A Decomposition-based EMOA for Set-based Robustness

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ABSTRACT

A large number of real-world multi-objective problems are subject to uncertainty, leading to the need to develop optimization methods that deal with it appropriately. In this work, we extend the decomposition-based algorithm to search for set-based robust solutions with the achievement scalarizing functions. Our preliminary work shows promising results that indicate our approach is able to outperform state-of-the-art algorithms that aim for set-based robust solutions.

CCS CONCEPTS

• **Theory of computation** → **Evolutionary algorithms**; **Non-convex optimization**; *Numeric approximation algorithms*;

KEYWORDS

Multi-objective optimization, robustness, decomposition

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1 INTRODUCTION

When facing a real-world problem, the decision maker may not always be interested in the best solutions, in particular, if these solutions are subject to uncertainty. Thus, there exists an additional challenge. One has to search not only for solutions with a good performance but also that the solutions are capable of coping with uncertainty, leading to the so-called set-based robustness (SBR) [1].

SBR is highly relevant when the decision maker has an aversion towards uncertainty and would like to study the problem from a worst-case perspective (e.g., critical applications).

To the best knowledge of the authors, there exist only two evolutionary approaches that aim for SBR. Namely, extension for NSGA-II [3] and SMS-EMOA [6]. In this work, we extend decomposition-based EMOAs to the problem at hand. Our preliminary results indicate that our approach can outperform the state-of-the-art algorithms in the Lamé superspheres problems (LSS). We measured the performance of the algorithms in terms of the Δ_2 indicator [4] in decision space.

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2 SET-BASED ROBUSTNESS

First, we define an uncertain multi-objective optimization problem (UMOP). Here, we assume that the decision variables are given from the set $Q \subset \mathbb{R}^n$ and the uncertainties in the problem formulation are given as scenarios from a known uncertainty set $U \subseteq \mathbb{R}^m$. It is also assumed that $F : Q \times U \to \mathbb{R}^k$.

Definition 2.1. An UMOP $P(U) := (P(\xi), \xi \in U)$ is defined as the family of parametrized problems $P(\xi) := \min_{x \in Q} F(x, \xi)$, where $F : Q \times U \to \mathbb{R}^k$ and $Q \subseteq \mathbb{R}^n$.

In [1], the authors proposed a set-based definition for min-max robust efficiency. Here, for a given feasible solution x, the worst case of the objective vector is interpreted as a set, namely the supremum of the maximization multi-objective problem of the objective function over the uncertainty set.

Definition 2.2 ([1]). Given an UMOP $P(U)$, a feasible solution $\bar{x} \in Q$ is called set-based minmax robust efficient (re) if there is no $x' \in Q \backslash \{\bar{x}\}$ such that $F_U(x') \subseteq F_U(\bar{x}) - \mathbb{R}^k_{\geq}$, where $F_U(x) = \{F(x, \xi) : \xi \in U\}$.

The robust counterpart of an UMOP is the problem of identifying all $\xi \in U$ which are re. Thus, the robust counterpart problem can be defined as

$$\min_{x \in Q} \sup_{\xi \in U} F(x, \xi), \tag{1}$$

In the following, we assume that the $\max_{\xi \in U} F(x, \xi)$ exists for all $x \in Q$.

3 OUR PROPOSED APPROACH

One of the most popular approaches to solve an MOP with EMOAs is to use a decomposition approach [5]. This allows using a family of scalarizing functions to solve the problem. In this work, we extend the decomposition methods to SBR by employing the achievement scalarizing function (AASF) [7]. Note that any other scalarizing function for set-based robustness could be used in our proposed framework.

The key aspect of our proposed set-based robustness decomposition EMOA (SBR-D-EMOA) is the evaluation of the individuals (Algorithm 1). To assess the fitness Py, we evaluate all individuals Px with each weight w for a reference point Z, a penalty factor $\rho = 1 \times 10^{-4}$, and then we select the best individual for each weight. Px corresponds to the classical population in a MOEA and each

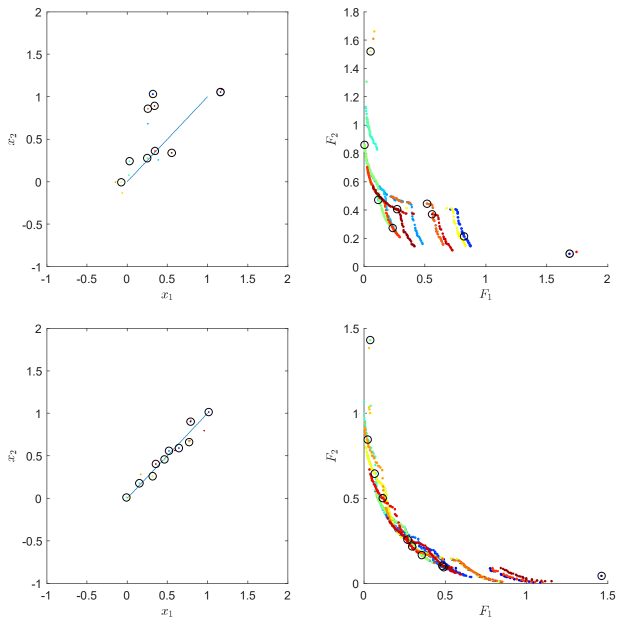

Figure 1: Step by step of the method in decision space (left) and objective space (right) for generations 25 and 50 (from top to bottom).

component $Py_{i,j}$ for $i, j = 1, \ldots, N$ measures the fitness of an individual i and the scalarizing function with the weight j.

Algorithm 1 Individuals Evaluation

Require: $F = [f_1, \ldots, f_k], Px \in \mathbb{R}^{N \times n}, \mathbb{R}^m, W \in \mathbb{R}^{N \times k}, Z \in \mathbb{R}^k$
Ensure: $Py \in \mathbb{R}^{N \times N}$
$Py = 0$
for all $x \in Px$ **do**
for all $w \in W$ **do**
$$Py_{i,j} = \max_{\xi \in U} \max_{i=1,\ldots,k} [w_i(f_i(x,\xi) - Z_i)] + \rho \sum_{i=1}^{k} (|F_i(x,\xi) - Z_i|)$$
end for
end for

4 NUMERICAL RESULTS

We compare our approach on the LSS problems with $k = 2$ [2] and decision uncertainty, $-0.2 \leq \xi_1, \xi_2, \leq 0.2$. For these problems, the nominal and the SBR efficient solutions are the same. For all algorithms, we used a budget of 5,000 function evaluations, a population of 10, uniform crossover and Gaussian mutation ($pm = 0.2$ and $\sigma = 0.2$). Further, we used 10 subproblems, and 20 independent executions.

Figure 1 shows the state of a representative execution of our approach for generations 25 and 50. Left shows decision space and the right shows objective space. Figure 2 shows the convergence graph of the execution in terms of Δ_2 in decision space and the mean of AASF.

Table 1 shows the mean and standard deviation of our approach and those from [3, 6] of the Δ_2 indicator in decision space to measure the distance of the real solution and the one from the algorithm. The arrows represent if our algorithm significantly outperforms (↑), is outperformed (↓) or is indistinct (↔) according to the Wilcoxon rank sum test with confidence of 95%.

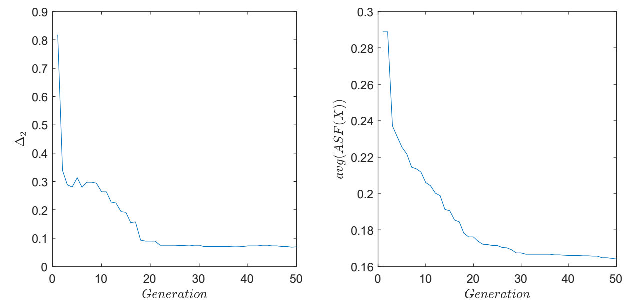

Figure 2: Convergence graph of the method measured with Δ_2 in decision space (left) and the mean of the AASF (right).

Table 1: Δ_2 comparison in decision space of SBR-D-EMOA.

Problem	SBR-D-EMOA	NSGA-II [3]	SMS-EMOA [6]
$\alpha = 0.25$	0.1264 (0.0154)	0.1139 (0.0272)↔	0.2054 (0.0595)↑
$\alpha = 0.5$	0.0825 (0.0092)	0.1011 (0.0112)↑	0.2106 (0.0601)↑
$\alpha = 1$	0.0702 (0.0070)	0.0878 (0.0161)↑	0.1964 (0.0567)↑

5 CONCLUSIONS AND FUTURE WORK

In this work, we presented a decomposition-based approach for SBR. Our approach has shown the benefits of using a decomposition based approach when dealing with UMOPs. Further, the approach has shown significantly better results than those in the start-of-the-art. However, it is still required to test the approach in more complex problems as well as to apply it to real-world applications.

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