Benchmarking a Variant of the CMAES-APOP on the BBOB Noiseless Testbed

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ABSTRACT

The CMAES-APOP algorithm tracks the median of the elite objective values in each S successive iterations to decide whether we should increase or decrease or keep the population size in the next slot of S iterations. This quantity could be seen as the 25th percentile of objective function values evaluated in each iteration on λ candidate points. In this paper we propose a variant of the CMAES-APOP algorithm, in which we will track some percentiles of the objective values simultaneously. Some numerical results will show the improvement of this approach on some ill-conditioned functions, and on some multi-modal functions with weak global structure in small dimensions.

CCS CONCEPTS

•Computing methodologies → Continuous space search;

KEYWORDS

Benchmarking, Black-box optimization, Evolutionary computation, CMA-ES, CMAES-APOP

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1 INTRODUCTION

Adapting population size seems to be a right way in the CMA-ES to optimize multi-modal functions [1, 2, 4, 11, 12]. In the CMAES-APOP algorithm [11], the non-decrease of the median of elite objective values in a slot of S successive iterations is tracked to adapt the population size for the next S successive iterations. The variation of the population size therefore takes a staircase form in iterations. In fact, the median quantity could be considered as the 25th percentile (the 25-percentile, for the convenience) of objective function values evaluated on λ candidate points. We think that this quantity should depend on the structure of objective function and/or even should be adapted during the evolution process. However, in the black-box context, we do not know in advance the structure of the considered function. Therefore, in this paper we introduce a variant of this algorithm in which we will track some percentiles of the objective values at the same time.

This approach comes from the observation that the performance of CMAES-APOP is quite stable when testing on some multi-modal functions below. The functions [9] have a high number of local optima, and have a minimal function value of 0. The known global minimum is located at $\boldsymbol{x} = 0$. The bound constraints for the Ackley function in $[-32.768, 32.768]^n$ are considered via quadratic penalty terms. That is $f_{\text{Ackley}}(\boldsymbol{x}) + \sum_{i=1}^{n} \theta(|x_i| - 32.768).(|x_i| - 32.768)^2$ will be minimized, where $\theta(x) = 1$ if $x > 0$ and $\theta(x) = 0$ if $x \leq 0$.

$$f_{\text{Rastrigin}}(\boldsymbol{x}) = 10n + \sum_{i=1}^{n} (x_i^2 - 10\cos(2\pi x_i))$$

$$f_{\text{Schaffer}}(\boldsymbol{x}) = \sum_{i=1}^{n-1} (x_i^2 + x_{i+1}^2)^{0.25}[\sin^2(50(x_i^2 + x_{i+1}^2)^{0.1}) + 1]$$

$$f_{\text{Ackley}}(\boldsymbol{x}) = 20 - 20 \cdot \exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) + \mathrm{e} - \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right)$$

$$f_{\text{Bohachevsky}}(\boldsymbol{x}) = \sum_{i=1}^{n-1} (x_i^2 + 2x_{i+1}^2 - 0.3\cos(3\pi x_i) - 0.4\cos(4\pi x_{i+1}) + 0.7)$$

Function	n	**25-p**	1-p	10-p	50-p	75-p	90-p
Rastrigin	10	**3.317e+04**	4.332e+04	3.527e+04	3.160e+04	3.069e+04	3.250e+04
	20	**9.077e+04**	1.189e+05	9.254e+04	9.212e+04	9.038e+04	9.286e+04
	40	**2.981e+05**	3.992e+05	3.163e+05	3.006e+05	3.034e+05	3.133e+05
Schaffer	10	**3.098e+04**	5.111e+04	3.334e+04	3.051e+04	3.012e+04	3.147e+04
	20	**8.175e+04**	1.663e+05	8.833e+04	8.024e+04	8.233e+04	8.646e+04
	40	**2.255e+05**	4.942e+05	2.266e+05	2.224e+05	2.348e+05	2.325e+05
Ackley	10	**1.403e+04**	2.280e+04	1.481e+04	1.369e+04	1.429e+04	1.498e+04
	20	**3.105e+04**	6.125e+04	3.263e+04	3.024e+04	3.144e+04	3.326e+04
	40	**7.204e+04**	1.275e+05	7.379e+04	6.761e+04	7.164e+04	7.617e+04
Bohachevsky	10	**1.002e+04**	1.494e+04	1.052e+04	1.015e+04	1.064e+04	1.085e+04
	20	**2.397e+04**	4.261e+04	2.533e+04	2.366e+04	2.378e+04	2.494e+04
	40	**5.536e+04**	9.881e+04	5.781e+04	5.627e+04	5.810e+04	6.101e+04

Table 1: The aRT of some variants of CMAES-APOP, where the 25-percentile is replaced by the other percentiles (aRT (average Running Time) = number of function evaluations divided by the number of successful trials)

For each function, 51 runs are conducted. Each run is stopped and regarded as successful, when the function value is smaller than $f_{\text{stop}} = 10^{-10}$ ($f_{\text{stop}} = 10^{-8}$ for the Schaffer function). Some additional conditions that are added to the Schaffer function are: `TolX` $= 10^{-30}$, `TolFun` $= 10^{-20}$, `TolHistFun` $= 10^{-20}$. In this test, the starting point for the functions Rastrigin, Schaffer, Ackley, Bohachevsky is $(5, \ldots, 5)$, $(55, \ldots, 55)$, $(15, \ldots, 15)$, and $(8, \ldots, 8)$ respectively; and the initial step-size σ for these functions is 2, 20, 5, 3 respectively.

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Indeed, when we run the CMAES-APOP algorithm with the small initial population size $\lambda = \lambda_{\text{default}}$ (i.e, set $k_n = 1$, see [11]) and without the upper bound for the population size in three dimensions $n = 10, 20, 40$, we obtain high success rates (more than 80%) for all tests (not reported in Table 1). From Table 1 we can see that the performance of CMAES-APOP does not change so much except for the 1-percentile case. This is because 1-percentile is a sensitive quantity.

2 A VARIANT OF CMAES-APOP

We recall some notations used in the paper [11] and introduce some new ones:

- k_n : the factor for setting the initial population size. It depends on the problem dimension n and is quite large to prevent a premature convergence.
- iter : number of iterations.
- S : number of iterations in each slot.
- P : a set of percentiles.
- $f^p := \text{percentile}(\{f(\mathbf{x}_{i:\lambda}), i = 1, ..., \lambda\}, p)$: the p-percentile of objective function of λ candidates in each iteration, where p can vary from 0 to 100 (in fact p will be chosen from the set of percentiles P); f^p_{prev} and f^p_{cur} denote the p-percentiles in the previous and current iteration respectively.
- n_{up} : the number of times "$f^p_{\text{cur}} - f^p_{\text{prev}} > 0$" occurs during a slot of S iterations.
- t_{up} : the history of n_{up} in each slot recorded.
- no_{up} : the number of most recent slots we do not see the non-decrease.
- $\lambda_{\max} := (20n + 30)\lambda_{\text{default}}$: the maximum number of the population, where $\lambda_{\text{default}} = \lfloor 4 + 3\log(n) \rfloor$.

The algorithm below is quite similar to the CMAES-APOP algorithm. It collects n_{up} during S iterations. After each S iterations, depending the information of n_{up}, it adapts the population size for the S next iterations.

A variant of the CMAES-APOP

Input: $\mathbf{m} \in \mathbb{R}^n$, $\sigma \in \mathbb{R}_+$
Initialize: $\mathbf{C} = \mathbf{I}$, $\mathbf{p}_c = 0$, $\mathbf{p}_\sigma = 0$, $\lambda = k_n \times \lambda_{\text{default}}$
Set: $\mu = \lfloor \lambda/2 \rfloor$, $w_i = \log(\mu + 0.5) - \log i$, $i = 1, \ldots, \mu$,
μ_w, c_c, c_σ, c_1, c_μ, $c_1 + c_\mu \le 1$, d_σ,
iter = 0, $S = 5$, $r_{\max} = 30$, $n_{\text{up}} = 0$, $t_{\text{up}} = [\ ]$.
While not terminate
  iter = iter + 1;
  $\mathbf{x_i} = \mathbf{m} + \sigma \mathbf{y_i}$, $\mathbf{y_i} \sim \mathbf{N}(\mathbf{0}, \mathbf{C})$, for $i = 1, \ldots, \lambda$
  Take p randomly from the set of percentiles P
  if iter > 1
    if $f^p_{\text{cur}} - f^p_{\text{prev}} > 0$ //Check if f^p increases
      $n_{\text{up}} = n_{\text{up}} + 1$;
    end
  end
  $\mathbf{m} \leftarrow \sum_{i=1}^{\mu} w_i \mathbf{x}_{i:\lambda} = \mathbf{m} + \sigma \mathbf{y}_w$, where $\mathbf{y}_w = \sum_{i=1}^{\mu} w_i \mathbf{y}_{i:\lambda}$
  $\mathbf{p}_c \leftarrow (1 - c_c)\mathbf{p}_c + \mathbf{1}_{\|\mathbf{p}_\sigma < 1.5\sqrt{n}\|}\sqrt{(1 - (1 - c_c)^2)}\sqrt{\mu_w}\mathbf{y}_w$
  $\mathbf{p}_\sigma \leftarrow (1 - c_\sigma)\mathbf{p}_\sigma + \sqrt{(1 - (1 - c_\sigma)^2)}\sqrt{\mu_w}\mathbf{C}^{-\frac{1}{2}}\mathbf{y}_w$
  $\mathbf{C} \leftarrow (1 - c_1 - c_\mu)\mathbf{C} + c_1 \mathbf{p}_c \mathbf{p}_c^T + c_\mu \sum_{i=1}^{\mu} w_i \mathbf{y}_{i:\lambda}\mathbf{y}_{i:\lambda}^T$
  $\sigma \leftarrow \sigma \times \exp\left(\frac{c_\sigma}{d_\sigma}\left(\frac{\|\mathbf{p}_\sigma\|}{E\|N(0,1)\|} - 1\right)\right)$
  if (mod(iter, S) = 1) & (iter > 1)   // Adapting pop-size
    $t_{\text{up}} = [t_{\text{up}}; n_{\text{up}}]$;   // History of n_{up}
    if $n_{\text{up}} > 1$
      $\lambda \leftarrow \left\lfloor \min\left(\exp\left(\frac{n_{\text{up}} \cdot (4 + 3\log(n))}{S \cdot \sqrt{\lambda - \lambda_{\text{default}} + 1}}\right), r_{\max}\right) \times \lambda \right\rfloor$;
      $\lambda \leftarrow \min(\lambda, \lambda_{\max})$
      $\sigma \leftarrow \sigma \times \exp\left(\frac{1}{n}\left(\frac{n_{\text{up}}}{S} - \frac{1}{5}\right)\right)$; // Enlarge σ a little bit
    elseif $n_{\text{up}} = 0$
      $\text{no}_{\text{up}} = \text{length}(t_{\text{up}}) - \max(\text{find}(t_{\text{up}} > 0))$;
      if $\lambda > 2\lambda_{\text{default}}$
        $\lambda \leftarrow \max\left(\lfloor \lambda \times \exp(-\text{no}_{\text{up}}/10)) \rfloor, 2\lambda_{\text{default}}\right)$;
      end
    end
    if λ is changed   // Only when $n_{\text{up}} > 1$ or $n_{\text{up}} = 0$
      Update μ, $w_{i=1\ldots\mu}$, μ_w w.r.t the new pop-size λ
      Update the parameters c_c, c_σ, c_1, c_μ, d_σ
    end
    $n_{\text{up}} \leftarrow 0$ // Reset n_{up}
  end

There are only two different points. The first one is that the upper bound of the population size is not constant but depends on the problem dimension. The second one is that instead of using the 25-percentile as in the CMAES-APOP during the evolution process, we will take (randomly) a percentile p from the set P. In this work, we will consider 3 variants corresponding to 3 sets: $P_1 = \{1, 25, 50\}, P_2 = \{1, 50\}, P_3 = \{1, 50, 75\}$. The 50-percentile is chosen to ensure the balance. Choosing the 1-percentile means that we want to track the change of the most elite part of the population. We think it makes sense for weak global structure functions since we sometimes try to ignore the contribution of other local optima and focus on the global ones. Another reason is in the case when the optimal solution of the problem is quite close to the corner of box constraints and a penalty-term for constraints is applied. In this case, when the evolution process approaches this global optimum, the number of penalized points could be large (especially in a large dimension), and dominate normal points. Consequently, the signal which we are tracking may not be accurate if we use a large percentile. Finally, we choose the 75-percentile because sometimes we want to hear the voice of non-elite candidates. In fact, the non-elite candidates (a half of the population) do not contribute to the evolution process in the CMA-ES. However, in this way they do indirectly.

3 EXPERIMENTAL PROCEDURE

We test the 3 variants with a budget of $2 \times 10^5 \times n$, where n is the problem dimension, on the BBOB noiseless functions in six different dimensions. We denote the variants corresponding to $P_1 = \{1, 25, 50\}, P_2 = \{1, 50\}$, and $P_3 = \{1, 50, 75\}$ by Var1, Var2 and Var3 respectively. We used the matlab implementation of CMA-ES [1], version 3.40.beta to make the variants of CMAES-APOP. Similar to CMAES-APOP, all variants use the restart strategies as in BIPOP-CMA-ES [4], but without the stagnation condition. Also, the parameter k_n will be tuned to 10, 20, 30, 40, 50, 60 for dimension $n = 2, 3, 5, 10, 20, 40$ respectively for all variants. Whenever the population size λ is updated, the parameters corresponding are also changed.

The experiment is tested on a MacBook Air Intel(R) Core(TM) i5-5250U CPU @ 1.60GHz, RAM 8G using MATLAB R2016b. Since

[1] https://www.lri.fr/ hansen/cmaes20091024.m

these variants are also developed for solving multi-modal functions, in the first run, we use the pure CMA-ES with the default population size $\lambda = \lambda_{\text{default}}$. The population size adaptation strategy in all variants of CMAES-APOP is applied with the initial population size $\lambda = k_n \times \lambda_{\text{default}}$ whenever the algorithm is restarted and then repeated until the budget is used up. We choose the starting point $\mathbf{m}^0$ uniformly in $[-4, 4]^n$ and set the initial step-size $\sigma_0 = 2$ for all runs. We will compare these variants with CMAES-APOP [11], IPOP-CMA-ES [14] and BIPOP-CMA-ES [4].

4 CPU TIMING

Besides studying the performance of the proposed algorithms, we are interested in evaluating the CPU timing of these algorithms. Thus we run Var1, Var2, Var3 on the BBOB test suite [8] with a small budget. Here, we set maximum budget equal to $1000 \times n$ function evaluations according to [10]. Table 2 shows the time per function evaluation of the variants in dimensions 2, 3, 5, 10, 20, 40.

	2	3	5	10	20	40
Var1	2.3e-06	1.5e-06	9.9e-07	6.7e-07	5.2e-07	4.7e-07
Var2	2.1e-06	1.4e-06	9.5e-07	6.8e-07	5.5e-07	4.7e-07
Var3	2.1e-06	1.4e-06	1.0e-06	7.2e-07	4.8e-07	4.8e-07

Table 2: The CPU time (in second) per function evaluation.

5 RESULTS

Results from experiments according to [10] and [5] on the benchmark functions given in [3, 8] are presented in Figures 1, 2 and 3 and in Tables 3 and 4. The experiments were performed with COCO [7], version 2.2, the plots were produced with version 2.2.1.

The **average runtime (aRT)**, used in the figures and tables, depends on a given target function value, $f_t = f_{\text{opt}} + \Delta f$, and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach f_t, summed over all trials and divided by the number of trials that actually reached f_t [6, 13]. **Statistical significance** is tested with the rank-sum test for a given target Δf_t using, for each trial, either the number of needed function evaluations to reach Δf_t (inverted and multiplied by -1), or, if the target was not reached, the best Δf-value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

Figure 1 and Tables 3 and 4 show that in 5-D, Var1, Var2, and Var3 solve 21, 22, and 22 respectively among 24 functions; and in 20-D the variants solve 19 among 24 functions. The performance of these variants is similar to the performance of CMAES-APOP except for some cases. It means that all variants still run faster about 2.5 times than the IPOP-CMA-ES does, and 3-4 times than the BIPOP-CMA-ES does on the well-structured multi-modal function f_{15} in 20-D, 40-D; and faster about 1.8 times than the IPOP-CMA-ES does, and 2.6 times than the BIPOP-CMA-ES does on the function f_{18} in dimension 20. Also, the variants run faster about 1.8 times than the IPOP-CMA-ES does, and 2.5 times than the BIPOP-CMA-ES does on the function f_7 in dimension 40.

All variants of CMAES-APOP improve slightly the performance on $f_7, f_{15}, f_{16}, f_{18}, f_{21}$ in 10-D, f_8 in 40-D,f_{13} in 20-D. Moreover, Var1, Var2, and Var3 can solve some hard functions in small dimension while CMAES-APOP does not, for instance f_4, f_{23} in 3-D, f_{24} in 2-D, 3-D (all variants) and in 5-D (just with Var2 and Var3). However, the variants do not work well as CMAES-APOP does on f_3 in 10-D, f_{16} in 40-D, on function f_{19} in 10-D, f_{20} in 20-D, and on f_{21} in dimensions 20, 40. However, all variants are still better than the IPOP-CMA-ES and BIPOP-CMA-ES on f_3 in 10-D; than the BIPOP-CMA-ES on f_{19} in dimensions 10, 40; and than BIPOP-CMA-ES on f_{20} in dimensions 10, 20.

From Figue 3 and Tables 3 and 4, we can see that Var1 and Var3 provide better empirical cumulative distribution functions (ECDFs) than Var2 does for the class of conditioned functions, and the class of multi-modal functions with adequate global structure in high dimensions. This means that tracking more percentiles in the context of CMAES-APOP can help us to make better decisions in adapting population size for these classes, especially when we focus on percentiles indicating the elite part of the population. Nevertheless, Var3 sometimes works slightly better than Var1 and Var2 do on ill-conditioned functions, for example on f_8 in 10-D, 40-D; on f_9 in 10-D, 20-D; on f_{12} in 10-D, 40-D; on f_{13} in 20-D; and on f_{20} in 5-D. It implies that using the information of non-elite individuals to adapt the population size in the context of CMAES-APOP is not bad idea.

6 CONCLUSION

In this work, we have proposed a variant of CMAES-APOP in which we track the change of some percentiles of objective values rather than one percentile, and set the upper bound of the population size depending on the problem dimension. Some versions of this approach are tested on the BBOB noiseless testbed. The numerical results show that this approach can improve the performance of CMAES-APOP in some cases when the set of percentiles P is chosen appropriately. Especially, the proposed versions can efficiently solve some multi-modal functions with weak global structure in small dimensions. However, the percentile p in this work is chosen uniformly from a set P. Thus, how to initialize a good set P and how to evaluate the importance of each percentile p in P during the evolution process are still questions that need to be answered. Moreover, we think that the information of percentiles not only plays a role in adapting population size but also could play a deeper role inside the evolution process of the CMA-ES. All that will be our direction in future work.

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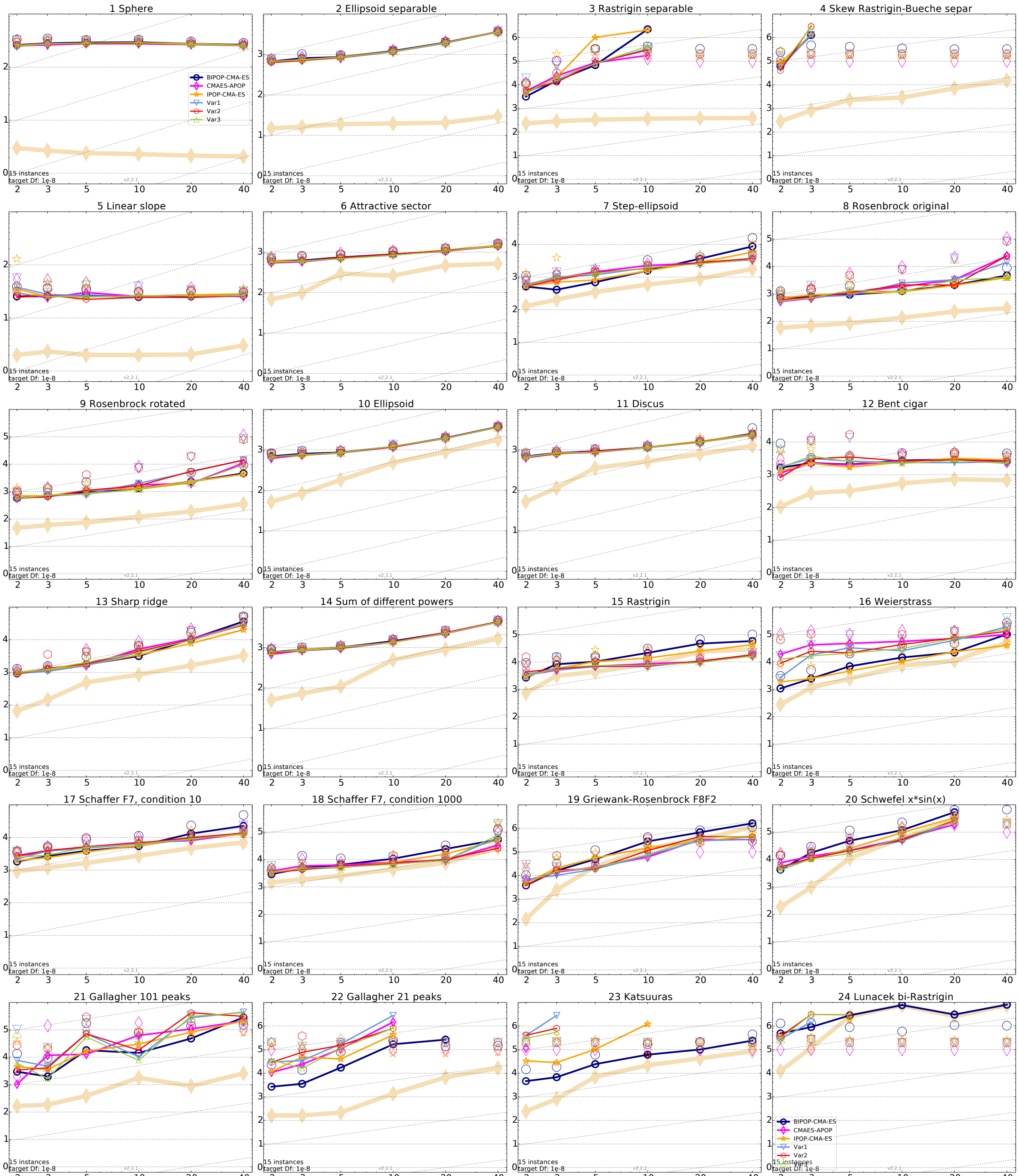

Figure 1: Average running time (aRT in number of f-evaluations as $\log_{10}$ value), divided by dimension for target function value 10^{-8} versus dimension. Slanted grid lines indicate quadratic scaling with the dimension. Different symbols correspond to different algorithms given in the legend of f_1 and f_{24}. Light symbols give the maximum number of function evaluations from the longest trial divided by dimension. Black stars indicate a statistically better result compared to all other algorithms with $p < 0.01$ and Bonferroni correction number of dimensions (six). Legend: ○: BIPOP-CMA-ES, ◇: CMAES-APOP, ⋆: IPOP-CMA-ES, ▽: Var1, ⬡: Var2, △: Var3

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f1	11	12	12	12	12	12	12	15/15
BIPOP-C	6.4(5)	18(3)	31(7)	43(11)	55(7)	82(6)	108(14)	15/15
CMAES-A	4.7(5)	**12**(4)	28(4)	**39**(7)	**52**(6)	77(6)	103(8)	15/15
IPOP-CM	5.0(1)	16(5)	29(8)	41(7)	55(4)	80(6)	104(6)	15/15
Var1	3.8(2)	15(4)	**26**(6)	39(9)	53(13)	**77**(8)	**101**(10)	15/15
Var2	4.1(3)	15(4)	28(11)	41(8)	55(8)	78(8)	103(11)	15/15
Var3	**3.4**(2)	15(6)	28(11)	40(8)	54(8)	80(10)	106(9)	15/15
f2	83	87	88	89	90	92	94	15/15
BIPOP-C	26(8)	32(8)	35(4)	37(4)	39(4)	42(3)	44(3)	15/15
CMAES-A	28(9)	33(2)	35(1)	37(1)	38(2)	41(3)	43(2)	15/15
IPOP-CM	27(9)	32(7)	36(4)	37(4)	39(3)	41(2)	43(5)	15/15
Var1	26(7)	31(4)	**34**(3)	36(5)	38(3)	41(4)	43(4)	15/15
Var2	**23**(8)	31(4)	34(3)	**36**(3)	**38**(4)	**40**(2)	**42**(3)	15/15
Var3	26(6)	**30**(9)	36(3)	37(3)	38(3)	41(3)	44(4)	15/15
f3	716	1622	1637	1642	1646	1650	1654	15/15
BIPOP-C	2.8(3)	**32**(41)	**205**(26)	**205**(163)	**205**(194)	**206**(214)	**206**(126)	14/15
CMAES-A	**2.7**(3)	80(53)	253(224)	256(357)	257(358)	259(195)	260(363)	12/15
IPOP-CM	4.5(8)	106(81)	3140(4119)	3131(2524)	3124(2261)	3117(5481)	3110(2827)	2/15
Var1	3.8(5)	45(20)	228(230)	240(295)	241(141)	242(159)	243(237)	15/15
Var2	2.8(5)	72(56)	252(380)	255(284)	257(254)	258(393)	260(165)	15/15
Var3	3.6(3)	52(55)	221(164)	231(171)	232(216)	234(364)	235(162)	15/15
f4	809	1633	1688	1758	1817	1886	1903	15/15
BIPOP-C	5.4(4)	∞	∞	∞	∞	∞	∞ *2e6*	0/15
CMAES-A	8.3(7)	∞	∞	∞	∞	∞	∞ *5e5*	0/15
IPOP-CM	**4.0**(6)	∞	∞	∞	∞	∞	∞ *9e5*	0/15
Var1	7.8(5)	∞	∞	∞	∞	∞	∞ *1e6*	0/15
Var2	5.2(6)	∞	∞	∞	∞	∞	∞ *1e6*	0/15
Var3	5.7(4)	**8575**(1e4)	∞	∞	∞	∞	∞ *1e6*	0/15
f5	10	10	10	10	10	10	10	15/15
BIPOP-C	9.0(3)	13(5)	13(4)	13(3)	13(5)	13(5)	13(3)	15/15
CMAES-A	12(6)	15(4)	15(7)	15(5)	15(7)	15(7)	15(5)	15/15
IPOP-CM	9.2(5)	12(5)	13(5)	13(4)	13(6)	13(3)	13(4)	15/15
Var1	9.1(5)	13(5)	13(5)	13(4)	13(4)	13(5)	13(6)	15/15
Var2	**8.1**(4)	**11**(5)	**11**(3)	**11**(4)	**11**(5)	**11**(2)	**11**(4)	15/15
Var3	8.5(3)	12(5)	12(1)	12(5)	12(5)	12(2)	12(1)	15/15
f6	114	214	281	404	580	1038	1332	15/15
BIPOP-C	4.7(2)	4.2(1)	4.4(0.9)	3.9(0.5)	3.3(0.4)	2.6(0.6)	2.5(0.4)	15/15
CMAES-A	**3.3**(0.7)	3.5(0.9)	4.1(0.9)	3.7(0.7)	3.2(0.6)	2.5(0.4)	2.4(0.5)	15/15
IPOP-CM	4.9(2)	4.2(1)	4.4(0.7)	4.0(0.8)	3.4(0.4)	2.5(0.2)	2.5(0.1)	15/15
Var1	3.3(2)	3.5(0.5)	4.0(1)	**3.6**(0.8)	**3.1**(0.4)	2.4(0.4)	2.4(0.3)	15/15
Var2	3.7(1)	**3.5**(1)	4.1(0.7)	3.7(0.3)	3.3(0.5)	2.5(0.3)	2.5(0.1)	15/15
Var3	3.8(1)	3.5(0.7)	**3.9**(0.8)	3.6(0.8)	3.1(0.4)	**2.4**(0.2)	**2.3**(0.3)	15/15
f7	24	324	1171	1451	1572	1572	1597	15/15
BIPOP-C	10(16)	**3.0**(3)	**2.0**(2)	**2.0**(0.6)	**2.0**(2)	**2.0**(1)	**2.0**(1)	15/15
CMAES-A	6.4(3)	4.4(5)	3.5(2)	3.7(2)	3.9(1)	3.9(1)	4.1(0.4)	15/15
IPOP-CM	8.7(6)	3.5(3)	2.3(2)	2.2(1.0)	2.4(1.0)	2.4(1)	2.4(1)	15/15
Var1	6.2(3)	3.5(3)	2.7(2)	3.2(1)	3.2(1)	3.2(2)	3.4(1)	15/15
Var2	**5.3**(3)	3.3(4)	2.9(2)	3.3(2)	3.7(0.8)	3.7(0.1)	3.9(0.3)	15/15
Var3	6.3(5)	4.6(5)	2.9(2)	3.8(1)	3.6(1)	3.6(1)	3.8(0.5)	15/15
f8	73	273	336	372	391	410	422	15/15
BIPOP-C	6.5(2)	7.4(6)	9.0(5)	9.4(5)	10(2)	10(8)	11(1)	15/15
CMAES-A	**5.2**(0.8)	8.9(2)	11(14)	11(15)	12(15)	12(15)	13(1.0)	15/15
IPOP-CM	7.0(6)	10(2)	11(2)	11(7)	11(4)	12(8)	12(8)	15/15
Var1	6.2(2)	**6.7**(2)	**8.3**(2)	**8.8**(1)	**9.1**(1)	**10**(2)	**10**(2)	15/15
Var2	5.2(4)	10(4)	11(2)	11(12)	12(2)	12(2)	13(13)	15/15
Var3	5.3(1)	12(12)	13(20)	14(11)	14(20)	15(20)	15(20)	15/15
f9	35	127	214	263	300	335	369	15/15
BIPOP-C	12(3)	17(15)	14(4)	13(3)	13(1)	13(3)	12(2)	15/15
CMAES-A	10(3)	13(4)	12(4)	12(4)	11(3)	11(3)	11(2)	15/15
IPOP-CM	12(3)	21(20)	17(12)	16(11)	15(10)	15(9)	14(4)	15/15
Var1	**9.2**(2)	**13**(4)	**12**(3)	11(2)	11(2)	11(1)	11(1)	15/15
Var2	10(5)	20(9)	17(5)	16(5)	15(24)	15(4)	14(12)	15/15
Var3	10(4)	13(5)	12(4)	**11**(3)	**11**(3)	**11**(3)	**11**(2)	15/15
f10	349	500	574	607	626	829	880	15/15
BIPOP-C	7.0(2)	5.7(0.8)	5.4(0.5)	5.5(0.6)	5.6(0.6)	4.6(0.2)	4.7(0.3)	15/15
CMAES-A	6.2(2)	5.4(1)	5.3(0.5)	5.3(0.9)	5.4(0.6)	4.5(0.4)	4.6(0.4)	15/15
IPOP-CM	7.3(2)	5.8(0.8)	5.4(1)	5.4(0.6)	5.5(0.4)	4.6(0.4)	4.6(0.3)	15/15
Var1	6.6(2)	5.5(1)	**5.2**(1)	**5.3**(0.8)	**5.4**(0.2)	**4.5**(0.3)	**4.6**(0.3)	15/15
Var2	6.6(3)	5.6(1)	5.3(0.9)	5.3(0.7)	5.4(0.7)	4.5(0.4)	4.6(0.3)	15/15
Var3	**6.0**(2)	**5.2**(1)	5.3(0.4)	5.4(0.3)	5.4(0.4)	4.5(0.2)	4.6(0.2)	15/15
f11	143	202	763	977	1177	1467	1673	15/15
BIPOP-C	17(5)	14(3)	4.4(0.4)	3.7(0.4)	3.2(0.4)	2.8(0.2)	2.7(0.2)	15/15
CMAES-A	16(3)	14(2)	4.2(0.6)	**3.5**(0.2)	**3.0**(0.4)	**2.6**(0.2)	**2.5**(0.3)	15/15
IPOP-CM	17(3)	15(2)	4.3(0.5)	3.6(0.3)	3.1(0.3)	2.7(0.2)	2.6(0.2)	15/15
Var1	17(6)	15(3)	4.3(0.3)	3.6(0.4)	3.2(0.3)	2.8(0.3)	2.6(0.3)	15/15
Var2	18(7)	16(2)	4.6(0.5)	3.8(0.4)	3.3(0.3)	2.9(0.3)	2.7(0.3)	15/15
Var3	**15**(5)	**14**(3)	**4.1**(0.5)	3.5(0.4)	3.0(0.2)	2.6(0.2)	2.5(0.3)	15/15
f12	108	268	371	413	461	1303	1494	15/15
BIPOP-C	21(18)	15(12)	15(9)	15(11)	15(6)	6.7(4)	6.6(4)	15/15
CMAES-A	16(12)	**10**(12)	**12**(8)	**13**(3)	13(6)	6.2(4)	6.3(3)	15/15
IPOP-CM	19(25)	12(10)	12(0.7)	13(10)	**13**(11)	**5.5**(4)	**5.5**(5)	15/15
Var1	**12**(8)	11(7)	12(7)	13(6)	14(12)	6.3(4)	8.3(20)	15/15
Var2	13(14)	11(6)	13(13)	14(10)	15(13)	10(4)	10(25)	15/15
Var3	14(1)	13(10)	14(11)	16(11)	17(8)	7.9(3)	8.0(3)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f13	132	195	250	319	1310	1752	2255	15/15
BIPOP-C	7.9(4)	11(6)	12(6)	11(3)	3.2(0.6)	**3.0**(0.7)	3.4(1)	15/15
CMAES-A	5.4(0.9)	8.9(6)	12(4)	11(2)	2.9(0.6)	3.0(0.5)	**3.2**(4)	15/15
IPOP-CM	6.3(5)	10(7)	**11**(4)	10(5)	**2.9**(1)	3.3(1)	3.3(1)	15/15
Var1	6.1(4)	10(5)	11(4)	**10**(3)	2.9(0.4)	3.4(0.7)	3.3(2)	15/15
Var2	5.5(2)	**8.8**(5)	13(13)	13(11)	3.7(0.9)	3.4(1)	3.8(0.7)	15/15
Var3	**4.8**(1)	10(7)	12(4)	10(6)	3.0(1)	3.7(0.5)	3.7(3)	15/15
f14	10	41	58	90	139	251	476	15/15
BIPOP-C	2.1(2)	5.6(2)	7.5(3)	7.9(3)	9.1(2)	11(2)	9.0(0.7)	15/15
CMAES-A	**1.3**(3)	**3.7**(3)	**6.3**(0.7)	**6.9**(1.0)	8.7(2)	**10**(2)	**8.5**(1)	15/15
IPOP-CM	4.2(4)	5.8(5)	7.7(2)	8.5(3)	9.3(3)	11(1)	8.9(1)	15/15
Var1	1.6(1)	4.5(2)	6.5(0.6)	7.3(2)	8.4(1)	11(1)	8.6(1)	15/15
Var2	1.6(4)	4.5(2)	7.2(1)	7.4(1)	**8.1**(3)	10(2)	8.6(1.0)	15/15
Var3	1.4(2)	4.5(2)	6.9(2)	7.7(1)	8.5(2)	11(2)	8.6(1)	15/15
f15	511	9310	19369	19743	20073	20769	21359	14/15
BIPOP-C	3.3(5)	3.0(1)	2.4(0.8)	2.4(1)	2.4(1)	2.4(1)	2.4(1)	15/15
CMAES-A	**3.0**(5)	2.3(0.4)	1.3(0.4)	1.4(0.4)	1.5(0.5)	1.5(0.3)	1.6(0.4)	15/15
IPOP-CM	4.5(5)	2.5(3)	2.3(2)	2.3(2)	2.3(2)	2.3(3)	2.4(3)	15/15
Var1	4.4(6)	2.0(2)	1.3(0.7)	**1.4**(0.9)	**1.4**(0.8)	**1.5**(0.6)	**1.5**(0.8)	15/15
Var2	4.9(7)	2.0(1)	**1.3**(0.7)	1.4(0.5)	1.5(0.5)	1.5(0.5)	1.6(0.5)	13/15
Var3	3.8(8)	**2.0**(0.5)	1.4(1)	1.5(0.7)	1.6(1)	1.6(1)	1.7(0.9)	15/15
f16	120	612	2662	10163	10449	11644	12095	15/15
BIPOP-C	6.0(6)	7.1(5)	5.2(17)	2.1(1)	2.7(4)	2.7(4)	2.8(4)	15/15
CMAES-A	3.9(4)	68(84)	55(85)	15(29)	21(22)	20(27)	19(34)	13/15
IPOP-CM	5.1(4)	**4.7**(6)	**3.4**(3)	**1.1**(2)	**1.9**(3)	**1.9**(1)	**1.9**(2)	15/15
Var1	**3.7**(4)	36(51)	48(43)	14(7)	14(15)	13(17)	13(6)	15/15
Var2	3.9(3)	47(77)	24(17)	7.6(7)	7.9(5)	8.7(5)	8.7(6)	15/15
Var3	4.9(5)	27(51)	29(38)	8.3(8)	9.0(6)	8.5(11)	8.5(10)	15/15
f17	5.0	215	899	2861	3669	6351	7934	15/15
BIPOP-C	7.0(5)	2.0(1)	2.0(2)	2.0(2)	2.0(0.7)	2.0(1)	2.4(1)	15/15
CMAES-A	**3.4**(4)	**1.9**(2)	2.5(7)	2.1(2)	3.1(2)	2.7(2)	3.1(0.5)	15/15
IPOP-CM	10(6)	2.1(1)	**1.9**(2)	**1.2**(2)	**1.5**(2)	**1.6**(0.6)	**2.0**(0.8)	15/15
Var1	4.7(3)	2.1(1)	2.8(3)	2.0(2)	2.8(2)	2.9(1)	2.7(0.9)	15/15
Var2	3.9(4)	4.3(0.6)	2.9(7)	3.0(3)	2.8(2)	3.0(0.6)	3.1(2)	15/15
Var3	5.3(9)	1.9(0.9)	4.0(6)	2.3(2)	2.4(2)	3.2(0.3)	3.0(0.2)	15/15
f18	103	378	3968	8451	9280	10905	12469	15/15
BIPOP-C	2.0(1)	6.9(6)	2.0(2)	2.0(0.5)	2.0(0.3)	2.3(1)	2.5(1)	15/15
CMAES-A	**1.5**(1)	**1.6**(0.5)	**1.3**(2)	1.6(1)	2.0(0.3)	2.3(0.5)	2.4(0.6)	15/15
IPOP-CM	2.4(2)	5.5(0.5)	1.7(2)	2.1(1.0)	2.1(0.6)	2.0(0.3)	**2.0**(0.6)	15/15
Var1	1.7(0.6)	10(20)	2.6(2)	1.9(0.9)	2.0(0.3)	2.0(0.8)	2.0(0.7)	15/15
Var2	1.6(1)	2.2(1)	2.3(2)	1.8(1)	2.0(0.2)	2.2(0.2)	2.2(0.3)	15/15
Var3	1.8(1)	5.3(14)	1.4(2)	**1.6**(0.9)	**1.9**(0.6)	**2.0**(0.2)	2.1(0.1)	15/15
f19	1	1	242	1.0e5	1.2e5	1.2e5	1.2e5	15/15
BIPOP-C	41(16)	5602(2008)	322(226)	2.0(2)	2.0(2)	2.0(2)	2.0(1)	15/15
CMAES-A	**2.0**(0)	**2.0**(0)	180(115)	0.81(0.4)	0.80(0.3)	0.86(0.3)	0.90(0.3)	15/15
IPOP-CM	42(51)	3439(2588)	250(217)	2.4(1)	2.2(1)	2.3(1)	2.3(1)	15/15
Var1	**2.0**(0)	**2.0**(0)	194(77)	0.69(0.2)	**0.66**(0.2)	**0.72**(0.3)	**0.75**(0.2)	15/15
Var2	**2.0**(0)	**2.0**(0)	**178**(102)	**0.65**(0.2)	0.69(0.2)	0.74(0.2)	0.78(0.2)	15/15
Var3	**2.0**(0)	**2.0**(0)	234(140)	0.76(0.5)	0.75(0.2)	0.82(0.5)	0.85(0.2)	15/15
f20	16	851	38111	51362	54470	54861	55313	14/15
BIPOP-C	6.6(3)	**16**(15)	5.5(6)	4.4(2)	4.3(4)	4.3(2)	4.4(2)	15/15
CMAES-A	**3.0**(3)	33(7)	2.1(2)	1.8(0.5)	1.8(1)	1.9(2)	1.9(2)	15/15
IPOP-CM	7.8(4)	22(15)	2.9(2)	2.2(1)	2.2(1)	2.2(2)	2.3(3)	15/15
Var1	3.3(2)	26(12)	2.2(0.8)	1.9(0.9)	1.8(2)	1.9(1)	2.0(2)	15/15
Var2	4.5(4)	31(15)	2.0(2)	1.7(0.5)	1.7(1)	1.8(1)	1.9(1)	15/15
Var3	3.5(2)	19(10)	**1.6**(1.0)	**1.3**(0.8)	**1.4**(0.7)	**1.4**(0.8)	**1.5**(0.7)	15/15
f21	41	1157	1674	1692	1705	1729	1757	14/15
BIPOP-C	4.5(2)	27(96)	48(118)	50(10)	51(157)	51(126)	51(130)	15/15
CMAES-A	3.6(3)	22(2)	**32**(224)	**32**(29)	**36**(37)	**36**(39)	**36**(90)	14/15
IPOP-CM	13(4)	**11**(3)	44(35)	45(72)	46(49)	47(81)	47(67)	14/15
Var1	3.6(2)	16(39)	151(14)	210(300)	208(315)	206(172)	203(149)	12/15
Var2	3.1(2)	46(39)	154(154)	210(365)	209(427)	207(422)	204(145)	13/15
Var3	**2.9**(2)	20(9)	161(476)	160(174)	159(2)	158(316)	156(286)	12/15
f22	71	386	938	980	1008	1040	1068	14/15
BIPOP-C	14(16)	40(9)	**90**(145)	**87**(200)	**84**(185)	**82**(131)	**81**(118)	15/15
CMAES-A	13(23)	86(181)	372(485)	543(1071)	530(471)	515(514)	503(594)	9/15
IPOP-CM	24(59)	95(250)	192(501)	198(434)	194(216)	190(144)	188(276)	11/15
Var1	**10**(22)	52(79)	558(682)	884(962)	860(1422)	834(502)	814(1171)	9/15
Var2	10(10)	45(78)	457(353)	744(623)	724(1098)	734(966)	716(1605)	9/15
Var3	10(21)	**13**(24)	408(909)	585(535)	569(794)	553(1101)	540(1068)	11/15
f23	3.0	518	14249	27890	31654	33030	34256	15/15
BIPOP-C	**3.4**(3)	26(19)	**7.4**(6)	**4.1**(4)	**3.7**(3)	**3.6**(4)	**3.5**(3)	15/15
CMAES-A	4.8(5)	48(77)	494(625)	∞	∞	∞	∞ *5e5*	0/15
IPOP-CM	4.4(5)	53(33)	35(23)	18(47)	16(30)	15(11)	15(2)	11/15
Var1	4.4(4)	**17**(15)	457(579)	∞	∞	∞	∞ *1e6*	0/15
Var2	5.6(6)	37(35)	457(369)	502(987)	∞	∞	∞ *1e6*	0/15
Var3	5.1(7)	47(65)	457(351)	∞	∞	∞	∞ *1e6*	0/15
f24	1622	2.2e5	6.4e6	9.6e6	9.6e6	1.3e7	1.3e7	3/15
BIPOP-C	4.2(3)	**3.1**(3)	1.3(1)	1.2(1)	1.2(1.0)	1.1(1)	1.1(0.6)	3/15
CMAES-A	4.8(6)	36(32)	∞	∞	∞	∞	∞ *5e5*	0/15
IPOP-CM	5.7(9)	19(24)	1.5(1)	**0.97**(2)	**0.97**(2)	**0.73**(1)	**0.73**(0.6)	2/15
Var1	4.5(6)	21(11)	∞	∞	∞	∞	∞ *1e6*	0/15
Var2	**3.9**(8)	69(46)	2.3(2)	1.6(1)	1.6(0.8)	1.2(0.7)	1.2(1)	1/15
Var3	6.4(4)	17(15)	**1.2**(1)	1.6(2)	1.6(2)	1.2(0.9)	1.2(1)	1/15

Table 3: Average runtime (aRT in number of function evaluations) divided by the respective best aRT measured during BBOB-2009 in dimension 5. The aRT and in braces, as dispersion measure, the half difference between 10 and 90%-tile of bootstrapped run lengths appear for each algorithm and target, the corresponding reference aRT in the first row. The different target Δf-values are shown in the top row. #succ is the number of trials that reached the (final) target $f_{\text{opt}} + 10^{-8}$. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries, succeeded by a star, are statistically significantly better (according to the rank-sum test) when compared to all other algorithms of the table, with $p = 0.05$ or $p = 10^{-k}$ when the number k following the star is larger than 1, with Bonferroni correction by the number of functions (24). A ↓ indicates the same tested against the best algorithm from BBOB 2009. Best results are printed in bold.

Data produced with COCO v2.2.1

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f1	43	43	43	43	43	43	43	15/15
BIPOP-C	16(4)	29(6)	40(5)	53(5)	65(4)	91(6)	115(7)	15/15
CMAES-A	13(2)	25(2)	**35**(4)	**49**(4)	**61**(5)	86(7)	**111**(8)	15/15
IPOP-CM	16(3)	28(3)	41(4)	53(4)	65(3)	91(4)	116(3)	15/15
Var1	**12**(1)	**24**(2)	37(2)	49(3)	62(4)	**86**(5)	112(7)	15/15
Var2	13(1)	26(3)	37(3)	49(2)	63(4)	88(3)	112(4)	15/15
Var3	13(2)	25(2)	37(1)	50(4)	62(5)	87(5)	112(9)	15/15
f2	385	386	387	388	390	391	393	15/15
BIPOP-C	71(10)	81(9)	87(8)	90(6)	93(4)	97(4)	99(4)	15/15
CMAES-A	**66**(8)	**78**(7)	86(9)	90(5)	91(5)	95(5)	97(5)	15/15
IPOP-CM	70(5)	81(8)	86(6)	**89**(6)	91(3)	95(2)	97(3)	15/15
Var1	67(8)	80(7)	**85**(7)	89(4)	**90**(4)	**93**(5)	**95**(3)	15/15
Var2	68(6)	81(5)	86(5)	90(4)	92(2)	95(5)	97(3)	15/15
Var3	67(7)	79(10)	87(5)	90(6)	92(2)	95(2)	98(2)	15/15
f3	5066	7626	7635	7637	7643	7646	7651	15/15
BIPOP-C	**24**(10)	∞	∞	∞	∞	∞	∞ *6e6*	0/15
CMAES-A	48(21)	∞	∞	∞	∞	∞	∞ *2e6*	0/15
IPOP-CM	25(16)	∞	∞	∞	∞	∞	∞ *3e6*	0/15
Var1	56(12)	∞	∞	∞	∞	∞	∞ *4e6*	0/15
Var2	60(21)	∞	∞	∞	∞	∞	∞ *4e6*	0/15
Var3	58(43)	∞	∞	∞	∞	∞	∞ *4e6*	0/15
f4	4722	7628	7666	7686	7700	7758	1.4e5	9/15
BIPOP-C	∞	∞	∞	∞	∞	∞	∞ *6e6*	0/15
CMAES-A	∞	∞	∞	∞	∞	∞	∞ *2e6*	0/15
IPOP-CM	∞	∞	∞	∞	∞	∞	∞ *3e6*	0/15
Var1	∞	∞	∞	∞	∞	∞	∞ *4e6*	0/15
Var2	∞	∞	∞	∞	∞	∞	∞ *4e6*	0/15
Var3	∞	∞	∞	∞	∞	∞	∞ *4e6*	0/15
f5	41	41	41	41	41	41	41	15/15
BIPOP-C	10(1)	12(3)	12(3)	12(2)	13(2)	13(2)	13(2)	15/15
CMAES-A	10(1)	13(2)	13(2)	13(1)	13(3)	13(2)	13(1)	15/15
IPOP-CM	11(2)	13(3)	13(3)	13(2)	13(3)	13(3)	13(4)	15/15
Var1	10(1.0)	**11**(2)	**12**(2)	**12**(2)	**12**(2)	**12**(3)	**12**(2)	15/15
Var2	**10**(2)	11(2)	12(2)	12(2)	12(3)	12(3)	12(3)	15/15
Var3	11(2)	12(2)	13(2)	13(1)	13(2)	13(2)	13(2)	15/15
f6	1296	2343	3413	4255	5220	6728	8409	15/15
BIPOP-C	3.1(0.5)	2.5(0.2)	2.3(0.2)	2.3(0.3)	2.2(0.3)	2.3(0.2)	2.3(0.3)	15/15
CMAES-A	3.2(0.5)	2.6(0.4)	**2.2**(0.3)	**2.2**(0.3)	**2.2**(0.3)	**2.3**(0.2)	2.3(0.3)	15/15
IPOP-CM	3.3(0.5)	2.6(0.2)	2.4(0.2)	2.4(0.2)	2.3(0.3)	2.5(0.2)	2.4(0.2)	15/15
Var1	**2.9**(0.5)	**2.4**(0.3)	2.2(0.3)	2.3(0.4)	2.3(0.4)	2.3(0.3)	2.3(0.3)	15/15
Var2	3.2(0.4)	2.6(0.5)	2.4(0.3)	2.4(0.4)	2.4(0.3)	2.4(0.3)	2.4(0.2)	15/15
Var3	3.1(0.4)	2.5(0.3)	2.3(0.3)	2.3(0.2)	2.2(0.4)	2.3(0.4)	**2.3**(0.3)	15/15
f7	1351	4274	9503	16523	16524	16524	16969	15/15
BIPOP-C	2.0(1.0)	10(5)	7.0(2)	4.4(0.6)	4.4(0.5)	4.4(0.4)	4.3(0.5)	15/15
CMAES-A	2.7(5)	10(0.3)	4.9(0.3)	3.1(0.3)	3.1(0.3)	3.1(0.3)	3.1(0.2)	15/15
IPOP-CM	3.8(3)	10(2)	5.4(3)	3.4(1)	3.4(1)	3.4(1)	3.3(1)	15/15
Var1	**1.5**(0.2)	9.1(0.9)	4.6(0.5)	3.0(0.5)	3.0(0.5)	3.0(0.7)	3.0(0.5)	15/15
Var2	4.3(5)	9.2(0.9)	4.9(0.6)	3.2(0.4)	3.2(0.5)	3.2(0.6)	3.2(0.5)	15/15
Var3	5.4(9)	**8.9**(1)	**4.5**(0.8)	**2.9**(0.5)	**2.9**(0.2)	**2.9**(0.2)	**2.9**(0.4)	15/15
f8	2039	3871	4040	4148	4219	4371	4484	15/15
BIPOP-C	8.0(3)	8.1(2)	8.7(3)	9.0(1)	9.1(1)	9.1(3)	9.2(1)	15/15
CMAES-A	7.1(0.9)	13(1)	14(0.5)	14(44)	14(0.6)	14(0.9)	15(22)	15/15
IPOP-CM	7.5(2)	**7.8**(2)	**8.5**(2)	**8.7**(0.6)	**8.8**(1)	**8.9**(0.9)	**9.0**(1)	15/15
Var1	7.8(3)	13(1)	14(2)	14(1)	14(2)	14(1)	15(1)	15/15
Var2	7.8(3)	8.2(2)	8.8(1)	9.0(1)	9.1(1)	9.2(2)	9.3(1)	15/15
Var3	**7.1**(1)	13(20)	14(20)	14(20)	14(20)	14(1)	14(20)	15/15
f9	1716	3102	3277	3379	3455	3594	3727	15/15
BIPOP-C	9.4(3)	11(6)	12(1)	12(2)	12(8)	12(5)	12(5)	15/15
CMAES-A	**8.0**(2)	**9.3**(1)	**10**(2)	**10**(2)	**10**(1)	**10**(1)	**10**(1.0)	15/15
IPOP-CM	9.2(1)	11(0.9)	12(0.7)	12(5)	12(0.8)	12(0.6)	12(0.6)	15/15
Var1	10(3)	27(23)	28(44)	29(44)	28(44)	28(22)	28(1)	15/15
Var2	10(3)	28(1)	29(24)	30(2)	30(49)	30(44)	29(47)	15/15
Var3	8.3(1)	10(0.4)	10(0.6)	11(0.7)	11(0.7)	11(0.6)	11(0.6)	15/15
f10	7413	8661	10735	13641	14920	17073	17476	15/15
BIPOP-C	3.8(0.4)	3.7(0.3)	3.1(0.3)	2.5(0.2)	2.4(0.1)	2.2(0.1)	2.2(0.1)	15/15
CMAES-A	3.5(0.5)	3.6(0.1)	3.1(0.2)	2.6(0.1)	2.4(0.1)	2.2(0.1)	2.2(0.1)	15/15
IPOP-CM	**3.5**(0.5)	**3.5**(0.4)	**3.1**(0.2)	**2.5**(0.1)	**2.4**(0.1)	2.2(0.1)	2.2(0.1)	15/15
Var1	3.7(0.4)	3.6(0.4)	3.1(0.1)	2.5(0.1)	2.4(0.0)	**2.2**(0.1)	**2.2**(0.1)	15/15
Var2	3.6(0.3)	3.7(0.2)	3.2(0.3)	2.6(0.1)	2.4(0.0)	2.2(0.1)	2.2(0.1)	15/15
Var3	3.7(0.4)	3.6(0.3)	3.2(0.3)	2.6(0.1)	2.4(0.1)	2.2(0.1)	2.2(0.1)	15/15
f11	1002	2228	6278	8586	9762	12285	14831	15/15
BIPOP-C	21(1)	10(0.3)	3.9(0.2)	3.0(0.1)	2.8(0.1)	2.4(0.1)	2.1(0.0)	15/15
CMAES-A	**20**(2)	**10**(0.9)	**3.8**(0.2)	**3.0**(0.1)	**2.7**(0.1)	2.3(0.0)	2.0(0.1)	15/15
IPOP-CM	22(6)	11(2)	4.1(0.2)	3.2(0.5)	2.9(0.5)	2.4(0.1)	2.1(0.1)	15/15
Var1	20(2)	10(0.7)	3.8(0.2)	3.0(0.1)	2.7(0.1)	**2.3**(0.1)	**2.0**(0.0)	15/15
Var2	21(2)	10(0.4)	3.9(0.1)	3.0(0.1)	2.8(0.2)	2.4(0.1)	2.1(0.1)	15/15
Var3	20(2)	10(0.3)	3.8(0.1)	3.0(0.1)	2.7(0.1)	2.3(0.1)	2.0(0.0)	15/15
f12	1042	1938	2740	3156	4140	12407	13827	15/15
BIPOP-C	6.1(0.2)	7.9(11)	9.0(5)	10(6)	8.9(5)	3.9(1)	4.1(2)	15/15
CMAES-A	5.2(0.2)	6.4(5)	9.0(6)	10(5)	8.9(2)	3.9(0.9)	4.0(1)	15/15
IPOP-CM	10(7)	11(8)	11(8)	12(5)	10(6)	4.3(2)	4.4(2)	15/15
Var1	**4.5**(3)	**4.7**(3)	**5.7**(5)	**6.7**(3)	**6.5**(1)	**3.0**(0.9)	**3.2**(0.7)	15/15
Var2	7.4(6)	9.0(8)	10(6)	10(4)	9.1(3)	3.9(2)	4.0(2)	15/15
Var3	5.5(4)	8.1(5)	8.9(4)	9.3(4)	8.5(3)	3.6(1.0)	3.8(0.8)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f13	652	2021	2751	3507	18749	24455	30201	15/15
BIPOP-C	8.5(7)	**5.5**(4)	**10**(11)	12(9)	3.0(1)	4.5(3)	6.0(3)	15/15
CMAES-A	17(27)	16(15)	18(12)	18(10)	5.5(3)	6.8(3)	6.6(4)	15/15
IPOP-CM	13(10)	10(10)	12(9)	**10**(7)	**2.7**(2)	**3.4**(2)	**4.5**(2)	15/15
Var1	7.0(1)	13(9)	18(10)	20(12)	5.8(2)	6.7(3)	6.0(0.7)	15/15
Var2	7.8(11)	21(16)	22(11)	22(4)	5.5(3)	5.4(2)	5.4(0.3)	15/15
Var3	**6.3**(6)	10(15)	16(10)	18(13)	4.7(3)	5.6(2)	5.5(0.6)	15/15
f14	75	239	304	451	932	1648	15661	15/15
BIPOP-C	7.8(1)	5.8(0.8)	7.4(1)	8.6(1)	8.3(0.6)	12(0.7)	2.3(0.1)	15/15
CMAES-A	5.3(2)	5.0(1.0)	**6.3**(1)	8.0(0.4)	8.0(1)	12(2)	2.3(0.2)	15/15
IPOP-CM	7.4(3)	5.7(1)	7.1(2)	8.5(2)	7.9(1.0)	12(1)	2.3(0.1)	15/15
Var1	5.4(3)	**4.8**(0.5)	6.5(0.7)	**7.7**(0.7)	**7.6**(1)	12(0.7)	2.3(0.2)	15/15
Var2	**5.0**(1.0)	4.9(0.7)	6.5(1)	8.0(2)	7.9(1)	12(1)	2.4(0.1)	15/15
Var3	6.4(2)	5.0(0.6)	6.8(0.5)	7.8(0.9)	7.9(0.6)	**12**(1)	**2.3**(0.2)	15/15
f15	30378	1.5e5	3.1e5	3.2e5	3.2e5	4.5e5	4.6e5	15/15
BIPOP-C	**2.0**(0.6)	4.1(1)	2.8(1.0)	2.8(1)	2.8(0.8)	2.0(0.7)	2.0(0.7)	15/15
CMAES-A	5.5(0.8)	1.2(0.1)	0.59(0.1)	0.59(0.1)	0.59(0.1)	0.43(0.1)	0.43(0.1)	15/15
IPOP-CM	2.1(1)	2.1(1)	1.4(0.3)	1.4(0.5)	1.4(0.7)	1.0(0.3)	1.1(0.5)	15/15
Var1	5.4(0.5)	1.2(0.1)	0.60(0.1)	0.61(0.1)	0.61(0.1)	0.45(0.1)	0.45(0.1)	15/15
Var2	5.5(0.4)	1.2(0.2)	0.61(0.1)	0.62(0.1)	0.62(0.1)	0.46(0.0)	0.46(0.0)	15/15
Var3	5.1(0.7)	**1.1**(0.2)	**0.55**(0.1)	**0.56**(0.1)	**0.56**(0.1)	**0.41**(0.1)	**0.42**(0.1)	15/15
f16	1384	27265	77015	1.4e5	1.9e5	2.0e5	2.2e5	15/15
BIPOP-C	**3.4**(0.9)	2.0(1)	2.3(1)	2.0(2)	2.0(1)	**2.0**(2)	**2.0**(1)	15/15
CMAES-A	43(151)	37(16)	16(10)	9.0(7)	6.7(4)	7.1(2)	6.4(2)	14/15
IPOP-CM	3.4(1)	**1.6**(1)	**1.8**(1)	**1.7**(0.7)	**1.7**(0.5)	2.1(0.7)	2.1(1.0)	15/15
Var1	65(233)	39(35)	15(8)	8.3(5)	6.4(3)	6.1(3)	5.5(4)	15/15
Var2	3.6(1)	47(31)	17(10)	10(7)	7.2(5)	7.6(5)	6.9(4)	15/15
Var3	16(0.9)	37(23)	16(10)	9.1(4)	7.5(4)	7.2(2)	6.5(3)	15/15
f17	63	1030	4005	12242	30677	56288	80472	15/15
BIPOP-C	4.3(3)	2.0(0.5)	**2.0**(2)	**2.0**(2)	2.5(3)	2.7(0.9)	2.7(2)	15/15
CMAES-A	2.7(2)	**1.7**(0.2)	2.5(12)	5.1(4)	3.3(0.5)	2.4(0.4)	**1.9**(0.3)	15/15
IPOP-CM	4.3(3)	1.9(0.5)	2.4(4)	2.0(0.4)	**1.5**(1.0)	**2.0**(0.7)	2.0(1)	15/15
Var1	**1.9**(1)	2.0(1.0)	6.3(10)	6.0(4)	3.4(0.3)	2.5(0.2)	2.0(0.3)	15/15
Var2	2.5(1)	1.8(0.9)	6.3(6)	7.7(1.0)	3.8(0.3)	2.8(0.5)	2.4(0.2)	15/15
Var3	2.8(4)	1.9(0.3)	4.1(0.4)	6.3(4)	3.3(2)	2.6(0.3)	2.1(0.4)	15/15
f18	621	3972	19561	28555	67569	1.3e5	1.5e5	15/15
BIPOP-C	2.1(0.6)	4.8(9)	2.5(2)	3.2(3)	2.2(0.7)	3.3(1)	3.2(0.8)	15/15
CMAES-A	1.8(0.4)	6.0(10)	3.6(0.7)	3.6(0.4)	**1.7**(0.3)	**1.1**(0.2)	**1.2**(0.4)	15/15
IPOP-CM	2.2(0.5)	3.6(4)	**2.2**(2)	**3.1**(1)	1.9(0.3)	2.1(0.7)	2.1(0.8)	15/15
Var1	1.7(0.4)	**2.1**(0.1)	4.3(0.3)	3.7(0.4)	1.8(0.3)	1.2(0.2)	1.3(0.1)	15/15
Var2	**1.5**(0.3)	3.1(4)	4.2(0.5)	3.7(0.2)	1.8(0.2)	1.3(0.1)	1.3(0.1)	15/15
Var3	1.7(0.2)	2.2(9)	4.4(1)	3.7(0.5)	1.8(0.2)	1.2(0.1)	1.3(0.1)	15/15
f19	1	1	3.4e5	4.7e6	6.2e6	6.7e6	6.7e6	15/15
BIPOP-C	338(128)	4.8e4(1e4)	2.4(2)	2.0(0.6)	2.0(0.6)	2.0(0.6)	2.0(0.3)	15/15
CMAES-A	**2.0**(0)	**2.0**(0)	1.8(0.4)	**0.51**(0.3)	**0.63**(0.4)	1.00(0.6)	0.99(0.9)	4/15
IPOP-CM	322(66)	5.5e4(4e4)	**1.4**(1.0)	0.90(0.3)	0.76(0.2)	**0.82**(0.2)	**0.83**(0.2)	15/15
Var1	**2.0**(0)	**2.0**(0)	2.0(0.5)	0.77(0.8)	0.78(2)	0.90(0.6)	0.90(1.0)	8/15
Var2	**2.0**(0)	**2.0**(0)	2.1(0.6)	0.78(0.5)	0.87(0.8)	1.4(2)	1.4(1)	6/15
Var3	**2.0**(0)	**2.0**(0)	1.7(0.4)	0.94(1)	0.86(1)	1.2(0.9)	1.2(2)	7/15
f20	82	46150	3.1e6	5.5e6	5.5e6	5.6e6	5.6e6	14/15
BIPOP-C	8.5(2)	18(8)	2.0(0.6)	1.9(1)	1.9(0.6)	1.9(0.6)	1.9(1)	14/15
CMAES-A	7.4(2)	25(7)	**0.69**(0.4)	**0.68**(0.3)	**0.68**(0.6)	**0.68**(0.2)	**0.68**(0.5)	9/15
IPOP-CM	9.1(3)	**13**(4)	1.3(0.4)	1.1(0.2)	1.1(0.4)	1.2(0.4)	1.2(0.4)	15/15
Var1	**7.1**(0.9)	22(9)	0.90(0.5)	0.88(0.5)	0.88(0.7)	0.87(0.9)	0.87(0.5)	12/15
Var2	7.7(0.9)	22(11)	0.89(0.3)	0.96(0.6)	0.96(0.6)	0.95(0.5)	0.95(0.9)	12/15
Var3	7.5(1)	24(7)	0.87(0.3)	0.85(0.5)	0.85(0.8)	0.85(0.6)	0.84(0.5)	11/15
f21	561	6541	14103	14318	14643	15567	17589	15/15
BIPOP-C	**6.3**(0.7)	**84**(168)	**67**(100)	**67**(136)	**65**(79)	**62**(182)	**55**(82)	13/15
CMAES-A	8.0(26)	255(331)	151(218)	149(76)	146(73)	138(161)	122(289)	8/15
IPOP-CM	7.5(18)	145(123)	114(235)	113(146)	110(179)	104(134)	92(137)	7/15
Var1	8.5(13)	664(1079)	392(748)	386(666)	378(747)	355(484)	315(523)	7/15
Var2	8.0(14)	748(1377)	594(780)	586(1499)	573(618)	539(784)	477(978)	5/15
Var3	54(24)	717(1206)	440(756)	433(369)	424(256)	399(527)	353(713)	7/15
f22	467	5580	23491	24163	24948	26847	1.3e5	12/15
BIPOP-C	**14**(28)	**26**(31)	**225**(101)	**219**(338)	**212**(281)	**197**(231)	**39**(76)	5/15
CMAES-A	164(890)	2353(1714)	∞	∞	∞	∞	∞ *2e6*	0/15
IPOP-CM	460(36)	312(402)	∞	∞	∞	∞	∞ *1e6*	0/15
Var1	28(31)	1437(1796)	∞	∞	∞	∞	∞ *4e6*	0/15
Var2	128(415)	1437(1613)	∞	∞	∞	∞	∞ *4e6*	0/15
Var3	23(33)	1080(2869)	∞	∞	∞	∞	∞ *4e6*	0/15
f23	3.0	1614	67457	3.7e5	4.9e5	8.1e5	8.4e5	15/15
BIPOP-C	9.2(9)	**64**(48)	**2.0**(2)$^{\star 2}$	**3.5**(4)$^{\star 4}$	**3.9**(3)$^{\star 4}$	**2.4**(1.0)$^{\star 4}$	**2.4**(0.8)$^{\star 4}$	15/15
CMAES-A	3.3(2)	367(800)	416(379)	∞	∞	∞	∞ *2e6*	0/15
IPOP-CM	9.1(14)	2.4e4(3e4)	∞	∞	∞	∞	∞ *3e6*	0/15
Var1	4.7(8)	205(21)	831(712)	∞	∞	∞	∞ *4e6*	0/15
Var2	**2.8**(2)	649(1245)	386(312)	∞	∞	∞	∞ *4e6*	0/15
Var3	5.8(3)	403(1249)	831(1039)	∞	∞	∞	∞ *4e6*	0/15
f24	1.3e6	7.5e6	5.2e7	5.2e7	5.2e7	5.2e7	5.2e7	3/15
BIPOP-C	**2.0**(2)	**1.6**(2)	**1.2**(1)	**1.2**(0.6)	**1.2**(1)	**1.2**(0.5)	**1.2**(0.8)	3/15
CMAES-A	∞	∞	∞	∞	∞	∞	∞ *2e6*	0/15
IPOP-CM	∞	∞	∞	∞	∞	∞	∞ *5e6*	0/15
Var1	∞	∞	∞	∞	∞	∞	∞ *4e6*	0/15
Var2	∞	∞	∞	∞	∞	∞	∞ *4e6*	0/15
Var3	∞	∞	∞	∞	∞	∞	∞ *4e6*	0/15

Table 4: Average runtime (aRT in number of function evaluations) divided by the respective best aRT measured during BBOB-2009 in dimension 20. The aRT and in braces, as dispersion measure, the half difference between 10 and 90%-tile of bootstrapped run lengths appear for each algorithm and target, the corresponding reference aRT in the first row. The different target Δf-values are shown in the top row. #succ is the number of trials that reached the (final) target $f_{\mathrm{opt}} + 10^{-8}$. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries, succeeded by a star, are statistically significantly better (according to the rank-sum test) when compared to all other algorithms of the table, with $p = 0.05$ or $p = 10^{-k}$ when the number k following the star is larger than 1, with Bonferroni correction by the number of functions (24). A ↓ indicates the same tested against the best algorithm from BBOB 2009. Best results are printed in bold.

Data produced with COCO v2.2.1

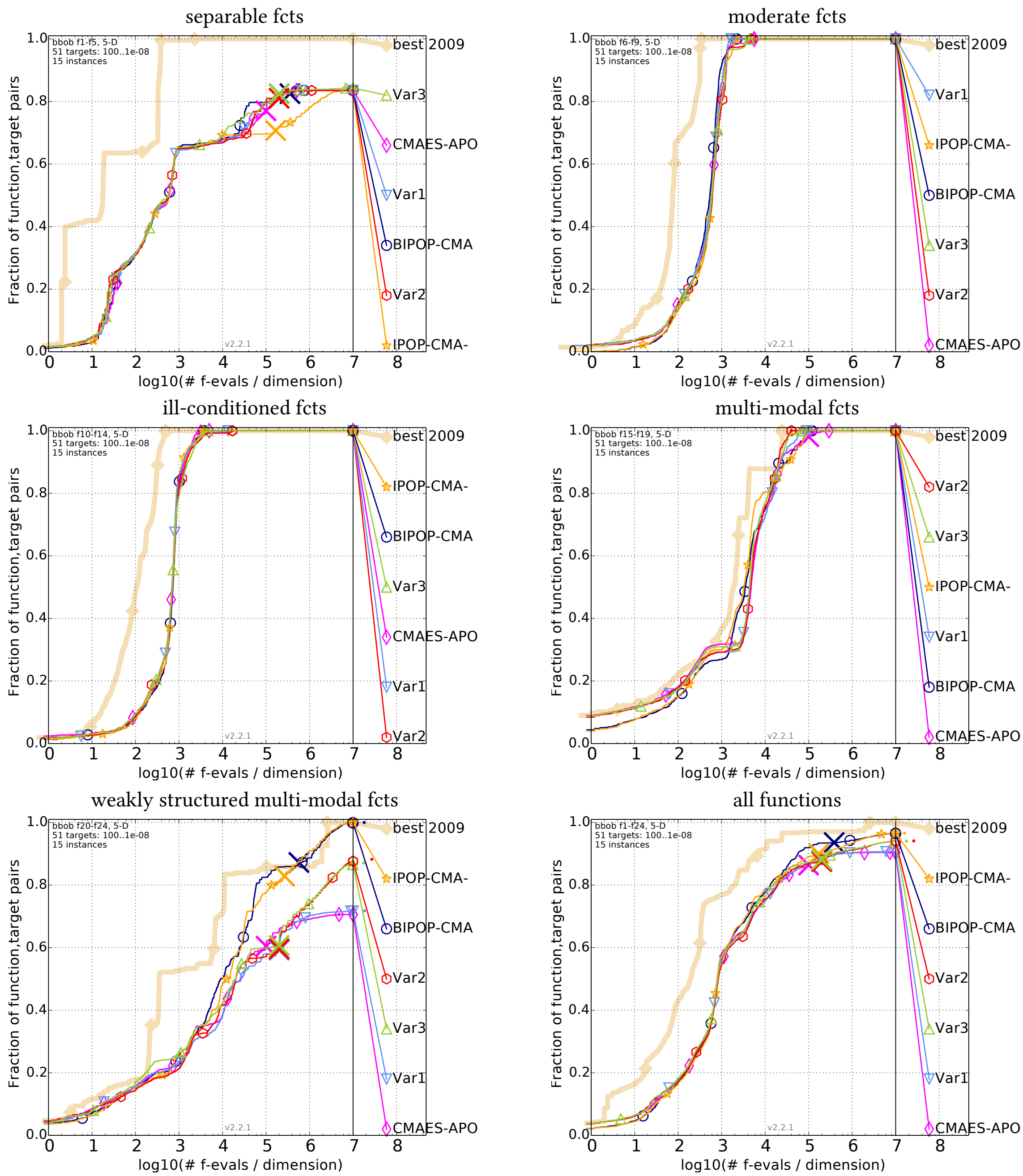

Figure 2: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 51 targets with target precision in $10^{[-8..2]}$ for all functions and subgroups in 5-D. As reference algorithm, the best algorithm from BBOB 2009 is shown as light thick line with diamond markers.

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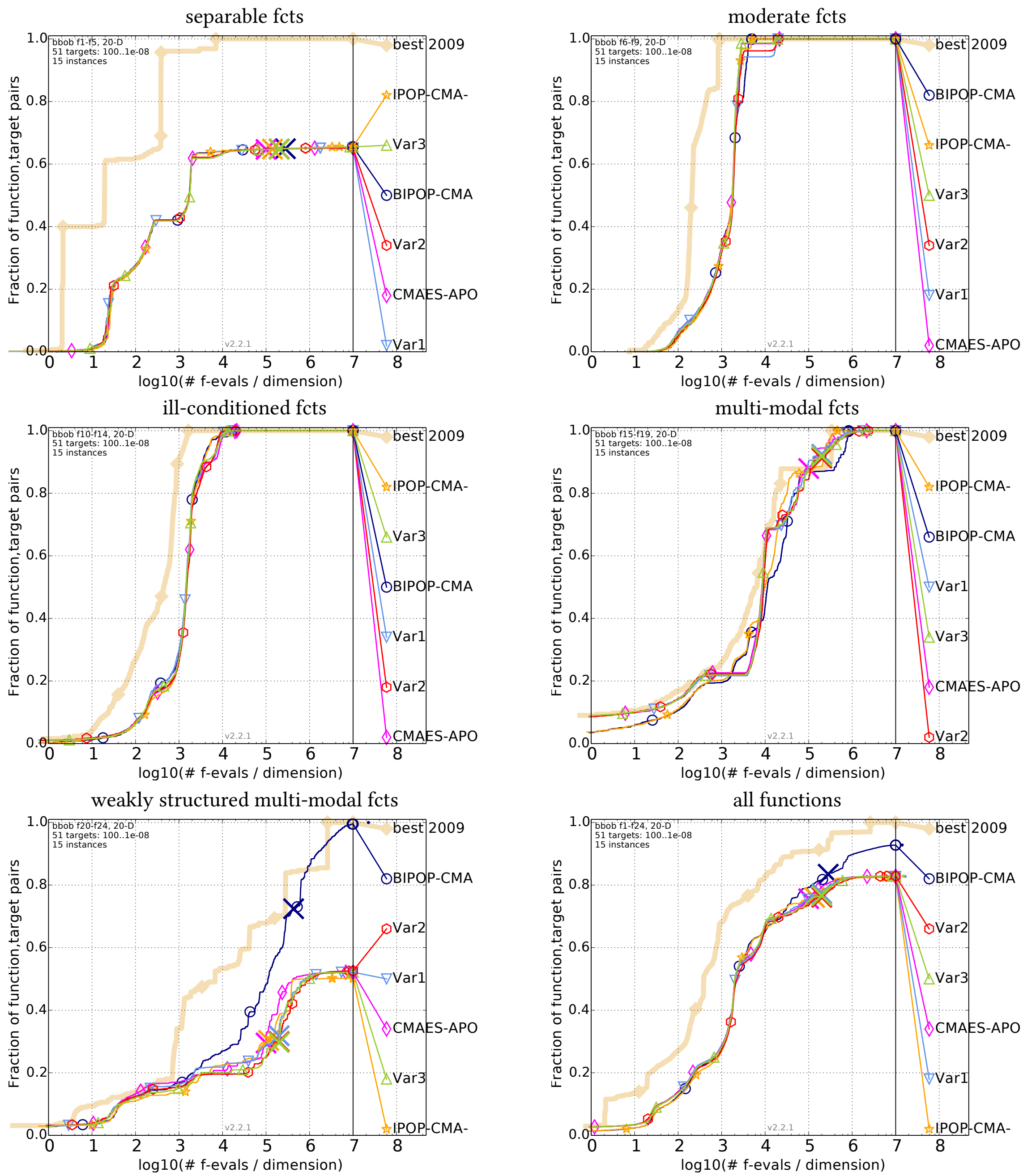

Figure 3: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 51 targets with target precision in $10^{[-8..2]}$ for all functions and subgroups in 20-D. As reference algorithm, the best algorithm from BBOB 2009 is shown as light thick line with diamond markers.

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