A Hybrid Between a Surrogate-Assisted Evolutionary Algorithm and a Trust Region Method for Constrained Optimization

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ABSTRACT

This paper develops a hybrid between an RBF-assisted Evolutionary Programming (EP) algorithm and the CONORBIT trust region method for constrained expensive black-box optimization. The proposed hybrid combines the advantages of each approach and results in better performance than either the RBF-assisted EP or CONORBIT alone on test problems from the CEC 2010 benchmark.

CCS CONCEPTS

• **Mathematics of computing → Nonconvex optimization**; **Bio-inspired optimization**;

KEYWORDS

Constrained optimization, surrogate-assisted evolutionary algorithm, trust region, hybrid methods, radial basis function

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1 INTRODUCTION

Many surrogate-assisted evolutionary and swarm algorithms have been proposed for solving optimization problems with computationally expensive simulation-based functions (e.g., [1, 4, 6, 7]). Commonly used surrogates include Kriging and Radial Basis Function (RBF) models, and they are used to approximate the objective and constraint functions globally or locally, or both.

This paper proposes a hybrid surrogate-assisted approach for solving a constrained optimization problem of the form:

$$\min \ f(x)$$
$$x \in \mathbb{R}^d, \ \ell \le x \le u, \ g_i(x) \le 0, \ i = 1, \ldots, m,$$

where $[\ell, u] \subset \mathbb{R}^d$ is a hypercube and $f, g_1, \ldots, g_m$ are black-box functions whose values come from computationally expensive but deterministic simulations. Here, one simulation yields the values of $f(x), g_1(x), \ldots, g_m(x)$ at a given $x \in \mathbb{R}^d$. The proposed hybrid combines the RBF-assisted Constrained Evolutionary Programming algorithm CEP-RBF [4] with the CONORBIT trust region method [5]. CEP-RBF uses global RBF models while CONORBIT uses local RBF models of the objective and constraint functions. Numerical results show that this hybrid generally outperforms the component algorithms on CEC 2010 benchmark problems.

2 PROPOSED HYBRID

A $(\mu + \mu)$-EP maintains a population of μ solutions (parents) and generates offspring without recombination and using only Gaussian mutations. The $(\mu + \mu)$-CEP-RBF [4] builds RBF surrogates for the objective and constraint functions using all previous sample points, including infeasible ones. In every generation, each of the μ parents generates a large number ν of trial offspring, and the RBF surrogates are used to identify the most promising among the trial offspring in terms of having the best RBF objective value among those with the minimum number of predicted constraint violations. The simulations are then carried out only on the promising trial offspring and the algorithm proceeds as in an EP.

CONORBIT [5] is a trust region method that also uses RBF models for the objective and constraint functions. In each iteration, the next sample point is typically obtained by minimizing an RBF model of the objective subject to RBF constraints with some small margin and within the current trust region. The margin for the RBF constraints is meant to facilitate the generation of feasible iterates.

The CEP-RBF-CONORBIT hybrid begins by running CEP-RBF from space-filling design points on $[\ell, u]$, and then after a fraction of the computational budget (simulations) has been exhausted, the sample points obtained are then passed on to CONORBIT for the remaining simulations. CEP-RBF performs global search while CONORBIT performs local search focused on promising regions obtained by CEP-RBF. One advantage of this pairing of algorithms is that CONORBIT can reuse the sample points obtained by CEP-RBF, thereby eliminating wasteful simulations.

A preliminary implementation of the hybrid allocates a fraction of the computational budget for CEP-RBF and the remainder for CONORBIT. A more adaptive transition from CEP-RBF to CONORBIT can be designed, including one that clusters a fraction of the promising sample points from CEP-RBF and then uses these clusters as initial points for multiple CONORBIT runs. One can also modify CEP-RBF so that several iterations of CONORBIT are performed from some of the parents, possibly resulting in local improvement, before proceeding with the CEP-RBF iterations.

The current version of CEP-RBF-CONORBIT requires a feasible point among the initial points as in [4]. However, the method can be extended to handle initial points that are all infeasible.

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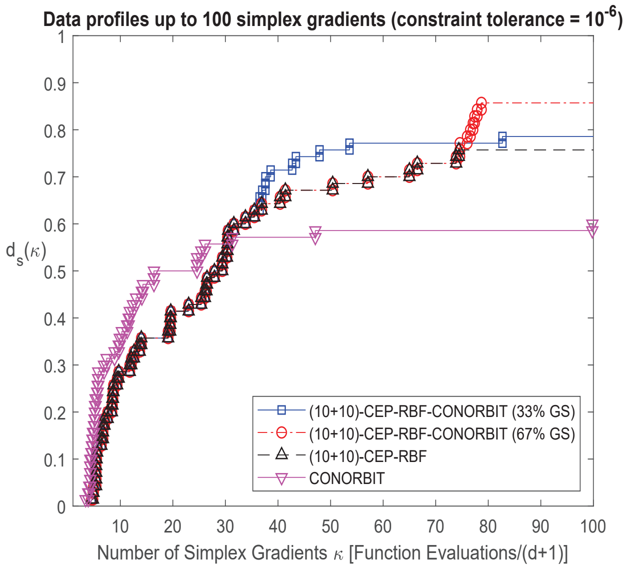

Figure 1: Data profiles for two variants of CEP-RBF-CONORBIT and for the component algorithms.

3 RESULTS AND DISCUSSION

Two variants of CEP-RBF-CONORBIT with different fractions of simulations allocated for global search are compared with CEP-RBF and CONORBIT on 10-D instances of seven test problems from the CEC 2010 benchmark [2], including modifications where the = constraint is replaced by $\leq$. CEP-RBF and CONORBIT outperformed several alternatives on many test problems [4, 5], so no additional methods are included in the comparisons.

CEP-RBF and the two variants of CEP-RBF-CONORBIT used a population of size $\mu = 10$ with $\nu = \min(10^3 d, 10^4)$ trial offspring for each parent and an initial standard deviation of $\sigma_{\text{init}} = 0.2L([\ell, u])$ for the Gaussian mutations. For the first variant of CEP-RBF-CONORBIT, 33% of the simulations are allocated for CEP-RBF (global search) with the rest for CONORBIT, while for the second variant 67% of the simulations are allocated for CEP-RBF. All algorithms used a cubic RBF model with a linear tail as in [4, 5]. In addition, CONORBIT uses the fmincon solver from Matlab to solve the trust region subproblems. Fmincon is applied to the RBF surrogates of the objective and constraints, so it does not evaluate the black-box functions. These algorithms are labeled as (10 + 10)-CEP-RBF and (10 + 10)-CEP-RBF-CONORBIT (33% GS or 67% GS).

The numerical experiments are performed using Matlab 9.4. Each algorithm is run for 10 trials on each test problem and used the same space-filling design for each trial. This design is an approximate maximin Latin hypercube design (LHD) with $2(d + 1)$ points that contains a subset of $d + 1$ affinely independent points. The initial parents for CEP-RBF and CEP-RBF-CONORBIT are the best μ points from among the LHD points and the given feasible point.

The two variants of CEP-RBF-CONORBIT hybrid are compared with the component algorithms using data profiles [3]. Figure 1 shows the data profiles of the algorithms up to 100 simplex gradients, where each simplex gradient is equivalent to $d + 1$ simulations. Hence, all algorithms are run to a maximum of $100(d + 1)$ simulations, where each simulation yields the values of the objective and all constraint functions at a given input.

Figure 1 shows that the two variants of the hybrid generally outperform the component algorithms after more than 30 simplex gradient estimates ($30(d + 1)$ simulations). In particular, (10 + 10)-CEP-RBF-CONORBIT (33% GS) solves about 75% of the problems within 50 simplex gradient estimates, while (10 + 10)-CEP-RBF and CONORBIT solves only about 70% and 60% of the problems, respectively, within the same computational budget. Here, a *problem* corresponds to a particular combination of test problem and initial points for a given trial. Hence, there are 70 problems involved (7 test problems × 10 trials). Moreover, (10+10)-CEP-RBF-CONORBIT (67% GS) solves about 85% of the problems within 100 simplex gradient estimates, while (10 + 10)-CEP-RBF and CONORBIT solves only about 75% and 60% of the problems, respectively, within the same budget. Between 50 and 100 simplex gradient estimates, one can see some advantage of using the hybrid approach over the individual component algorithms. The data profile for the CONORBIT algorithm rises faster compared to that of the others within 30 simplex gradient estimates, but quickly flattens out because the method is really meant for local optimization. CEP-RBF has better balance between global and local search, but can be improved by switching to the CONORBIT algorithm after a certain number of simulations.

4 CONCLUSIONS AND FUTURE WORK

This paper developed the CEP-RBF-CONORBIT hybrid for constrained expensive black-box optimization that combines an RBF-assisted EP and an RBF-based trust region method. Numerical experiments on test problems from the CEC 2010 benchmark show that the hybrid generally outperforms the component algorithms CEP-RBF and CONORBIT on these test problems. Hence, CONORBIT has the potential to improve the performance of a surrogate-assisted metaheuristic for constrained expensive black-box optimization. The preliminary hybrid method presented uses a simple procedure for switching from CEP-RBF to CONORBIT where a fraction of the computational budget is allocated for each component. Future work will explore adaptive procedures for switching between these two methods as well as applications to a simulation-based problem.

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