An a priori Knee Identification Multi-objective Evolutionary Algorithm Based on α-Dominance

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ABSTRACT

In the preference-based multi-objective optimization, the lack of priori-knowledge makes it difficult for the decision maker to specify an informed preference. Thus, the knees are regarded as the naturally preferred solutions on the Pareto optimal front. However, most research is based on a given large number of solutions and *a posteriori* identifies the knee candidates among them.

Based on the α-dominance relationship, this paper proposes a new framework to *a priori* search the knee regions. Firstly, a number of reference vectors are generated in the objective space. During the environmental selection, all solutions are associated to their closest reference vectors. The solutions associated to different reference vectors are deemed to be non-α-dominated with each other. If they are correlated with the same reference vector, the α-dominance relationship is adopted to sort the solutions into different frontiers. Therefore, the knee candidates are assigned to the first layer and selected with a higher priority, so that more knee information from the previous generation will be preserved and more potential knee regions will be explored. The comparative experiments demonstrate that the proposed method is competitive in identifying convex knee regions.

CCS CONCEPTS

• **Theory of computation** → **Evolutionary algorithms**;
• **Computing methodologies** → *Genetic algorithms*;

KEYWORDS

Preference, Knee, α-dominance, evolutionary algorithm

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1 INTRODUCTION

Recently, increasing attention has been paid to the search of knee points (regions) of multiobjective optimization problems (MOPs). Branke et al. [5] identify the knees with the largest reflex angle. Deb and Gupta [6] propose a bend angle to characterize the knees. Das et al. [4] identify the knees with the maximum distance to a hyperplane constructed by the extreme points of the population. Branke et al. [5] characterize the knees with highest expected marginal utility (EMU). Its extension [1] recursively uses the EMU to find most-likely knee candidate in a knee region. The method [7] uses the min-max utility to find the knee regions. Besides, a niching-based method [8] is proposed to identify both the concave and convex knee regions.

However, most of them are *a posteriori* and based on the assumption given a set of well-distributed solutions approximating to the Pareto optimal front (PoF). Thus, this paper proposes a new strategy to apply the α-dominance relation in different objective spaces to sort the population so as to get good knee candidates for the decision maker.

The rest paper is organized as follows. The proposed method is introduced in Section 2. The experiments and analysis are presented in Section 3. Section 4 concludes the paper.

2 PROPOSED METHOD

The proposed algorithm is named by α-MOEA-KI[1], following the framework of NSGA-II [2]. The difference is to predefine a set of reference vectors and the environmental selection. The reference vectors are applied to partition the population according to the perpendicular distance from the solutions to their closet reference vectors. In the environmental selection, the α-dominance [3] based non-dominated sorting is introduced to sort the population.

In the sorting, we only change the comparator of the conventional Pareto dominance based non-dominated sorting. Specifically, the $f_i(\overrightarrow{x}) \prec f_i(\overrightarrow{y})$ is replaced with $g_i(\overrightarrow{x}, \overrightarrow{y}) < 0 \wedge W_x == W_y$, which means only the solutions in the same subregion ($W_x == W_y$) are compared by means of the $\alpha-$dominance relationship. With the same reason, $f_i(\overrightarrow{y}) \prec f_i(\overrightarrow{x})$ is replaced with $g_i(\overrightarrow{x}, \overrightarrow{y}) < 0 \wedge W_x == W_y$. If they are from different subregions, then they are non-dominated. Thus, after the comparison on m objectives, if $Domination = -1$,

[1] https://github.com/LursonkjGuo/Code

$\overrightarrow{x} \prec_\alpha \overrightarrow{y}$; if *Domination* $= 1$, $\overrightarrow{y} \prec_\alpha \overrightarrow{x}$; otherwise, $\overrightarrow{x} \nprec_\alpha \overrightarrow{y}$.

3 EXPERIMENTS AND ANALYSIS

The problems, including DO2DK [5] with $K = 4$, DEB2DK [5] with $K = 5$, CKP [8] with $K = 5$, DEB3DK [5] with $K = 3$ are chosen as the benchmarks, where $s = 1$ and $n = 30$ are set. In the comparative experiments, KneeDis [4] with $\triangle = 0.35$, KneeWD [7] with $\delta = 0.2$, KneeEMU [5] with $\triangle = 0.8$, KneeDEA [8] with $diff = 0.001$ are selected. $\alpha = 0.75$ is set in the proposed method.

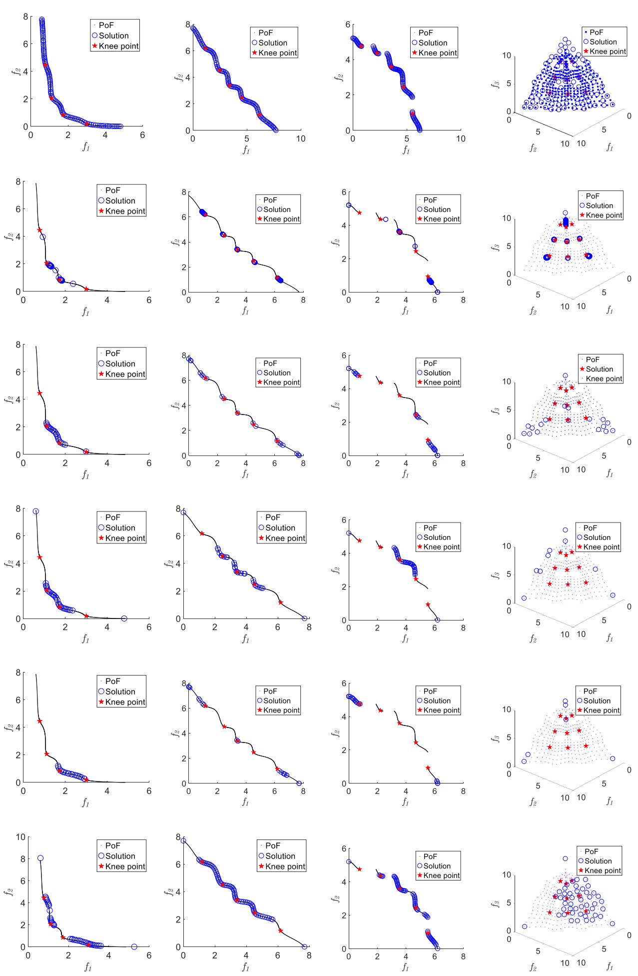

Figure 1: The first row plots the reference points on PoFs. The rest rows are the results obtained by α-MOEA-KI, KneeWD, KneeDis, KneeEMU, and KneeDEA, in sequence.

Fig. 1 visualizes the knee candidates obtained by the five knee identification methods. The results indicate that α-MOEA-KI is competitive in approximating to the true knee points on all problems. Specifically, α-MOEA-KI, KneWD, and KneeDEA have better performance on DO2DK problem with asymmetric PoF. On DEB2DK problem with symmetrical PoF, α-MOEAKI and KneeWD can find all the knee regions with good proximity. In dealing with the discontinuous CKP problem, α-MOEAKI and KneeDEA have the best performance in searching the knee regions. α-MOEA-KI offers the best performance with good candidates to the knee regions of DEB3DK with discontinuous and complex PoF.

4 CONCLUSION

In *a posteriori* identification of knee points, a large well-distributed solution set along the PoF is required, however it is expensive to gain such a set of reference solutions. Based on the α-dominance relationship, this paper proposed an *a priori* algorithm to search the knee regions. It applies the α-dominance to sort the possible knee candidates from different sub-regions into the fronts with a higher priority to be selected to next generation. Thus more information of the possible knee regions will be preserved and more possible knee regions are explored. The comparative experiments demonstrate that the proposed method is competitive in identifying knee regions. One future research is to investigate the performance of the proposed method in dealing with high-dimensional problems by means of self-adjusting trade-off rates and reference vectors.

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REFERENCES

[1] Kalyan Bhattacharjee, Hemant Singh, Michael Ryan, and Tapabrata Ray. 2017. Bridging the Gap: Many-objective Optimization and Informed Decision-making. *IEEE Transactions on Evolutionary Computation* 21, 5 (2017), 813–820. https://doi.org/10.1109/TEVC.2017.2687320

[2] Kalyanmoy Deb, Amrit Pratap, Sameer Agarwal, and TAMT Meyarivan. 2002. A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation* 6, 2 (2002), 182–197. https://doi.org/10.1109/4235.996017

[3] Kokolo Ikeda, Hajime Kita, and Shigenobu Kobayashi. 2001. Failure of Pareto-based MOEAs: Does non-dominated really mean near to optimal?. In *Proceedings of the 2001 Congress on Evolutionary Computation*, Vol. 2. IEEE, 957–962.

[4] Das Indraneel. 1999. On characterizing the "knee" of the Pareto curve based on normal-boundary intersection. *Structural and Multidisciplinary Optimization* 18, 2 (1999), 107–115. https://doi.org/10.1007/BF01195985

[5] Branke Jürgen, Deb Kalyanmoy, Dierolf Henning, and Osswald Matthias. 2004. Finding knees in multi-objective optimization. In *Proceedings of International Conference on Parallel Problem Solving from Nature*, Vol. 3242. Springer, Berlin, Heidelberg, 722–731. https://doi.org/10.1007/978-3-540-30217-9_73

[6] Deb Kalyanmoy and Gupta Shivam. 2011. Understanding knee points in bicriteria problems and their implications as preferred solution principles. *Engineering Optimization* 43, 11 (2011), 1175–1204. https://doi.org/10.1080/0305215X.2010.548863

[7] Rachmawati Lily and Srinivasan Dipti. 2009. Multiobjective evolutionary algorithm with controllable focus on the knees of the Pareto front. *IEEE Transactions on Evolutionary Computation* 13, 4 (2009), 810–824. https://doi.org/10.1109/TEVC.2009.2017515

[8] Guo Yu, Yaochu Jin, and Markus Olhofer. 2018. A Method for *a Posteriori* Identification of Knee Points Based on Solution Density. In *Proceedings of 2018 IEEE Congress on Evolutionary Computation (CEC)*. IEEE, Rio de Janeiro, Brazil, 1–8. https://doi.org/10.1109/CEC.2018.8477885