefore, the PSA mechanism is not helpful for weakly-structured multimodal functions as long as it is used with a simple restart strategy.

8.2 PSA-CMA-ESwRS

All of the unimodal functions could be solved at the first run with the default population size. Therefore, we observed almost identical aRT

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f1	11	12	12	12	12	12	12	15/15
BIPOP-C	**3.2**(3)	**9.1**(5)	**15**(4)	**21**(4)	**28**(5)	**41**(5)	**54**(3)	15/15
PSA-CMA	3.6(1)	16(7)	31(6)	44(8)	54(12)	80(13)	103(9)	15/15
PSA-CMA	3.4(4)	11(4)	18(5)	24(4)	31(4)	44(4)	56(5)	15/15
f2	83	87	88	89	90	92	94	15/15
BIPOP-C	**13**(3)	**16**(3)	**18**(2)	**19**(2)	20(2)	21(3)	**22**(2)	15/15
PSA-CMA	18(2)	20(2)	22(2)	23(3)	24(2)	27(2)	30(3)	15/15
PSA-CMA	16(3)	17(1)	18(1)	19(1)	**19**(1)	**21**(0.5)	22(1)	15/15
f3	716	1622	1637	1642	1646	1650	1654	15/15
BIPOP-C	1.4(0.9)	16(18)	139(312)	139(299)	139(91)	139(546)	140(110)	14/15
PSA-CMA	1.4(0.5)	**13**(8)	84(139)	84(78)	84(137)	84(91)	84(124)	15/15
PSA-CMA	**1.3**(1.0)	17(18)	**61**(44)	**61**(32)	**61**(40)	**61**(27)	**61**(84)	15/15
f4	809	1633	1688	1758	1817	1886	1903	15/15
BIPOP-C	2.7(2)	∞	∞	∞	∞	∞	∞ *2e6*	0/15
PSA-CMA	**2.1**(1)	**1.3e4**(2e4)	∞	∞	∞	∞	∞ *5e6*	0/15
PSA-CMA	2.2(2)	1.3e4(2e4)	∞	∞	∞	∞	∞ *5e6*	0/15
f5	10	10	10	10	10	10	10	15/15
BIPOP-C	4.5(2)	6.5(2)	6.6(3)	6.6(2)	6.6(2)	6.6(2)	6.6(2)	15/15
PSA-CMA	4.6(2)	6.7(5)	7.0(3)	7.0(4)	7.0(3)	7.0(4)	7.0(3)	15/15
PSA-CMA	**4.2**(1)	**5.7**(2)	**6.0**(1)	**6.1**(2)	**6.1**(2)	**6.1**(2)	**6.1**(2)	15/15
f6	114	214	281	404	580	1038	1332	15/15
BIPOP-C	**2.3**(1)	**2.1**(0.4)	**2.2**(0.8)	**1.9**(0.4)	**1.7**(0.2)	1.3(0.2)	**1.3**(0.2)	15/15
PSA-CMA	5.2(2)	5.5(1)	6.4(2)	6.0(1)	5.2(0.7)	4.1(0.5)	4.1(0.4)	15/15
PSA-CMA	2.6(2)	2.2(0.3)	2.2(0.3)	2.0(0.3)	1.7(0.6)	**1.3**(0.4)	1.3(0.3)	15/15
f7	24	324	1171	1451	1572	1572	1597	15/15
BIPOP-C	4.9(2)	**1.5**(2)	1.0(0.8)	1.00(0.5)	1.0(0.6)	1.0(0.6)	1.00(0.7)	15/15
PSA-CMA	6.7(5)	1.5(0.5)	**0.63**(0.2)	**0.69**(0.2)	**0.67**(0.1)	**0.67**(0.1)	**0.66**(0.1)	15/15
PSA-CMA	**4.1**(3)	1.9(1)	1.1(0.3)	1.0(0.2)	1.0(0.4)	1.0(0.3)	0.99(0.3)	15/15
f8	73	273	336	372	391	410	422	15/15
BIPOP-C	**3.2**(3)	**3.7**(5)	**4.5**(1)	**4.7**(0.6)	**4.8**(2)	**5.1**(1)	**5.4**(0.8)	15/15
PSA-CMA	7.6(3)	7.4(4)	8.2(1)	8.4(1.0)	8.6(5)	9.2(0.7)	10(3)	15/15
PSA-CMA	3.9(2)	5.7(6)	6.2(6)	6.3(5)	6.4(3)	6.6(6)	6.9(5)	15/15
f9	35	127	214	263	300	335	369	15/15
BIPOP-C	**5.8**(2)	8.7(9)	**7.2**(3)	**6.7**(3)	**6.4**(1)	**6.3**(2)	**6.2**(3)	15/15
PSA-CMA	14(6)	13(2)	11(3)	10(1)	10(2)	10(0.7)	10(1)	15/15
PSA-CMA	5.9(2)	**8.4**(4)	7.3(9)	6.8(4)	6.4(0.7)	6.4(0.6)	6.3(1)	15/15
f10	349	500	574	607	626	829	880	15/15
BIPOP-C	3.5(0.7)	2.9(0.5)	2.7(0.3)	2.7(0.3)	2.8(0.4)	2.3(0.1)	2.4(0.1)	15/15
PSA-CMA	3.7(0.7)	3.3(0.2)	3.2(0.3)	3.3(0.3)	3.5(0.1)	3.0(0.3)	3.2(0.3)	15/15
PSA-CMA	**3.1**(1)	**2.7**(1.0)	**2.7**(0.3)	**2.7**(0.2)	**2.8**(0.3)	**2.3**(0.1)	**2.3**(0.1)	15/15
f11	143	202	763	977	1177	1467	1673	15/15
BIPOP-C	8.3(2)	**7.1**(2)	**2.2**(0.3)	1.8(0.1)	**1.6**(0.2)	1.4(0.1)	**1.3**(0.1)	15/15
PSA-CMA	**8.2**(2)	7.3(1)	2.3(0.3)	1.9(0.2)	1.7(0.1)	1.6(0.1)	1.6(0.1)	15/15
PSA-CMA	9.4(2)	7.8(0.6)	2.2(0.2)	**1.8**(0.1)	1.6(0.1)	**1.4**(0.1)	1.3(0.1)	15/15
f12	108	268	371	413	461	1303	1494	15/15
BIPOP-C	11(9)	**7.4**(0.6)	**7.4**(6)	**7.5**(4)	**7.7**(5)	**3.3**(2)	**3.3**(2)	15/15
PSA-CMA	16(9)	9.0(6)	8.1(5)	8.5(5)	9.0(3)	4.1(1)	4.2(2)	15/15
PSA-CMA	**10**(17)	7.6(8)	7.5(3)	7.8(4)	7.8(6)	3.5(2)	3.5(1)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f13	132	195	250	319	1310	1752	2255	15/15
BIPOP-C	3.9(3)	5.4(2)	5.9(2)	5.4(1)	**1.6**(0.4)	**1.5**(0.3)	1.7(0.8)	15/15
PSA-CMA	5.0(0.8)	5.6(1)	6.0(0.6)	5.9(0.4)	1.7(0.1)	1.7(0.1)	1.7(0.1)	15/15
PSA-CMA	**3.4**(1)	**4.3**(2)	**5.0**(2)	**5.2**(2)	1.7(0.3)	1.6(0.6)	**1.5**(0.3)	15/15
f14	10	41	58	90	139	251	476	15/15
BIPOP-C	**1.1**(1)	2.8(1)	3.7(1)	4.0(0.9)	4.5(0.5)	5.4(0.6)	4.5(0.6)	15/15
PSA-CMA	2.4(4)	5.4(3)	8.2(3)	7.9(2)	7.6(1)	7.4(0.8)	5.5(0.5)	15/15
PSA-CMA	1.8(1)	**2.7**(0.8)	**3.5**(0.5)	**3.9**(0.8)	**4.5**(0.7)	**5.1**(0.7)	**4.3**(0.4)	15/15
f15	511	9310	19369	19743	20073	20769	21359	14/15
BIPOP-C	**1.6**(2)	1.5(1)	1.2(0.6)	1.2(0.7)	1.2(0.5)	1.2(0.6)	1.2(0.7)	15/15
PSA-CMA	2.4(1)	**0.62**(0.2)	**0.42**(0.3)	**0.43**(0.3)	**0.43**(0.3)	**0.43**(0.2)	**0.43**(0.2)	15/15
PSA-CMA	1.8(2)	0.86(0.2)	0.70(0.8)	0.69(0.2)	0.69(0.5)	0.68(0.6)	0.68(0.3)	15/15
f16	120	612	2662	10163	10449	11644	12095	15/15
BIPOP-C	3.0(2)	**3.6**(2)	**2.6**(0.9)	**1.1**(1.0)	**1.3**(1)	**1.4**(1)	**1.4**(2)	15/15
PSA-CMA	4.5(5)	25(33)	7.0(7)	2.0(2)	2.0(2)	1.8(2)	1.8(2)	15/15
PSA-CMA	**2.0**(2)	7.9(12)	5.2(5)	1.4(0.8)	1.4(1)	1.6(1)	1.6(2)	15/15
f17	5.0	215	899	2861	3669	6351	7934	15/15
BIPOP-C	**3.5**(3)	1.00(0.5)	**1.0**(1)	1.00(1.0)	1.00(0.8)	1.00(0.5)	1.2(0.7)	15/15
PSA-CMA	3.7(2)	1.8(0.8)	1.0(0.1)	**0.51**(0.1)	**0.56**(0.1)	**0.53**(0.0)$^{\star 2}$	**0.57**(0.0)$^{\star 4}$	15/15
PSA-CMA	4.5(5)	**1.00**(0.3)	1.1(0.2)	0.94(0.9)	0.94(0.7)	1.0(0.1)	0.97(0.1)	15/15
f18	103	378	3968	8451	9280	10905	12469	15/15
BIPOP-C	**1.0**(0.8)	3.4(13)	1.0(1.0)	1.0(0.3)	1.0(0.3)	1.2(0.5)	1.3(0.6)	15/15
PSA-CMA	2.2(1)	2.2(0.4)	**0.35**(0.1)	**0.35**(0.4)	**0.39**(0.0)	**0.45**(0.2)*	**0.48**(0.3)$^{\star 2}$	15/15
PSA-CMA	1.1(0.3)	**0.99**(0.4)	0.87(0.8)	0.64(0.4)	0.73(0.4)	0.76(0.3)	0.84(0.2)	15/15
f19	1	1	242	1.0e5	1.2e5	1.2e5	1.2e5	15/15
BIPOP-C	**20**(12)	2801(5126)	161(92)	**1.00**(0.7)	**1.00**(0.6)	**1.0**(0.5)	**1.00**(0.6)	15/15
PSA-CMA	36(21)	**1481**(1031)	**132**(125)	1.8(2)	6.8(10)	6.8(10)	6.8(18)	14/15
PSA-CMA	23(21)	2094(1372)	233(685)	6.4(14)	12(14)	12(19)	12(14)	13/15
f20	16	851	38111	51362	54470	54861	55313	14/15
BIPOP-C	**3.3**(2)	8.2(9)	2.8(2)	2.2(1)	2.1(2)	2.2(1.0)	2.2(1)	15/15
PSA-CMA	4.5(5)	**4.0**(1)	2.7(3)	2.1(1)	1.9(1)	1.9(2)	1.9(3)	15/15
PSA-CMA	4.3(3)	7.8(7)	**1.8**(3)	**1.4**(1)	**1.3**(0.5)	**1.3**(0.4)	**1.3**(1)	15/15
f21	41	1157	1674	1692	1705	1729	1757	14/15
BIPOP-C	2.3(1)	14(50)	24(3)	25(62)	25(79)	25(4)	25(64)	15/15
PSA-CMA	4.1(4)	10(15)	**12**(22)	**12**(7)	**12**(13)	**12**(3)	**11**(13)	15/15
PSA-CMA	**2.0**(1)	**8.4**(10)	14(33)	14(7)	14(21)	14(21)	14(39)	15/15
f22	71	386	938	980	1008	1040	1068	14/15
BIPOP-C	6.9(10)	20(10)	45(105)	43(69)	42(25)	41(61)	40(92)	15/15
PSA-CMA	**2.7**(2)	30(25)	57(142)	55(23)	54(28)	52(58)	51(93)	15/15
PSA-CMA	12(30)	**14**(14)	**14**(10)	**13**(16)	**13**(16)	**13**(7)	**13**(16)	15/15
f23	3.0	518	14249	27890	31654	33030	34256	15/15
BIPOP-C	**1.7**(2)	**13**(15)	**3.7**(4)$^{\star 2}$	**2.1**(2)$^{\star 2}$	**1.8**(1)$^{\star 2}$	**1.8**(1)$^{\star 2}$	**1.8**(0.7)$^{\star 2}$	15/15
PSA-CMA	2.7(4)	57(52)	∞	∞	∞	∞	∞ *5e6*	0/15
PSA-CMA	2.7(2)	44(69)	968(1226)	496(450)	437(240)	419(491)	404(621)	5/15
f24	1622	2.2e5	6.4e6	9.6e6	9.6e6	1.3e7	1.3e7	3/15
BIPOP-C	**2.1**(1)	**1.6**(0.7)$^{\star 2}$	**1**(1)	**1.0**(0.8)	**1.0**(1)	**1**(1)	**1**(1)	3/15
PSA-CMA	2.1(1)	68(58)	5.5(4)	7.6(7)	7.6(12)	5.7(6)	5.7(6)	1/15
PSA-CMA	3.5(2)	10(8)	1.6(2)	3.8(4)	3.8(5)	2.9(3)	2.9(3)	2/15

Table 1: Average runtime (aRT in number of function evaluations) divided by the respective best aRT measured during BBOB-2009 in dimension 5. The aRT and in braces, as dispersion measure, the half difference between 10 and 90%-tile of bootstrapped run lengths appear for each algorithm and target, the corresponding reference aRT in the first row. The different target Δf-values are shown in the top row. #succ is the number of trials that reached the (final) target $f_{opt} + 10^{-8}$. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries, succeeded by a star, are statistically significantly better (according to the rank-sum test) when compared to all other algorithms of the table, with $p = 0.05$ or $p = 10^{-k}$ when the number k following the star is larger than 1, with Bonferroni correction by the number of functions (24). A ↓ indicates the same tested against the best algorithm from BBOB 2009. Best results are printed in bold.

Data produced with COCO v0.0.0

values for PSA-CMA-ESwRS and BIPOP-CMA-ES. The performance is improved over the PSA-CMA-ES with a simple restart. However, on most well-structured multimodal functions, the first runs failed to locate the global optimum and it wasted the function evaluations. Nevertheless, compared to the best 2009 portfolio, the performance is still competitive and sometimes better. On Weierstrass function (f_{16}), the aRT is improved mainly because of the upper bound for the population size. Otherwise, the population size tends to increase too much on this function. From Table 1 and 2, we observe that the proposed restart strategy improves the number of successful trials on weakly-structured multimodal functions. However, on the Katsuuras function ($n > 10$), it still failed to reach the target by any trial. Since this function has a repetitive landscape with a lot of global optima, the PSA-CMA-ES is likely to increase the population size excessively and cause the same problem as on the Weierstrass function (f_{16}).

9 CONCLUSION

We have evaluated the PSA-CMA-ES with the proposed restart strategy on the BBOB noiseless testbed. It has been revealed that the PSA-CMA-ES works well on well-structured multimodal functions but not very effective on weakly-structured multimodal functions. On most unimodal functions, it has also shown that the higher the dimension is, the relatively worse the PSA-CMA-ES performs than the BIPOP-CMA-ES. However, with the proposed restart strategy, the performance on unimodal functions and weakly-structured functions is improved by interlacing multiple search regimes and introducing an upper bound for the population size. On the other hand, the performance on well-structured multimodal functions have a little worse than the plain PSA-CMA-ES, though it is still competitive with the best 2009 portfolio.

We will investigate the hyper-parameter setting for the PSA mechanism in the future work. The parameter values might need to be set depending on the dimension of the search space. Moreover, as mentioned in this paper, the upper bound for the population

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f1	43	43	43	43	43	43	43	15/15
BIPOP-C	**7.9**(1)	**14**(3)	**20**(1)	**26**(3)	**33**(3)	**45**(3)	**57**(4)	15/15
PSA-CMA	37(13)	58(6)	79(6)	102(23)	124(25)	171(19)	217(22)	15/15
PSA-CMA	8.3(1)	14(1)	21(1)	27(2)	33(2)	46(2)	58(3)	15/15
f2	385	386	387	388	390	391	393	15/15
BIPOP-C	35(4)	40(5)	**44**(3)	**45**(4)	47(2)	48(1)	50(2)	15/15
PSA-CMA	58(7)	64(4)	67(7)	70(6)	73(5)	77(5)	82(5)	15/15
PSA-CMA	**34**(5)	**39**(6)	44(3)	45(4)	**46**(1)	**48**(4)	**49**(3)	15/15
f3	5066	7626	7635	7637	7643	7646	7651	15/15
BIPOP-C	**12**(6)$^{\star 3}$	∞	∞	∞	∞	∞	∞ *6e6*	0/15
PSA-CMA	35(7)	∞	∞	∞	∞	∞	∞ *2e7*	0/15
PSA-CMA	34(7)	∞	∞	∞	∞	∞	∞ *2e7*	0/15
f4	4722	7628	7666	7686	7700	7758	1.4e5	9/15
BIPOP-C	∞	∞	∞	∞	∞	∞	∞ *6e6*	0/15
PSA-CMA	∞	∞	∞	∞	∞	∞	∞ *2e7*	0/15
PSA-CMA	∞	∞	∞	∞	∞	∞	∞ *2e7*	0/15
f5	41	41	41	41	41	41	41	15/15
BIPOP-C	5.0(1.0)	6.1(1)	6.2(1)	6.2(1)	6.3(1)	6.3(1)	6.3(0.9)	15/15
PSA-CMA	4.9(2)	5.9(2)	6.1(2)	6.1(2)	6.1(0.7)	6.1(0.9)	6.1(1)	15/15
PSA-CMA	**4.8**(0.3)	**5.7**(0.9)	**5.8**(0.9)	**5.8**(0.5)	**5.8**(0.5)	**5.8**(1.0)	**5.8**(0.8)	15/15
f6	1296	2343	3413	4255	5220	6728	8409	15/15
BIPOP-C	**1.5**(0.4)	**1.3**(0.1)	**1.2**(0.2)	**1.1**(0.2)	**1.1**(0.2)	**1.2**(0.1)	**1.2**(0.1)	15/15
PSA-CMA	41(9)	31(2)	27(2)	26(2)	25(2)	26(1)	25(2)	15/15
PSA-CMA	1.8(0.4)	1.4(0.2)	1.3(0.2)	1.2(0.1)	1.2(0.1)	1.2(0.1)	1.2(0.1)	15/15
f7	1351	4274	9503	16523	16524	16524	16969	15/15
BIPOP-C	**1.00**(0.3)	4.9(2)	3.5(0.5)	2.2(0.2)	2.2(0.3)	2.2(0.3)	2.1(0.2)	15/15
PSA-CMA	4.4(0.6)	**2.2**(0.6)$^{\star 2}$	**1.2**(0.3)	**0.85**(0.2)	**0.85**(0.2)	**0.85**(0.2)	**0.83**(0.2)	15/15
PSA-CMA	2.6(2)	2.9(0.4)	1.5(0.2)	1.1(0.2)	1.1(0.1)	1.1(0.2)	1.0(0.2)	15/15
f8	2039	3871	4040	4148	4219	4371	4484	15/15
BIPOP-C	**4.0**(0.9)	**4.0**(1)	**4.3**(0.4)	**4.5**(1.0)	**4.5**(0.4)	**4.6**(0.4)	**4.6**(0.4)	15/15
PSA-CMA	14(2)	14(2)	15(2)	16(1)	16(1)	16(0.8)	16(2)	15/15
PSA-CMA	4.3(0.8)	6.3(4)	6.7(0.4)	6.9(4)	6.9(5)	6.9(8)	7.0(4)	15/15
f9	1716	3102	3277	3379	3455	3594	3727	15/15
BIPOP-C	**4.7**(2)	**5.7**(3)	**6.0**(0.9)	**6.1**(4)	**6.1**(0.7)	**6.1**(0.7)	**6.1**(0.7)	15/15
PSA-CMA	16(2)	17(1)	18(1.0)	19(1.0)	19(1)	19(0.9)	19(0.5)	15/15
PSA-CMA	4.8(2)	8.8(10)	9.2(11)	9.5(11)	9.5(10)	9.4(6)	9.3(5)	15/15
f10	7413	8661	10735	13641	14920	17073	17476	15/15
BIPOP-C	1.9(0.2)	1.8(0.1)	**1.6**(0.1)	**1.3**(0.1)	1.2(0.0)	1.1(0.0)	1.1(0.0)	15/15
PSA-CMA	2.9(0.3)	2.8(0.2)	2.4(0.2)	2.0(0.1)	1.9(0.0)	1.8(0.1)	1.8(0.1)	15/15
PSA-CMA	**1.8**(0.3)	**1.8**(0.2)	1.6(0.1)	1.3(0.1)	**1.2**(0.1)	**1.1**(0.1)	**1.1**(0.1)	15/15
f11	1002	2228	6278	8586	9762	12285	14831	15/15
BIPOP-C	10(0.4)	**5.1**(0.3)	**1.9**(0.1)	**1.5**(0.0)	**1.4**(0.0)	**1.2**(0.0)	**1.0**(0.0)	15/15
PSA-CMA	16(1)	7.5(0.5)	2.9(0.2)	2.2(0.1)	2.1(0.1)	1.8(0.1)	1.6(0.1)	15/15
PSA-CMA	**10**(1)	5.1(0.2)	2.0(0.1)	1.5(0.0)	1.4(0.0)	1.2(0.0)	1.0(0.0)	15/15
f12	1042	1938	2740	3156	4140	12407	13827	15/15
BIPOP-C	**3.0**(6)	4.0(4)	4.5(4)	4.9(2)	4.5(2)	1.9(0.9)	2.0(0.8)	15/15
PSA-CMA	14(11)	12(5)	11(9)	13(6)	12(5)	6.0(2)	6.6(2)	15/15
PSA-CMA	4.1(1)	**3.9**(4)	**4.4**(4)	**4.5**(2)	**4.0**(2)	**1.7**(0.2)	**1.8**(0.2)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f13	652	2021	2751	3507	18749	24455	30201	15/15
BIPOP-C	**4.3**(6)	**2.7**(5)	**5.1**(4)	6.2(4)	1.5(0.6)	2.3(2)	3.0(2)	15/15
PSA-CMA	9.4(2)	4.6(1.0)	5.4(2)	5.7(1)	**1.3**(0.3)	**1.3**(0.2)	**1.4**(0.1)	15/15
PSA-CMA	5.0(5)	4.7(3)	6.3(4)	**5.6**(3)	1.4(0.7)	1.5(0.2)	1.7(0.6)	15/15
f14	75	239	304	451	932	1648	15661	15/15
BIPOP-C	**3.9**(1)	**2.9**(0.4)	**3.7**(0.6)	**4.3**(0.5)	**4.1**(0.3)	**6.2**(0.5)	1.2(0.1)	15/15
PSA-CMA	41(18)	26(4)	28(5)	28(6)	18(2)	16(1)	2.3(0.2)	15/15
PSA-CMA	5.3(2)	3.4(1.0)	4.0(0.6)	4.7(0.7)	4.3(0.4)	6.3(0.7)	**1.2**(0.0)	15/15
f15	30378	1.5e5	3.1e5	3.2e5	3.2e5	4.5e5	4.6e5	15/15
BIPOP-C	**1.0**(0.3)$^{\star 4}$	2.0(0.6)	1.4(0.5)	1.4(0.5)	1.4(0.4)	1.0(0.4)	1.00(0.4)	15/15
PSA-CMA	3.6(1)	**0.75**(0.3)	**0.36**(0.1)	**0.36**(0.1)	**0.36**(0.1)	**0.26**(0.1)	**0.26**(0.1)	15/15
PSA-CMA	4.4(0.6)	0.91(0.2)	0.43(0.1)	0.43(0.1)	0.43(0.1)	0.31(0.0)	0.31(0.0)	15/15
f16	1384	27265	77015	1.4e5	1.9e5	2.0e5	2.2e5	15/15
BIPOP-C	**1.7**(0.3)	**1.0**(0.7)$^{\star 4}$	**1.2**(0.7)$^{\star 4}$	**1.0**(0.8)$^{\star 4}$	**1.00**(0.6)$^{\star 4}$	**1.0**(0.8)$^{\star 4}$	**1.00**(0.5)$^{\star 4}$	15/15
PSA-CMA	2.7e4(2e4)	2350(2093)	832(1109)	460(612)	341(331)	724(866)	651(716)	2/15
PSA-CMA	2.1(1)	31(8)	11(3)	6.1(2)	4.6(1)	4.5(2)	4.0(1)	15/15
f17	63	1030	4005	12242	30677	56288	80472	15/15
BIPOP-C	**2.2**(1)	**1.0**(0.2)	**1.0**(0.1)	**1.0**(0.7)	1.2(0.4)	1.3(0.8)	1.4(0.4)	15/15
PSA-CMA	17(11)	8.8(2)	3.6(1)	1.7(0.4)	**0.91**(0.1)	**0.85**(0.1)	**0.69**(0.1)*	15/15
PSA-CMA	2.7(0.8)	1.0(0.3)	2.8(4)	2.5(0.2)	1.4(0.1)	1.1(0.1)	0.86(0.1)	15/15
f18	621	3972	19561	28555	67569	1.3e5	1.5e5	15/15
BIPOP-C	1.0(0.2)	2.4(7)	1.2(1.0)	1.6(1)	1.1(0.9)	1.7(0.7)	1.6(0.6)	15/15
PSA-CMA	8.1(2)	2.9(0.4)	**0.91**(0.2)	**0.89**(0.1)	**0.49**(0.1)$^{\star 2}$	**0.43**(0.0)$^{\star 2}$	**0.48**(0.2)*	15/15
PSA-CMA	**0.96**(0.2)	**0.89**(0.2)	1.3(1)	1.5(0.3)	0.73(0.2)	0.65(0.1)	0.64(0.6)	15/15
f19	1	1	3.4e5	4.7e6	6.2e6	6.7e6	6.7e6	15/15
BIPOP-C	**169**(62)	**2.4e4**(3e4)	**1.2**(1)	1.0(0.4)	1.0(0.3)	1.0(0.3)	1(0.3)	15/15
PSA-CMA	1188(484)	5.3e5(6e4)	1.6(0.2)	0.52(2)	0.67(1.0)	0.64(0.8)	0.63(2)	13/15
PSA-CMA	215(32)	1.5e5(3e5)	1.6(0.4)	**0.26**(0.2)	**0.25**(0.1)	**0.39**(0.7)	**0.39**(0.4)	15/15
f20	82	46150	3.1e6	5.5e6	5.5e6	5.6e6	5.6e6	14/15
BIPOP-C	**4.3**(0.9)	**9.2**(4)	**1.00**(0.3)	**1.00**(0.5)	**1.00**(0.3)	**1.00**(0.7)	**1.00**(0.9)	14/15
PSA-CMA	33(7)	22(12)	2.2(2)	4.7(6)	4.6(5)	4.6(9)	4.6(11)	8/15
PSA-CMA	5.2(0.9)	13(3)	2.5(3)	6.9(11)	6.9(3)	6.8(13)	6.8(8)	6/15
f21	561	6541	14103	14318	14643	15567	17589	15/15
BIPOP-C	**3.2**(6)	**55**(39)	48(113)	47(166)	46(70)	43(82)	39(110)	13/15
PSA-CMA	14(19)	1145(1546)	532(1073)	524(1049)	512(1717)	482(972)	427(570)	11/15
PSA-CMA	9.2(4)	74(207)	**42**(39)	**41**(73)	**40**(126)	**38**(119)	**33**(13)	15/15
f22	467	5580	23491	24163	24948	26847	1.3e5	12/15
BIPOP-C	**6.8**(10)	**13**(28)	215(316)	209(227)	202(293)	188(177)	37(68)	5/15
PSA-CMA	16(24)	1065(1351)	∞	∞	∞	∞	∞ *2e7*	0/15
PSA-CMA	9.1(13)	128(138)	**104**(50)	**101**(115)	**98**(127)	**91**(122)	**18**(34)	15/15
f23	3.0	1614	67457	3.7e5	4.9e5	8.1e5	8.4e5	15/15
BIPOP-C	**4.6**(3)	**32**(28)	**1.00**(0.9)$^{\star 4}$	**1.7**(1.0)$^{\star 4}$	**2.0**(1)$^{\star 4}$	**1.2**(0.8)$^{\star 4}$	**1.2**(0.8)$^{\star 4}$	15/15
PSA-CMA	16(4)	4.4e4(2e4)	∞	∞	∞	∞	∞ *2e7*	0/15
PSA-CMA	5.2(8)	1631(7783)	2123(2224)	810(465)	∞	∞	∞ *2e7*	0/15
f24	1.3e6	7.5e6	5.2e7	5.2e7	5.2e7	5.2e7	5.2e7	3/15
BIPOP-C	**1.00**(0.6)$^{\star 3}$	1.00(1)	1.0(1)	1(0.7)	1.00(1)	1(0.5)	1.00(0.9)	3/15
PSA-CMA	∞	∞	∞	∞	∞	∞	∞ *2e7*	0/15
PSA-CMA	5.2(5)	**0.93**(0.4)	**0.48**(0.8)	**0.48**(0.5)	**0.48**(0.3)	**0.48**(0.2)	**0.48**(0.4)	9/15

Table 2: Average runtime (aRT **in number of function evaluations) divided by the respective best** aRT **measured during BBOB-2009 in dimension** 20**. The** aRT **and in braces, as dispersion measure, the half difference between 10 and 90%-tile of bootstrapped run lengths appear for each algorithm and target, the corresponding reference** aRT **in the first row. The different target Δf-values are shown in the top row. #succ is the number of trials that reached the (final) target $f_{\mathrm{opt}} + 10^{-8}$. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries, succeeded by a star, are statistically significantly better (according to the rank-sum test) when compared to all other algorithms of the table, with $p = 0.05$ or $p = 10^{-k}$ when the number k following the star is larger than 1, with Bonferroni correction by the number of functions (24). A ↓ indicates the same tested against the best algorithm from BBOB 2009. Best results are printed in bold.**

Data produced with COCO v0.0.0

size used in this paper is excessively large intuitively. It will also be meaningful to investigate about this value.

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