Identifying Variable Interaction Using Mutual Information of Multiple Local Optima

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ABSTRACT

Identifying the interaction of search variables of black-box optimization problem is beneficial for optimization task. However, very little research pay attention to the quality of information source, i.e. what information is beneficial for identifying the interactions between variables. In this paper, we propose a new method that utilizes multiple local optima as information sources to identify the interaction between variables. First, a multimodal optimization algorithm is used to search for multiple local optima of the optimization problem. Then, hierarchical clustering is used to cluster and discretize local optima. Finally, the interaction between variables is quantified using the mutual information of local optima. Experimental results on three 12-dimensional multimodal problems show that the proposed method can effectively identify the interactions among decision variables.

CCS CONCEPTS

• **Theory of computation** → *Bio-inspired optimization.*

KEYWORDS

variables interaction identification, multimodal optimization, local optima, mutual information

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1 INTRODUCTION

The interaction between variables in optimization problem is ubiquitous, and the complex variable interaction structure makes optimization problem more difficult to solve. If the interaction structure between search variables is identified, it can be applied to black-box optimization task as prior knowledge to improve the performance and efficiency of the optimization algorithm. For instance, the accurately identified interaction structure can bring positive effects to cooperative co-evolutionary (CC) algorithm and Bayesian optimization algorithm.

Some researches on the design of interaction measurement and interaction structure search method have been proposed. However, very little research pay attention to the quality of information source, i.e. what information is beneficial for identifying interactions between variables. Local optima are important features of the objective function, which may help reveal the hidden relations of the problem under study [3]. We find that the local optima of the optimization objective function are reliable information source which can be utilized to accurately identify the interaction between variables.

In this paper, a variable interaction identification method using only multiple optima is proposed. First, a multimodal optimization algorithm is employed to search for problem's multiple local optima. Then, using multiple local optima as information source, we calculate the mutual information which quantifies the interaction between variables. Experiments are carried out on test functions. Experimental results demonstrate the effectiveness of using multiple local optima to identify variables interaction.

2 INTERACTION STRUCTURE IN OPTIMIZATION PROBLEM

Given an optimization task, the interaction between variables refers that the setting of one variable can affect the shape of fitness landscape of other variables, including the location of the optimal solution. To identify the interaction structure between variables, techniques such as perturbation [4] and statistical learning [2] are usually used.

For optimization task, we have equation 1, which means the global optimum of a additive separable function is the optimum of every subcomponent.

$$\arg\min_{X} f(X) = (\arg\min_{X[P_1]} f_1(X[P_1]), \cdots, \arg\min_{X[P_i]} f_i(X[P_i])) \quad (1)$$

$X[Pi]$ denotes that a certain part of the full dimension of X. Decision variables in different partitions of the structure have no interaction, while the decision variables in the same partition are interactive.

3 THE PROPOSED METHOD

In our proposed method, one multimodal optimization algorithm is utilized to obtain multiple local optima which are regarded as information source to identify interaction structures. Then, multiple local optima are clustered and discretized in terms of single variable. Finally, we compute the mutual information between the

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local optima, and rank them according to mutual information. The ranking list updates when a new local optimum is found.

The framework of the proposed method is given in Algorithm 1. The Niching migratory multi-Swarm Optimizer (NMMSO)[1] is a multimodal optimization method which constantly executes and outputs the local optima found. When a new local optimum is found, multiple local optima will be clustered using hierarchical cluster. Then the mutual information of the multiple optima will be calculated and ranked. The top 10 interactions will be selected as identified interactions to generate a interactive graph.

Algorithm 1: Framework

```
while FE < MaxFes do
    [swarm, optima] ←NMMSO(swarm);
    if |optima| has changed then
        [labeled_optima] ←HierarchicalCluster(optima);
        [MI] ←MutualInfor(labeled_optima);
        [interaction_graph] ←TruncationSelection(MI);
    end
end
```

4 EXPERIMENTS

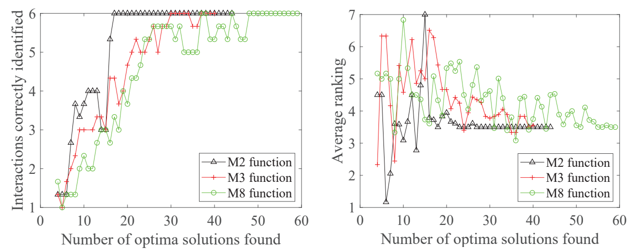
We consider the maximum of two quadratics (MTQ) function as benchmark function to verify our proposed method. The basic MTQ function in our experiment is formulated as:

$$MTQ(x_1, x_2) = \max \begin{cases} 70 * (1 - (x_1 - 0.2)^2 - (x_2 - 0.2)^2) \\ 75 * (1 - 7 * (x_1 + 0.6)^2 - 7 * (x_2 + 0.6)^2) \end{cases} \quad (2)$$

The MTQ function has one global optimum $(-0.6, -0.6)$ and one local optimum$(0.2, 0.2)$. The x_1 and x_2 are interactive. In our experiments, we add up six two-dimensional MTQ functions to form a 12-dimensional function and name it $M2$. In this way, there are 6 pairs of variables interacting with each other in the $M2$ function. In addition to the basic MTQ function, we extend the MTQ function to have more local optima. The extended MTQ functions have 3 and 8 local optima respectively, which are named $M3$ and $M8$. They are also 12-dimensional. $M3$ and $M8$ are used to verify the validity of our identification method for functions with more local optima in subspace.

We analyzed the performance of our method by the number of authentic interactions in the top 10 interactions and their average ranking. Figure 1(a) shows the curves of the number of authentic interaction ranked in top 10 by our method. We can see from the curves that the number of authentic interactions ranked in the top 10 basically increases, as the number of local optima we found increases. For all test functions, all six interactions can be successfully identified with a certain number of local optima.

Figure 1(b) gives the authentic interactions' average ranking in the top 10 of the list of all interactions. The higher the ranking, the stronger the interaction we suggest. When we get sufficient local optima information, the top-ranked interaction are authentic. In other words, with sufficient local optima, our approach is able



(a) Number of interactions correctly identified on different test functions

(b) Number of average ranking of authentic interactions identified on different test functions

Figure 1: Curves of results: the horizontal axis is the number of local optima found

to put the authentic interactions at 1st, 2nd, 3rd, 4th, 5th, and 6th, i.e. average ranking 3.5. The interactions between variables can be accurately identified by our method.

The performance of our proposed approach differs in different test functions. Compared with $M2$ and $M3$, $M8$ function needs more local optima to fully identify all variable interactions. We believe that when the number of local optima in a subspace increases, the whole function becomes more complex, and the local optima obtained becomes more diversified. Therefore, more information is needed before the interaction can be revealed.

5 CONCLUSIONS

We pioneered the issue that local optima are promising information sources for variable interactions identifying in black-box continuous optimization problems. We proposed a method to identify the interaction between variables using the mutual information of local optima in the black-box continuous problems. The experiment verifies that the proposed method can gradually identify the interaction structure between variables as more and more local optima are discovered.

As for future research, our method may interact with the optimization algorithm to improve each other's performance. Moreover, this approach is now only suitable for discovering the interactions structure of lower-dimensional problems. Further research is needed to extend its application to higher dimensional problems.

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