Selective Pressure in Constrained Differential Evolution*

Abstract†

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ABSTRACT

Differential Evolution (DE) is a highly competitive numerical optimization method for constrained optimization problems. In this study, a new selective pressure technique is applied in DE, which considers both function values and constraint violations with respect to ε-constraint level. The new modification was tested against known variants of constrained DE, and it is shown that selective pressure allows significant improvement of algorithm performance in various application scenarios.

CCS CONCEPTS

• **Mathematics of computing~Bio-inspired optimization** • **Theory of computation~Evolutionary algorithms** • **Computing methodologies~Search methodologies;**

KEYWORDS

Differential evolution, optimization, constrained optimization, selective pressure, evolutionary algorithm.

1 INTRODUCTION

The development 1 of evolutionary algorithms and heuristic methods based on the idea of evolution followed several key principles, such as selection of individuals with higher fitness, crossover between individuals and random mutations. Today most nature-inspired and evolutionary algorithms (EA) use these or other similar operations to find better solutions. In this study, the selective pressure is applied to choose the individuals for mutation operation in DE. There have been only few studies, where the selective pressure was introduced in DE, for example, [1]. The constrained optimization setup is considered, so that the probability of an individual being chosen for mutation depends not only on fitness value, but also on constraints violation.

2 SELECTIVE PRESSURE FOR CONSTRAINED DIFFERENTIAL EVOLUTION

The goal of a constrained optimization problem (COP) is to find an optimum of the real-valued function $f : X \longrightarrow R$ inside the search space $X \subseteq R^n$, while satisfying a set of q inequality and m-q equality constraints:

$$\begin{cases} g_j(x) \le 0, j = 1, \dots, q \\ h_j(x) = 0, j = q+1, \dots, m \end{cases} \tag{1}$$

The minimization COP is considered, so that the goal is to find feasible solutions. There are several constraints handling techniques developed for EA. For classic DE, the constraints handling could be applied on the selection step, for example, by applying Deb's rules [2], however, the best results were achieved by applying the ε-constraint method, described in [3]. The selection procedure with ε-constraint method also uses Deb's rules to determine if the new vector should replace the old one:

$$x_{i,j}^{G+1} = \begin{cases} u_j^G \text{ if } \varphi(u_j^G) < \varphi(x_{i,j}^G) \text{ and } \varphi(x_{i,j}^G) \ge \varepsilon \\ u_j^G \text{ if } f(u_j^G) < f(x_{i,j}^G) \text{ and } \varphi(x_{i,j}^G) < \varepsilon \\ x_{i,j}^G \text{ otherwise} \end{cases} \tag{2}$$

where ε level is a relaxation parameter determining the acceptable violation, so that all individuals with violations smaller than ε are considered as feasible. The ε value is controlled as described in [3], but that number of function evaluations NFE is used instead of generation number; ε is updated every generation:

$$\varepsilon_0(G) = \varphi(x_\theta), \theta = \theta_0 \cdot \left(1 - \frac{NFE}{NFE_{max}}\right)^{cp}$$
$$\varepsilon(G) = \begin{cases} \varepsilon_0(G), NFE \le NFE_c \\ 0, NFE > NFE_c \end{cases} \tag{3}$$

where $\varepsilon_0(G)$ is the initial ε-level at generation G, determined as the violation value of θ-th top individual in the population sorted with Deb's rules; NFE_c is the limit, after which no violations are allowed (ε=0).

In genetic algorithms, typical selection mechanisms are: fitness proportional selection, rank-based selection and tournament selection, one of the early works on these methods is [4]. The rank-based selection defines an individuals' probability p_i using its rank in a sorted fitness array:

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[1]It is a datatype.

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$$p_i = \frac{\text{rank}_i}{\sum_{j=1}^{NP} \text{rank}_j} \quad \text{rank}_i = e^{\frac{k \cdot i}{NP}} \tag{4}$$

where $k>0$ is a parameter controlling the exponential function. The selective pressure is introduced before the mutation step, i.e. it changes the uniform probabilities used in DE to select indexes of individuals considering fitness and constraint violation. The DE mutation with exponential rank selection is written as follows:

DE/current-to-pbest/er/4:

$$v_j = x_{i,j} + F * (x_{pb,j} - x_{i,j} + x_{er1,j} - x_{er2,j}) \tag{5}$$

where *er/4* means that exponential function is used for ranking with *k=4*. The parameter adaptation scheme for *F*, *Cr* and *NP* was the same as described in the L-SHADE algorithm [5].

3 EXPERIMENTAL SETUP

The performance of the proposed L-SHADE-ReC algorithm with selective pressure was evaluated on the CEC 2017 constrained optimization test suite [6]. There were totally six variants of selective pressure used, including linear ranking and exponential ranking with k=1, 2, 3, 5, 7. The same computational resource was used, i.e. 20000*D* function evaluations. For each function, 25 independent runs were performed. The population size was NP = 10D, archive size NA = NP, initial index for ε calculation θ_0 = 0.8NP, cp = 3, NFE_c = 0.8$NFE_{\max}$. For parameter adaptation, initial memory cells values for F and Cr were set to 0.3 and 0.8 respectively, number of memory cells H = 5, Linear Population Size Reduction was used with $NP_{\min}$ = 4, p = 0.15.

Table 1: Comparison of algorithms with and without rank-based selective pressure, Mann-Whitney statistical test

SP type	10D	30D	50D	100D
LR	3/0/25	3/1/24	3/1/23	5/2/21
ER1	3/0/25	3/1/24	3/1/24	5/2/21
ER2	3/1/24	5/1/22	4/0/24	4/1/23
ER3	3/0/25	5/0/23	5/0/23	7/0/21
ER5	5/0/23	10/0/18	7/0/21	8/0/20
ER7	6/0/22	11/1/16	9/0/19	11/0/17

Table 2: Ranking of L-SHADE-ReC compared to other methods, ER7

DIM	CAL-SHADE [7]	LSHADE 44-IDE [10]	LSHADE 44 [9]	UDE [8]	LSHADE -ReC
10D	220.5	165.5	167.5	148.5	**138**
30D	217.5	183.5	155	148.5	**135.5**
50D	229	189.5	143.5	151.5	**126.5**
100D	233	171	151	173	**112**
Total	900	709.5	617	621.5	**512**

Table 1 presents the comparison of basic L-SHADE with ε-constraint handling and no selective pressure with algorithms with rank-based selective pressure. The values in Table 1 show a statistically significant difference between results obtained by algorithms, which is checked by Mann-Whitney two-tailed rank sum test with significance level p=0.01 and tie correction.

The three values in Table 1 are the number of statistically significant wins/losses/ties. To compare with other participants of the CEC 2017 the ranking procedure [6] was used. Table 2 contains final rankings for all dimensions with k=7. For comparison, four methods from the CEC 2017 competition were chosen, namely CAL-SHADE [7], UDE [8], LSHADE44 [9] and LSHADE44 + IDE [10]. L-SHADE-ReC-ER7 was able to achieve the best results for all dimensions.

4 CONCLUSIONS

In this paper the application of selective pressure for Differential Evolution was proposed for the case of solving constrained optimization problems. The implementation of selective pressure is relatively simple and does not require large amount of computations, however, as the presented results show, it allows significant improvements in various scenarios. Thus, the proposed rank-based selection mechanism, as well as any other, could be incorporated in any modern DE implementation for constrained optimization. Directions of further work include development of novel mutation strategies, which combine various selection mechanisms, as well as self-adaptation of selective pressure.

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