Simulated Annealing for the Single-Vehicle Cyclic Inventory Routing Problem

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ABSTRACT

This paper studies the Single-Vehicle Cyclic Inventory Routing Problem (SV-CIRP) with the objective of simultaneously minimizing distribution and inventory costs for the customers and maximizing the collected rewards. A subset of customers is selected for the vehicle, including the quantity to be delivered to them. Simulated Annealing (SA) is proposed for solving the problem. Experimental results on 50 benchmark instances show that SA is comparable to the state-of-the-art algorithms. It is able to obtain 12 new best known solutions.

CCS CONCEPTS

• **Theory of computation → Approximation algorithms analysis**; **Simulated annealing**;

KEYWORDS

Metaheuristics, Simulated Annealing, Cyclic Inventory Routing Problem

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1 INTRODUCTION

In the traditional supply chain context, supplier and customers independently make their decisions on replenishing the inventory. Vendor Managed Inventory (VMI) was introduced later [5] to synchronize different activities. The Inventory Routing Problem (IRP), which is a VMI problem [4], considers both managing the inventories at the customers and distributing products from a central depot to the customers. The supplier manages the routing of vehicles distributing the products and the timely replenishment of inventories at the customers as well [1]. Cyclic IRP (CIRP) is a variant of IRP where customer demand rates are stable with an infinite planning horizon. The main objective is to find a cyclic distribution plan for a set of customers with the objective of minimizing long-term transportation and inventory costs [3]. The vehicles are allowed to perform multiple trips on a single day.

In Single-Vehicle CIRP (SV-CIRP), a special case of CIRP, visiting all customers is not compulsory and a vehicle is allowed to make multiple trips from the depot within one cycle. The cycle time represents the time between two deliveries to each customer. A reward occurs for each visited customer. The objective is to simultaneously minimize transportation and inventory costs and maximize the collected rewards from visited customers [7]. Simulated Annealing (SA) is designed to solve the problem.

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2 THE SV-CIRP

We consider an undirected network graph $G = (S^+, A)$, where $S^+ = \{0, 1, 2, \ldots, |S|\}$ is the set of nodes, and $A = \{(i, j) : i \neq j \in S^+\}$ refers to the set of arcs connecting two different nodes i and j. Let $S = S^+ \setminus \{0\}$ be a set of potential customers. Node 0 represents the depot. Each customer j has an inventory cost η_j (price/(time×quantity)), a handling cost ϕ_j (price), a demand rate d_j (quantity/time) and a fixed reward λ_j (price/time). The reward corresponds to the collected profit when the customer is selected for replenishment. The travel time from customer i to customer j is represented by t_{ij}. The vehicle is assumed to have a fixed operating cost ψ (price/time), a fixed average vehicle speed v (distance/time), the travel cost δ (price/distance) and the vehicle capacity κ (quantity). It is assumed that each customer has an infinite inventory capacity. The largest possible quantity that can be delivered to customer j is denoted as ${Q_j}^{max}$, which is calculated as:

$$ {Q_j}^{max} = d_j \times \frac{\kappa}{\min_{j \in S}\{d_j\}} \tag{1} $$

The objective is to minimize the total cost minus the total collected reward. The total cost consists of the transportation, delivery and holding costs. Due to the multiple trips a vehicle can make during one cycle, each one starting from the depot, the total travel time per cycle T is determined by the sum of all trip travel times. Each trip should respect the vehicle capacity.

3 SIMULATED ANNEALING

We calculate $ratio_{jj'}$ between nodes j and j' (Equation 2), which represents the collected reward per distance travelled. We then create a solution representation - a permutation of $|S|$ customers (Figure 1). Customers are sorted in descending order according to ratio values, starting from $j = 0$, until all customers are included. Since it is not mandatory to visit all customers, we select a subset of the visited customers, represented by shaded cells in Figure 1.

$$ratio_{jj'} = \frac{\lambda_{j'}}{t_{jj'} \times v} \quad \forall j, j' \in S^+ \tag{2}$$

Figure 1: An example of a solution representation

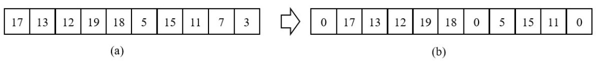

Figure 2: Decoding results (a); final route (b)

The solution representation is decoded into a sequence of possible visited customers (2(a)). T is determined by finding the lower and upper bound values, T_{min} and T_{max}[4]. T_{min} is the total travel time required to visit all selected customers, while T_{max} is calculated by Equation 3. Let S' be the set of selected customers. When $T_{min} > T_{max}$, we recalculate T_{min} and T_{max} after removing the customer with the highest demand rate until $T_{min} < T_{max}$. The ideal cycle time (the "economic-order-quantity"), T_{EOQ} [7], is calculated by Equation 4. There are three possible values of T: $T = T_{EOQ}$ if $T_{min} \leq T_{EOQ} \leq T_{max}$. If $T_{EOQ} < T_{min}$, then $T = T_{min}$. If $T_{EOQ} > T_{max}$, then $T = T_{max}$. The vehicle starts the delivery from the depot and sends as much as $d_j T$ to the selected customer j. Due to the vehicle capacity constraint, the vehicle may return to the depot to replenish its capacity and continue the trip, as long as the total trip time does not exceed T. Figure 2(b) illustrates the final route. Customers 7 and 3 cannot be visited due to the time limitation, therefore, they are not shaded.

$$T_{max} = \frac{\kappa}{max_{j \in S'} \, d_j} \tag{3}$$

$$T_{EOQ} = \sqrt{\frac{\delta \times v \times T_{min} + \sum_{j \in S'} \phi_j}{\sum_{j \in S'} \frac{d_j \times \eta_j}{2}}} \tag{4}$$

We propose SA with a random neighborhood structure that features various types of moves: Swap, Insert, Inverse, Remove and Add. Swap is performed by selecting two customers and exchanging their positions. Insert is done by selecting one customer and inserting it into the position before another randomly selected customer. Inverse is conducted by selecting two customers and reversing the positions of customers between them. Add is applied by selecting one non-shaded customer randomly and converting it into shaded customer. Remove is the opposite of Add. Other SA parameters, e.g. initial temperature, are determined to ensure the solution quality.

4 EXPERIMENTAL RESULTS

Five sets of benchmark instances are available on https://www.mech.kuleuven.be/en/cib/op#section-35. SA was coded in C++ and all experiments were executed on a computer with Intel Core i7-6700 CPU @ 3.40 GHz processor, 16.0 GB RAM. Each instance is solved five times. The results are compared with the best known solutions (BKs), taken from state-of-the-art algorithms: the Steepest Descent Hybrid Algorihm (SDHA) [6], Iterated Local Search (ILS) [7], Iterated Local Search (ILS) [4] and the convex optimization [2].

Dataset	BKs		SA			
	Average Cost	Average CPU (s)	Average Cost	Average CPU (s)	Average Gap (%)	New BKS
Set 1	-346.16	1.6	-346.16	12.8	0.00	0
Set 2	-740.73	144.7	-739.26	18.9	0.21	0
Set 3	-2182.35	1501.2	-2171.97	85.3	0.48	1
Set 4	-1041.24	6097.3	-1057.86	405	-1.46	5
Set 5	-1366.04	5858.9	-1372.32	414.1	-0.39	6

Table 1: Experimental Results

We only summarize the average results in Table 1. For Set 1, SA is able to obtain the optimal solutions. For Set 2, SA results are slightly worse than BKs. SA performs better in solving some instances compared to the results of ILS [4], at the cost of the computational time. For Set 3, SA performs better than ILS [4] in solving four instances: A25-1, A25-2, A25-6 and A25-8. SA improves one best known solution, A25-8. The BKs for Set 4 are obtained by the convex optimization [2]. SA is able to improve 5 BKs. For Set 5, SA also improves 6 solutions of the convex optimization. The best known solutions are further improved by 0.39%, on average, with 6 new BKs.

5 CONCLUSIONS

This work proposes Simulated Annealing to solve the SV-CIRP. The experimental results show that SA is comparable to the state-of-the-art algorithms. It finds 12 new BKs. We observe that the computational time remains limited, especially for larger instances. Some possible paths for further research include split deliveries and multiple vehicles.

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