Using Diversity as a Priority Function for Resource Allocation on MOEA/D

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ABSTRACT

The key characteristic of the Multi-Objective Evolutionary Algorithm Based on Decomposition (MOEA/D) is that a multi-objective problem is decomposed into multiple single-objective subproblems. In standard MOEA/D, all subproblems receive the same computational effort. However, as each subproblem relates to different areas of the objective space, it is expected that some subproblems are more difficult than others. Resource Allocation techniques allocates computational effort proportional to each subproblem's difficulty. This difficulty is estimated by a priority function. Using Resource Allocation, MOEA/D could spend less effort on easier subproblems and more on harder ones, improving efficiency. We propose that using diversity as the priority criteria results in better allocation of computational effort. Therefore we propose a new priority function: decision space diversity. We compare the proposed diversity based priority with previous approaches on the UF benchmarks. The proposed decision space priority achieved high IGD values, excellent rate of non-dominated solutions on the benchmark problem.

CCS CONCEPTS

• **Applied computing → Multi-criterion optimization and decision-making**; • **Theory of computation** → *Evolutionary algorithms*; *Continuous optimization*;

KEYWORDS

Resource Allocation, Diversity assessment, Multiobjective Optimization, Priority Functions.

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1 MOTIVATION

We propose a new priority function for estimating difficulty and calculating priority in Resource Allocation (RA) for MOEA/D. Our approach uses the idea of *diversity* in decision space to calculate the priority of solutions. Our motivation for this choice is that the quality of MOEA/Ds is often evaluated by diversity, such as Inverted Generational Distance and Hypervolume (combined with convergence). By assigning higher priority for regions with lower diversity, we encourage the algorithm to spend more computational effort in regions that are not yet well explored. Our new priority function measures the difference between the current solution and its parent using the 2-Norm. The new priority function guides the search behavior of the algorithm, by monitoring diversity.

Few studies have been concerned with Resource Allocation. We highlight: MOEA/D-GRA [7], MOEA/D-DRA [6] and in the Two-Level Stable Matching-Based Selection in MOEA/D [5], EAG-MOEA/D [1] and MOEA/D-CRA [4]. These studies indicate that it is worth monitoring the algorithm behavior and guiding its search. In all RA works mentioned above the choice of priority function was just one of multiple changes applied to the base framework. That is, in Zhang et al. used a 10-tournament selection in MOEA/D-DRA [6], while Zhou and Zhang used a new replacement strategy in MOEA/D-GRA [7]. Chiang in MOEA/D-AMS proposes an adaptive mating selection mechanism to dynamically adjusts the mating pools of individuals [3]. Cai and Lai in EAG-MOEA/D [1] and Kang et al. in MOEA/D-CRA [4] used an archive population.

We compare the new approach with the Relative Improvement and with the standard MOEA/D (with no priority function). The results show that the priority function focused on decision space lead to better results on the Inverted Generational Distance (IGD) metric and also lead to a higher percentage of non-dominated solutions.

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2 MOEA/D WITH PRIORITY FUNCTIONS

We use the basic framework in algorithm with priority functions of MOEA/D-GRA. In contrast to MOEA/D-GRA we only consider the basic algorithm and no other variant. We initialize the value of the vector $u = 1$, as in MOEA/D-DRA. As in DRA and GRA we have a learning period of ΔT iterations. $\Delta T = 20$ as in MOEA/D-GRA [7].

2.1 Priority Functions

Algorithm 1 2-Norm

```
Input: X^t decision vectors of solutions; X^{t-1}, decision vectors
  from the previous solutions; N, the population size.
for i=1 to N do
    u[i] = ||X_i^t - X_i^{t-1}||
u = scale (u) // between 0 and 1
return u
```

2.1.1 Norm of the difference of current solutions and its parents. To consider diversity on the decision space, we propose a priority function based on the 2-Norm of the difference between the current solution and its parent. The idea of using the Norm as priority function is that by considering diversity more resources are given to incumbent solutions that are similar to their parents, forcing them to update more often and leading to a higher exploration of the decision space. Algorithm 1 gives the details on implementation.

Algorithm 2 Relative Improvement

Input: Y^t, objective function values from the incumbent solutions; $Y^{t-\Delta T}$, objective function values from incumbent solution of iteration $t - \Delta T$, u from the previous ΔT iteration;
for i=1 to N **do**
  $\delta[i] = \frac{Y^t[i] - Y^{t-1}[i]}{Y^t[i]}$
  if $\delta[i] > 0.001$ **then**
    u[i] = $(0.95 + 0.05 \cdot \frac{\delta[i]}{0.001}) \cdot u[i]$
  else
    u[i] = 1
u / (max(u) + $1.0x10^{-50}$)
return u

2.1.2 Priority Function - Relative Improvement. R.I. was introduced in the context of the unconstrained MOEA competition in the CEC 2009 [6]. Algorithm 2 gives the details on implementation.

2.1.3 Priority Function - Random. Priority vector u sampled from a uniform distribution.

3 EXPERIMENTAL ANALYSIS

We perform a comparative experiment on benchmark functions under the UF function set (with 100 dimensions). We use the MOEA/D-DE implemented by the MOEADr package [2], modified to include RA. For reproducibility purposes, all the code and data used in these experiments are available at. We compare no Resource Allocation, Norm and Relative Improvement (R.I.) based on their Inverted Generational Distance (IGD) metric (Lower values of the IGD indicate better approximations). We evaluate the proportion of non-dominated solutions and the number of feasible solutions. For every strategy/function pair we perform 21 repetitions with 70000 function evaluations and population size $N = 350$.

IGD	None	Norm	R.I.	Random
UF1	0.140 (0.013)	0.109 (0.016)	**0.090 (0.012)**	0.093 (0.014)
UF2	0.082 (0.006)	0.060 (0.005)	0.060 (0.005)	**0.060 (0.004)**
UF3	0.260 (0.012)	**0.168 (0.025)**	0.183 (0.335)	0.214 (0.030)
UF4	0.100 (0.023)	**0.095 (0.002)**	0.095 (0.003)	**0.095 (0.002)**
UF5	1.759 (0.080)	**0.972 (0.056)**	1.056 (0.064)	1.085 (0.073)
UF6	0.121 (0.027)	0.100 (0.016)	**0.078 (0.014)**	0.079 (0.016)
UF7	0.125 (0.018)	**0.061 (0.006)**	0.068 (0.005)	0.074 (0.005)
UF8	0.286 (0.012)	**0.229 (0.014)**	0.257 (0.020)	0.232 (0.006)
UF9	0.451 (0.012)	**0.385 (0.020)**	0.420 (0.017)	0.400 (0.018)
UF10	3.693 (0.200)	2.380 (0.241)	**2.364 (0.272)**	2.639 (0.253)

Non-dominated	None	Norm	R.I.	Random
UF	0.34 (0.04)	**0.84 (0.06)**	0.58 (0.10)	0.69 (0.05)

Table 1: IGD medians and Proportion of non-dominated solutions. Best values are in bold Standard deviation (in parenthesis) was used as tie breaker.

[1]https://github.com/yclavinas/MOEADr/tree/gecco-poster

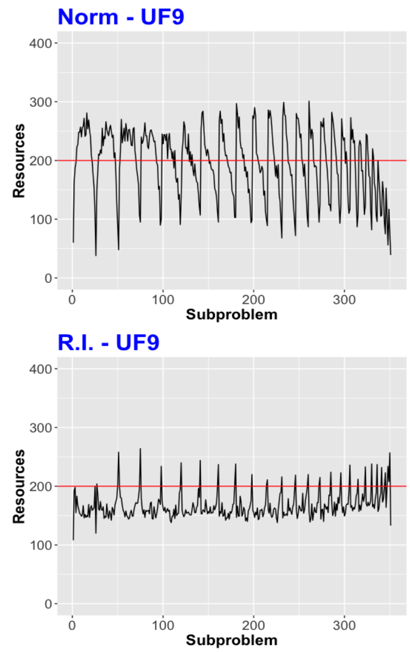

Figure 1: Resource Allocation by subproblem.

Table 1 (IGD values) shows that Norm had the best median in UF3, UF4, UF5, UF7, UF8 and UF9 functions. The R.I. was the best in 3 functions and Random in 2. Table 1 (Non-dominated) indicates that Norm leads to a very high rate of non-dominated solutions in the final solution set. Figure 1 illustrates the amount resource allocated by Norm, R.I to every subproblem on UF9 problem.

4 DISCUSSION

We showed that 2-Norm as priority function effectively improves the performance of MOEA/D, since it achieved IGD values and excellent rates of non-dominated solutions on the benchmark problems. Results indicate that 2-Norm leads to more diversity of the final solution set, being an effective priority function.

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